Lecture 11: Mechanics of Materials II

- Announcements:
- HW#3 online and due next Wednesday morning
- Prof. Nguyen's Wednesday Office Hours have changed to 12:30-1:30 p.m.
- ------------------------------------
- Reading: Senturia, Chpt. 8

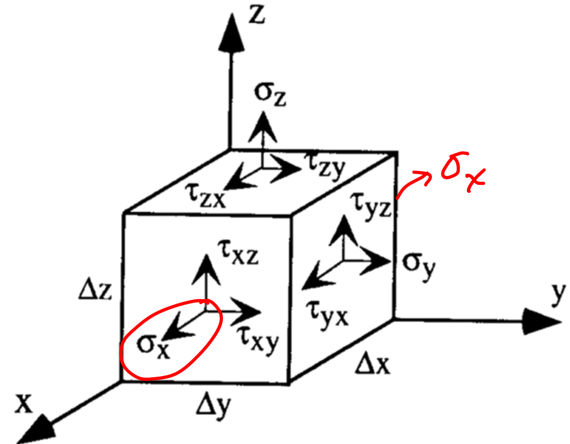
6 now
10 evening 5:P-7

- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
- ---------------------------------
- Last Time:

over

Example. Exercise the "terms"

⇒ Determine the volume change ΔV for a uniaxial stress (along the x-direction)



Upon application of σ_x, what is ΔV?

$\Delta x \longrightarrow \Delta x(1+\epsilon_x)$ (as a result of application of σ_x)

$\Delta y \longrightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \longrightarrow \Delta z(1-\nu\epsilon_x)$

} assuming isotropic material → same ν along y & z

The resulting change in volume: ΔV

$$\Delta V = \underbrace{\Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2}_{\text{volume after application of } \sigma_x} - \Delta x \Delta y \Delta z$$

$= \Delta x \Delta y \Delta z\left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$

[Assume small strains] $\Rightarrow (1+mx)^n \approx 1+nmx$

$\Delta V = \Delta x \Delta y \Delta z\left[(1+\epsilon_x)(1-2\nu\epsilon_x)-1\right]$

$$\boxed{\Delta V = \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

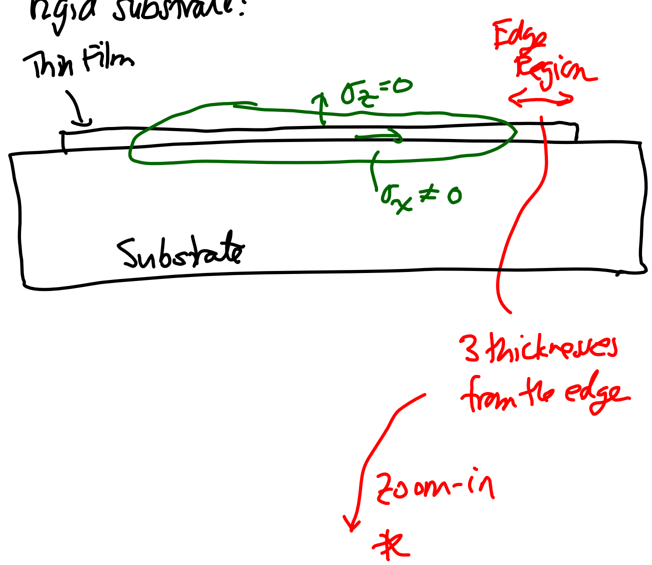
For $\nu = 0.5$ (rubber) $\rightarrow$ no ΔV!

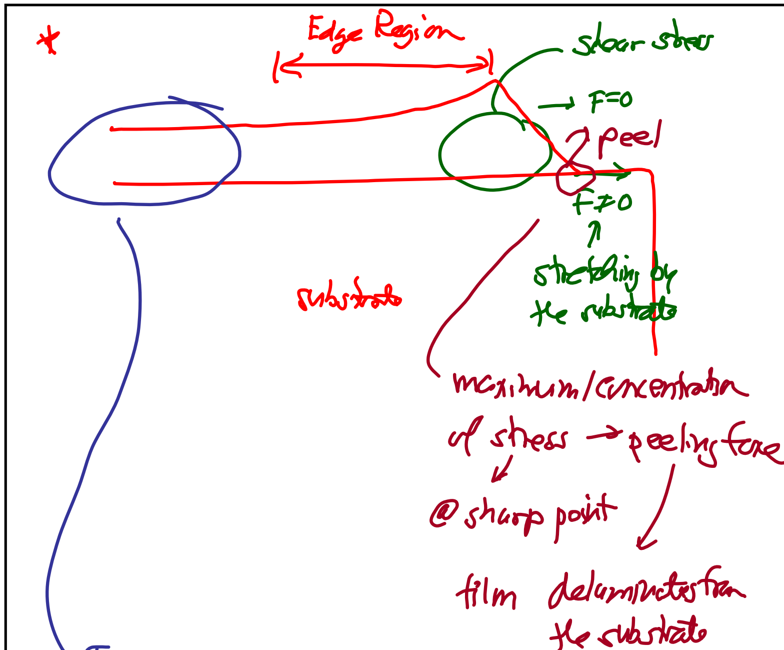
$\nu < 0.5 \rightarrow$ finite ΔV

Module 7, pg. 13

Important Case: Plane Stress

$\Rightarrow$ common case for a thin-film coating on a rigid substrate:





Take a closer look @ this region: $\sigma_z = 0$

Get two components of stress: (strain)

$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + 0)\right]$ (assuming isotropic material)

$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + 0)\right]$

Assume: Plane stress $\rightarrow$ isotropic $\rightarrow \sigma_x = \sigma_y = \sigma$
(symmetry in the xy-plane) $\downarrow$ $\epsilon_x = \epsilon_y = \epsilon$

$\epsilon_x = \frac{1}{E}\left[\sigma - \nu\sigma\right]$

$= \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} \Rightarrow \epsilon_x = \frac{\sigma}{E'} \leftarrow$ Biaxial Modulus where $\boxed{E' \triangleq \text{Biaxial Modulus} = \frac{E}{1-\nu}}$

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition. Linear Thermal Expansion Coefficient

$$\boxed{\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Expansion Coeff.}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{Kelvin}^{-1}]}$$

Remarks.

① α_T values tend to be in the 10^{-6} to 10^{-7} range.

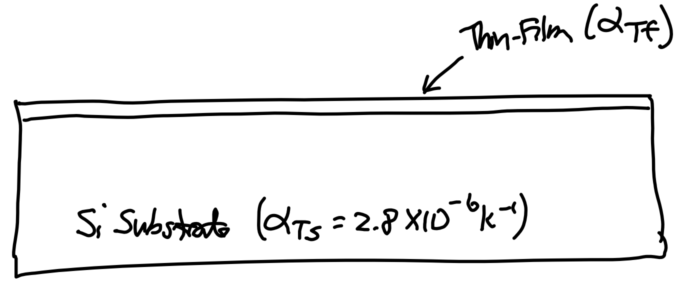
② $10^{-6}\,K^{-1} = 1\,\mu\text{strain}/K$

③ In 3D, get a volume thermal expansion coeff.

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

④ For moderate ΔT's → $\alpha_T \approx$ constant
↓
for large ΔT, then $\alpha_T = f(T)$

Example. Thin-Film Thermal Stress



Assume.

① Substrate is much thicker than the film.

② Film is deposited stress free @ T_d ← deposition temperature

③ Then, the whole thing is cooled to room temperature: T_r

Thermal Strain of the Substrate: (in one plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \quad \text{where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But if the film is attached to the substrate:
thickness substrate >> thickness of the film
∴ substrate wins!

→ The substrate dictates the actual strain experienced by the film: $\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$

Thus.

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

Note: This is biaxial strain (assuming the film is deposited isotropically onto the substrate)

$$\boxed{\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{E'} \epsilon_{f,mismatch}}$$

Ex. Thin film is polyimid → $\alpha_{Tf} = 70 \times 10^{-6} K^{-1}$

$E' = 4 GPa$

deposited @ $\underbrace{250°C\text{, then cool to room temp} = 25°C}_{\Delta T = 225K}$

$$\epsilon_{f,mismatch} = (70 - 2.8)\mu(225) = 1.5 \times 10^{-2}$$

$[\mu = 10^{-6}, m = 10^{-3}, k = 10^{3}, G = 10^{9}]$

$$\sigma_{f,mismatch} = (\underset{10^9}{4G})(1.5 \times 10^{-2}) = 60.5 MPa$$

stress is (+) → tensile

[(-) would be compressive]