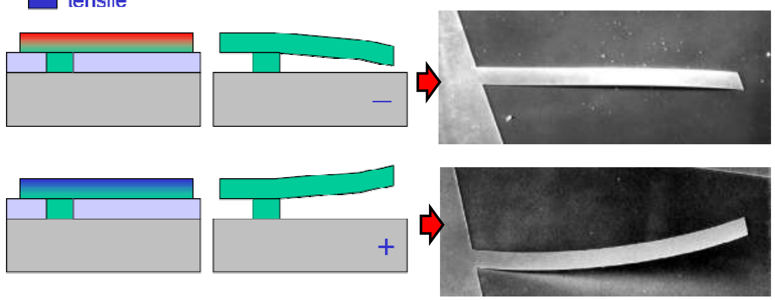
Measurement of Stress Gradient

- Use cantilever beams
  - Strain gradient (Γ = slope of strain-thickness curve) causes beams to deflect up or down
  - Assuming linear strain gradient Γ, $z = \Gamma L^2/2$

compressive
tensile

[P. Krulevitch Ph.D.]

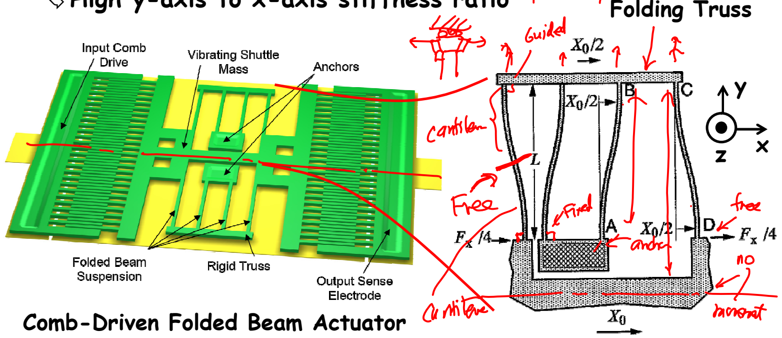


Folded-Flexure Suspensions

Folded-Beam Suspension

- Use of folded-beam suspension brings many benefits
  - Stress relief: folding truss is free to move in y-direction, so beams can expand and contract more readily to relieve stress
  - High y-axis to x-axis stiffness ratio

Folding Truss



Comb-Driven Folded Beam Actuator

Beam End Conditions

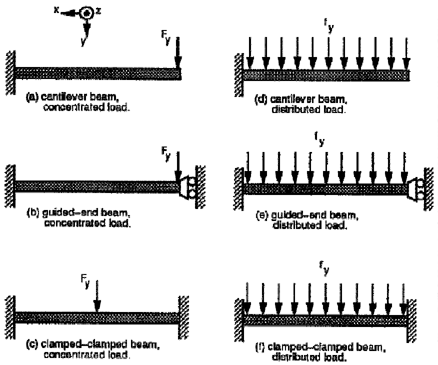
TABLE 4.1
Types of commonly used support conditions for beams and frames

Type of support	Displacement boundary conditions	Force boundary conditions
FREE	None	All, as specified
PINNED	$u = 0$, $w = 0$	Moment is specified
ROLLER (vertical)	$u = 0$	Transverse force and moment are specified
ROLLER (horizontal)	$w = 0$	Horizontal force and bending moment are specified
FIXED or CLAMPED	$u = 0$, $w = 0$, $dw/dx = 0$	None specified

[From Reddy, Finite Element Method]

Common Loading & Boundary Conditions

- Displacement equations derived for various beams with concentrated load F or distributed load f
- Gary Fedder Ph.D. Thesis, EECS, UC Berkeley, 1994



cantilever	guided-end	fixed-fixed
$x = \frac{F_x L}{Ehw}$	$x = \frac{F_x L}{Ehw}$	$x = \frac{F_x L}{4Ehw}$
$y = 4\frac{F_y}{Eh}\frac{L^3}{w^3}$	$y = \frac{F_y}{Eh}\frac{L^3}{w^3}$	$y = \frac{1}{16}\frac{F_y}{Eh}\frac{L^3}{w^3}$
$z = 4\frac{F_z}{Ew}\frac{L^3}{h^3}$	$z = \frac{F_z}{Ew}\frac{L^3}{h^3}$	$z = \frac{1}{16}\frac{F_z}{Ew}\frac{L^3}{h^3}$

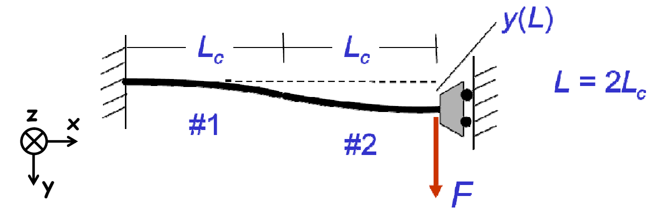
(a) Concentrated load.

cantilever	guided-end	fixed-fixed
$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{4E}$
$y = \frac{3}{2}\frac{f_y}{Eh}\frac{L^4}{w^3}$	$y = \frac{1}{2}\frac{f_y}{Eh}\frac{L^4}{w^3}$	$y = \frac{1}{32}\frac{f_y}{Eh}\frac{L^4}{w^3}$
$z = \frac{3}{2}\frac{f_z}{Ew}\frac{L^4}{h^3}$	$z = \frac{1}{2}\frac{f_z}{Ew}\frac{L^4}{h^3}$	$z = \frac{1}{32}\frac{f_z}{Ew}\frac{L^4}{h^3}$

(b) Distributed load.

Series Combinations of Springs

- For springs in series w/ one load
  - Deflections add
  - Spring constants combine like "resistors in parallel"



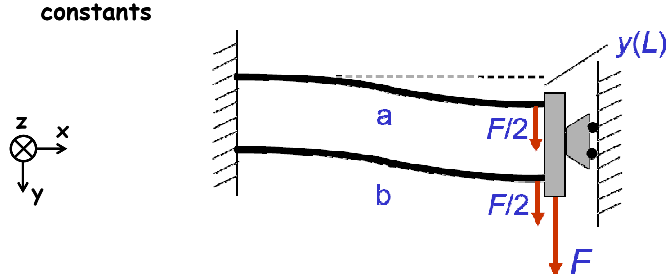
$Y(L) = F/k = 2\,y(L_c) = 2\,(F/k_c) = F(1/k_c + 1/k_c)$

Compliances effectively add:

$1/k = 1/k_c + 1/k_c$ → $k = k_c || k_c$

Parallel Combinations of Springs

- For springs in parallel w/ one load
  - Load is shared between the two springs
  - Spring constant is the sum of the individual spring constants



$Y(L) = F/k = F_a/k_a = F_b/k_b = (F/2)\,(1/k_a)$

$k = 2\,k_a$