**Quality Factor (or Q)**

- **Measure of the frequency selectivity of a tuned circuit**
- **Definition:**
  \[ Q = \frac{f_r}{BW_{3dB}} \]
- **Example:** series LCR circuit
  \[ Q = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{\omega L}{R} = \frac{1}{\omega C R} \]
- **Example:** parallel LCR circuit
  \[ Q = \frac{\text{Im}(Y)}{\text{Re}(Y)} = \frac{\omega C}{G} = \frac{1}{\omega L G} \]

---

**Selective Low-Loss Filters: Need Q**

- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - heavy insertion loss for resonator Q < 10,000

**Examples:**
- Resonator Tank
- Coupler
- Resonator Tank
- General BPF Implementation

---

**Clamped-Clamped Beam μResonator**

- **Resonator Beam**
- **Electrode**
- **Energy Lost**: \[ Q = V_p \frac{dC}{dt} \]
- **Density**
  - Smaller mass ⇔ higher freq. range and lower series \( R_s \)

---

**Example Definition**

- **Selectivity of a tuned circuit**
- Measure of the frequency...
**Oscillator: Need for High Q**

- **Main Function:** provide a stable output frequency
- **Difficulty:** superposed noise degrades frequency stability

\[ v_o(t) = V_o \sin(2\pi f t) \]

\[ v(t) = (V_o + \eta \sin(2\pi f t + \theta(t))) \]

**Energy Dissipation and Resonator Q**

\[ Q = \frac{1}{Q_{\text{defects}}} + \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{viscous}}} + \frac{1}{Q_{\text{support}}} \]

**Thermoelastic Damping (TED)**

- Occurs when heat moves from compressed parts to tensioned parts \( \rightarrow \) heat flux = energy loss

\[ \zeta = \frac{\Gamma(T)\Omega(f)}{2Q} = \frac{1}{2Q} \]

\[ \Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p} \]

\[ \Omega(f_o) = 2\left[ \frac{f_{TED}}{f_{TED}^2 + f^2} \right] \]

\[ f_{TED} = \frac{\pi K}{2\rho C_p h^2} \]

**Attaining High Q**

- **Problem:** IC’s cannot achieve Q’s in the thousands
  - transistors \( \Rightarrow \) consume too much power to get Q
  - on-chip spiral inductors \( \Rightarrow \) Q’s no higher than \( \sim 10 \)
  - off-chip inductors \( \Rightarrow \) Q’s in the range of 100’s
- **Observation:** vibrating mechanical resonances \( \Rightarrow \) Q > 1,000
- **Example:** quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q’s \( \sim 10,000 \) or higher (Q ~ \( 10^{8} \) possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode

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TED Characteristic Frequency

\[ f_{TED} = \frac{\pi K}{2 \rho \rho h^2} \]

- \( \rho \) = material density
- \( C_p \) = heat capacity at const. pressure
- \( K' \) = thermal conductivity
- \( h \) = beam thickness
- \( f_{TED} \) = characteristic TED frequency

* Governed by
  - Resonator dimensions
  - Material properties

**TABLE 1. MATERIAL PROPERTIES**

<table>
<thead>
<tr>
<th>Property</th>
<th>Silicon</th>
<th>Quartz</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal expansion</td>
<td>2.60</td>
<td>13.70</td>
<td>ppm/K</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>1.75</td>
<td>0.78</td>
<td>1312 GPa/1%</td>
</tr>
<tr>
<td>Material density</td>
<td>5.33</td>
<td>2.60</td>
<td>g/cm³</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>0.70</td>
<td>0.75</td>
<td>J/gK</td>
</tr>
<tr>
<td>Thermal conduct-</td>
<td>1.50</td>
<td>0.10</td>
<td>187 J/gK/s</td>
</tr>
<tr>
<td>wivity</td>
<td>1.00</td>
<td>11.34</td>
<td>10⁻⁴</td>
</tr>
</tbody>
</table>

[from Roszhat, Hilton Head 1990]

Q vs. Temperature

Quartz Crystal
- \( Q \approx 300,000,000 \text{ at } 4K \)
- Mechanism for \( Q \) increase with decreasing temperature thought to be linked to less hysteretic motion of material defects or less energy loss per cycle

Aluminum Vibrating Resonator
- \( Q \approx 500,000 \text{ at } 30K \)
- \( Q \approx 1,250,000 \text{ at } 4K \)
- Even aluminum achieves exceptional \( Q \)'s at cryogenic temperatures

1.51-GHz, \( Q=11,555 \) Nanocrystalline Diamond Disk \( \mu \)Mechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- \( Q \approx 11,555 \text{ (vacuum); } Q \approx 10,100 \text{ (air) } \)
- Below: 20 µm diameter disk

Resonator Data
- \( R = 32 \mu m, h = 3 \mu m \)
- \( d = 80 \text{ nm, } V_p = 3 \text{ V} \)

Design Performance:
- \( R = 10 \mu m, \theta = 2.2 \mu m, d = 800 \AA, V_p = 7 \text{ V} \)
- \( f_s = 1.51 \text{ GHz (2nd mode, } Q = 11,555 \)
Disk Resonator Loss Mechanisms

- Gas Damping
- Electronic Carrier Drift Loss
- Hysteretic Motion of Defect
- Substrate Loss Thru Anchors
- Substrate / Anchors
- No motion along the nodal axis, but motion along the finite width of the stem
- Stem / Height
- \( \frac{1}{4} \) helps reduce loss, but not perfect
- Substrate / Anchors

MEMS Material Property Test Structures

- Substrate / Anchors
- Gas Damping
- Substrate / Anchors
- Hysteretic Motion of Defect
- Electronic Carrier Drift Loss
- Substrate / Anchors

Stress Measurement Via Wafer Curvature

- Compressively stressed film \( \rightarrow \) bends a wafer into a convex shape
- Tensile stressed film \( \rightarrow \) bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:

\[
\sigma = \frac{E' h^2}{6 R t}
\]

- \( \sigma \) = film stress [Pa]
- \( E' = E/(1-\nu) \) = biaxial elastic modulus [Pa]
- \( h \) = substrate thickness [m]
- \( t \) = film thickness
- \( R \) = substrate radius of curvature [m]

MEMS Stress Test Structure

- Simple Approach: use a clamped-clamped beam
- Compressive stress causes buckling
- Arrays with increasing length are used to determine the critical buckling load, where

\[
\sigma_{\text{critical}} = -\frac{\pi^2 E h^2}{3 L^2}
\]

- \( E \) = Young’s modulus [Pa]
- \( I = (1/12) W h^3 \) = moment of inertia
- \( L, W, h \) indicated in the figure
- Limitation: Only compressive stress is measurable
**More Effective Stress Diagnostic**

- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress

**Folded-Beam Comb-Drive Resonator**

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator

**Comb-Drive Resonator in Action**

- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach
### Measurement of Young’s Modulus

- Use micromechanical resonators
  - Resonance frequency depends on $E$
  - For a folded-beam resonator:
    \[
    f_o = \sqrt{\frac{4Eh(W/L)^3}{M_{eq}}} \]
    - Equivalent mass
    - Extract $E$ from measured frequency $f_o$
    - Measure $f_o$ for several resonators with varying dimensions
    - Use multiple data points to remove uncertainty in some parameters

### Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix
  \[
  \begin{bmatrix}
  \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xz} \\ \tau_{xy} \\ \tau_{yz}
  \end{bmatrix} =
  \begin{bmatrix}
  C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
  C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
  C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\
  C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\
  C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\
  C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}
  \end{bmatrix}
  \begin{bmatrix}
  \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xz} \\ \gamma_{xy} \\ \gamma_{yz}
  \end{bmatrix}
  \]
  \[
  \text{Stresses} \quad \text{Stiffness Coefficients} \quad \text{Strains}
  \]

### Anisotropic Materials
Stiffness Coefficients of Silicon

• Due to symmetry, only a few of the 21 coefficients are non-zero.
• With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{xz} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix}
\]

where \( C_{11} = 165.7 \text{ GPa}, \)
\( C_{12} = 63.9 \text{ GPa}, \)
\( C_{44} = 79.6 \text{ GPa}. \)

Young’s Modulus in the (001) Plane

Anisotropic Design Implications

• Young’s modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures.
• E.g., disk or ring resonators, which rely on isotropic properties in the radial directions.
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken.
• E.g., ring vibratory rate gyroscopes.
  - Mode matching is required, where frequencies along different axes of a ring must be the same.
  - Not okay to ignore anisotropic variations, here.