PROBLEM SET #1B

Issued: Tuesday, Jan. 19, 2016

Due: Wednesday, Feb. 10, 2016, at 8:00 a.m. in the EE C247B homework box near 125 Cory.

This homework assignment continues from Problem Set #1A to provide early practice playing with dimensions and exploring how scaling can greatly improve or degrade certain performance characteristics of mechanical systems. Again, don't worry at this point if you do not understand fully some of the physical expressions used. They will be revisited later in the semester. This assignment just gives you a chance to play with them a bit.

Use the material parameters given in Table PS1B.1 wherever needed.

TYPE	SILICON NITRIDE	INTERCONNECT POLYSILICON	STRUCTURAL POLYSILICON	UNIT
DENSITY	3200	2300	2300	$\mathrm{kg/m^3}$
YOUNG'S MODULUS	200	150	150	GPa
POISSON RATIO	0.280	0.226	0.226	–
ELECTRICAL RESISTIVITY	10^{21}	10^{-5}	10^{-5}	$\Omega.\mathrm{m}$
THERMAL CONDUCTIVITY	43	30	30	$\mathrm{W/m.K}$
SPECIFIC HEAT	1.10	0.77	0.77	$\mathrm{J/g.K}$
MICRO-OVEN THICKNESS	1.0	0.4	2.0	μm

Table PS1B.1

(h) One important metric for time-keeping devices is frequency stability with respect to temperature, T. For mechanical resonators this instability is mainly based on temperature coefficient of Young's modulus, α_E defined as

$$\alpha_E = \frac{1}{E}\frac{\partial \mathrm{E}}{\partial T} \tag{1}$$

where E is Young's modulus, and thermal expansion coefficient, α_p defined as

$$\alpha_p = \frac{1}{L}\frac{\partial \mathrm{L}}{\partial T} \tag{2}$$

where L is length. Using the first mode resonance frequency expression found in Problem Set #1A part (d) for a polysilicon free-free beam resonator give a formula and numerical answer with units for the fractional frequency change in parts per million (ppm). Fractional frequency change in ppm is defined as

$$\left(\frac{\Delta f}{f_0}\right)_{ppm} = 10^6\left(\frac{1}{f_0}\frac{\partial f_0}{\partial T}\right) \tag{3}$$

where f_0 is resonance frequency. Use the following parameters for this part: $\alpha_E = -30x10^{-6}$K^{-1} and $\alpha_p = 2x10^{-6}$K^{-1}.

(i) One method to resolve the frequency instability issue is to utilize a micro-oven, in which a resonator is fabricated on a thermally isolating micro-platform equipped with heater and sense resistors, and its frequency is stabilized via the feedback control electronics. In other words, the temperature coefficient of the center frequency is reduced.

To achieve a decent frequency stability with minimal power dissipation, heat loss must be minimized via thermally isolating micro-resonator from the surrounding substrate. A perspective view schematic of the micro-oven control system is presented in Fig. PS1B.1. Here, a polysilicon free-free beam resonator is suspended above a nitride platform, which in turn, is suspended above the substrate via long, thin struts. The suspending struts provide thermal isolation of the platform from the substrate, as well as conductive interconnect between these media. Devices on the platform include the resonator and two polysilicon resistors to heat the platform and sense its temperature. The feedback electronics is also shown in the figure for the sake of completeness.

The main concern in this design is the degree of thermal isolation achieved, since this dictates the power required to sustain a given temperature and time required to change it at start-up. Given the resonator operates in vacuum, draw the thermal equivalent circuit. Geometric dimensions and material properties are given in Fig. PS1B.2 and Table PS1B.1, respectively.

(j) What is the total steady-state power dissipation required to maintain a constant temperature of 110°C on the platform? Assume a polysilicon thermal emissivity of 0.7.

(k) The micro-oven is hooked up as shown in Fig. PS1B.2. With what time constant will the platform reach its steady-state temperature after the voltage V_A steps from 0V to 1V? Give a formula and a numerical answer with units.

(l) If the final step function value of V_A is 1V, what is the steady-state temperature of the platform? Give a formula and a numerical answer with units.

(m) Scale all resonator, platform, and supporting struts dimensions by a factor of 10. What is the total steady-state power dissipation required to maintain a constant temperature of 110°C on the platform now? What is the new time constant? Compare with part (l) and comment.

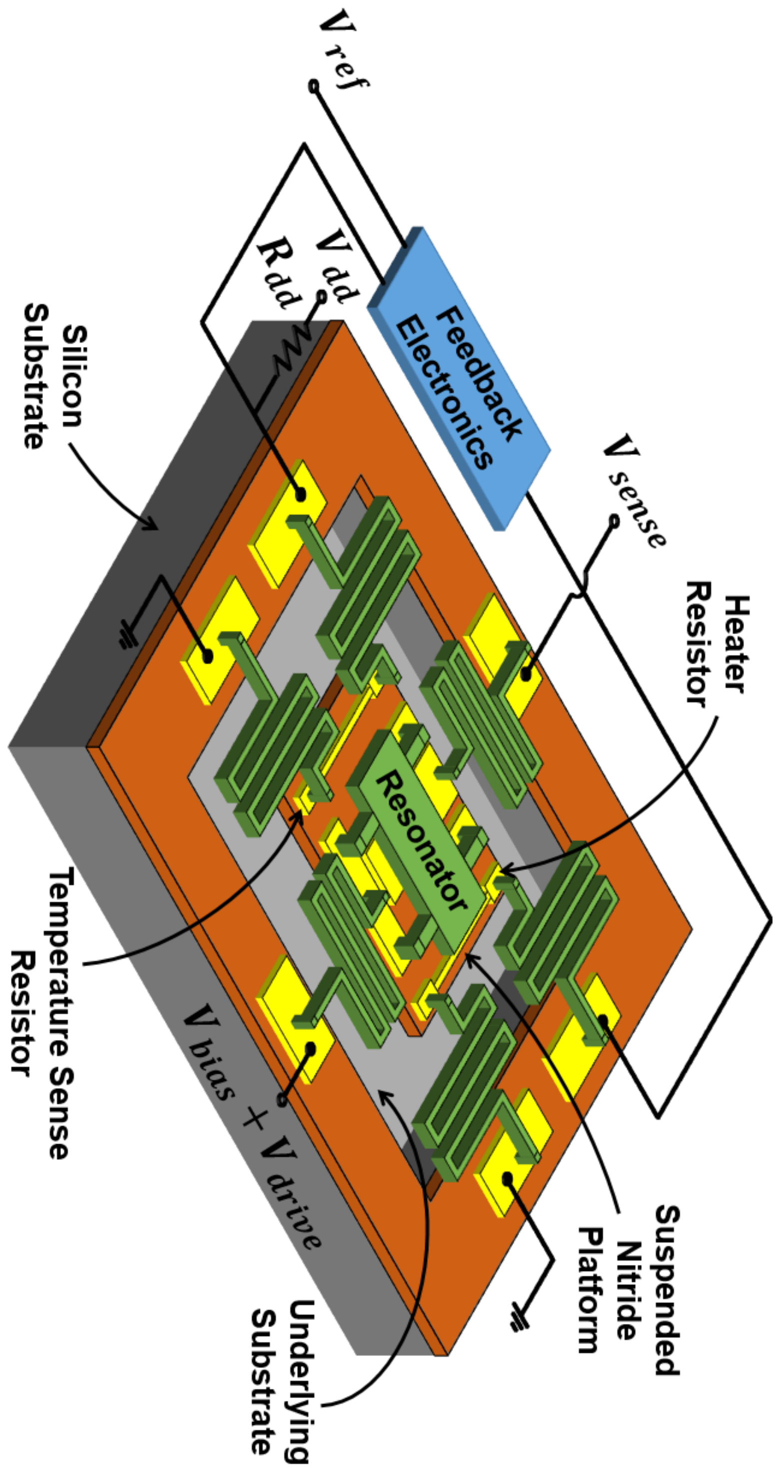

Fig. PS1B.1

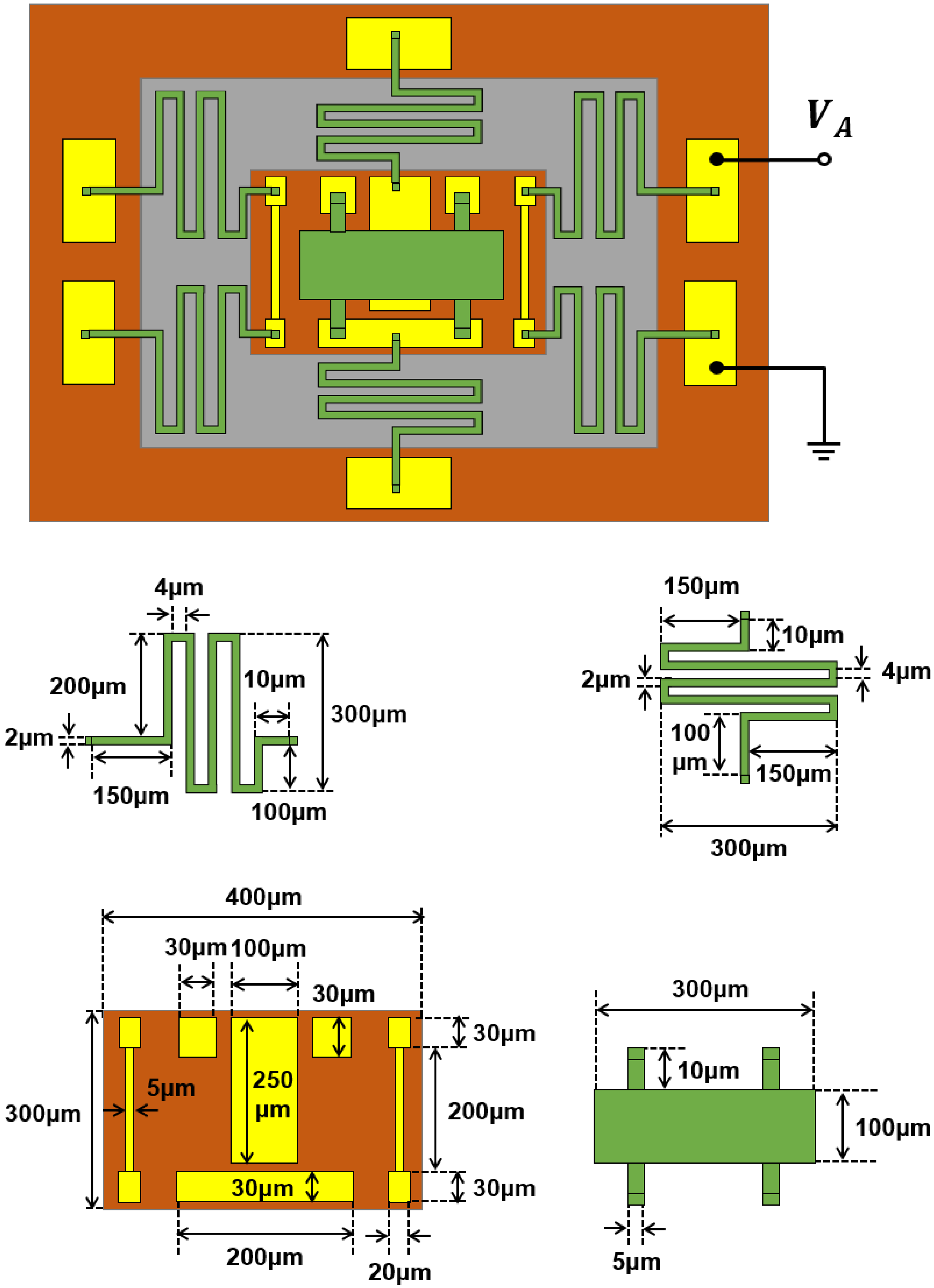
V_A
4μm
200μm
10μm
300μm
2μm
150μm
100μm
150μm
10μm
2μm
4μm
100 μm
150μm
300μm
400μm
30μm
100μm
30μm
30μm
5μm
250 μm
300μm
200μm
30μm
30μm
200μm
20μm
300μm
10μm
100μm
5μm

Fig. PS1B.2