Lecture 14: Beam Combos

- Announcements:
- HW#4 online and due Friday, March 11, 8 a.m.
- Midterm Exam two weeks away, Thursday, March 17, 5:10-7 p.m. in 3107 Etcheverry
- -----------------------------------
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
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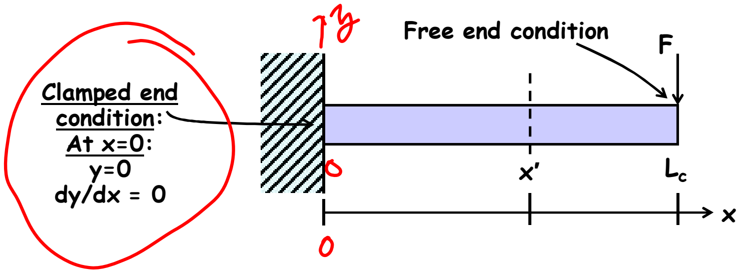
- Last Time:

Problem: Bending a Cantilever Beam



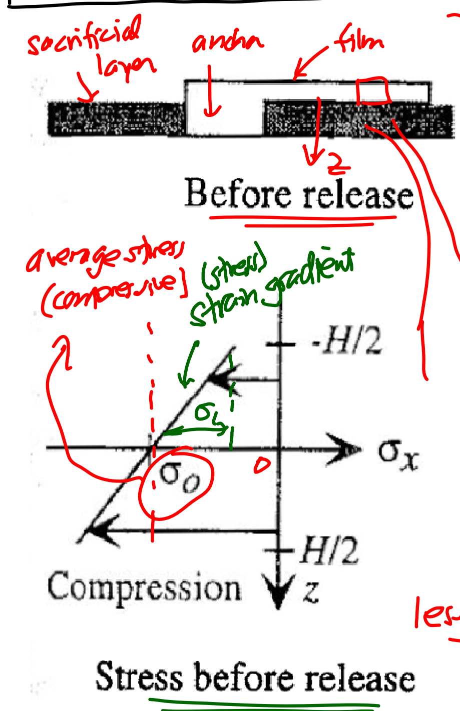
- Objective: Find relation between tip deflection $y(x=L_c)$ and applied load F
- Assumptions:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
  3. Shear stresses are negligible

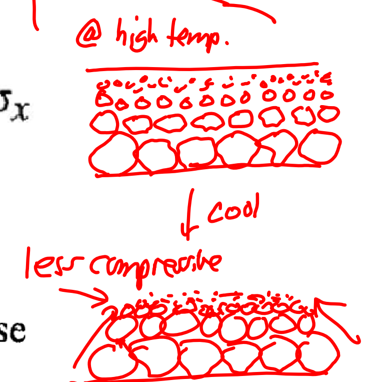
$$\boxed{w = \frac{FL}{2EI}x^2\left(1-\frac{x}{3L}\right)}$$

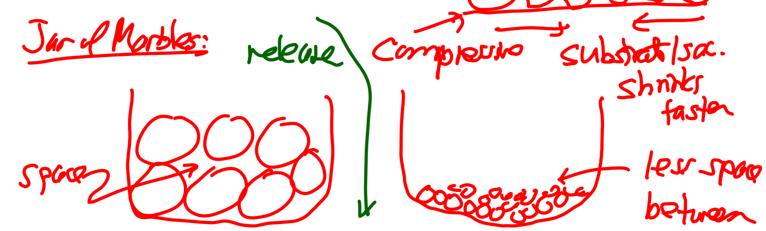
Deflection @ x due to a point load F applied @ x=L

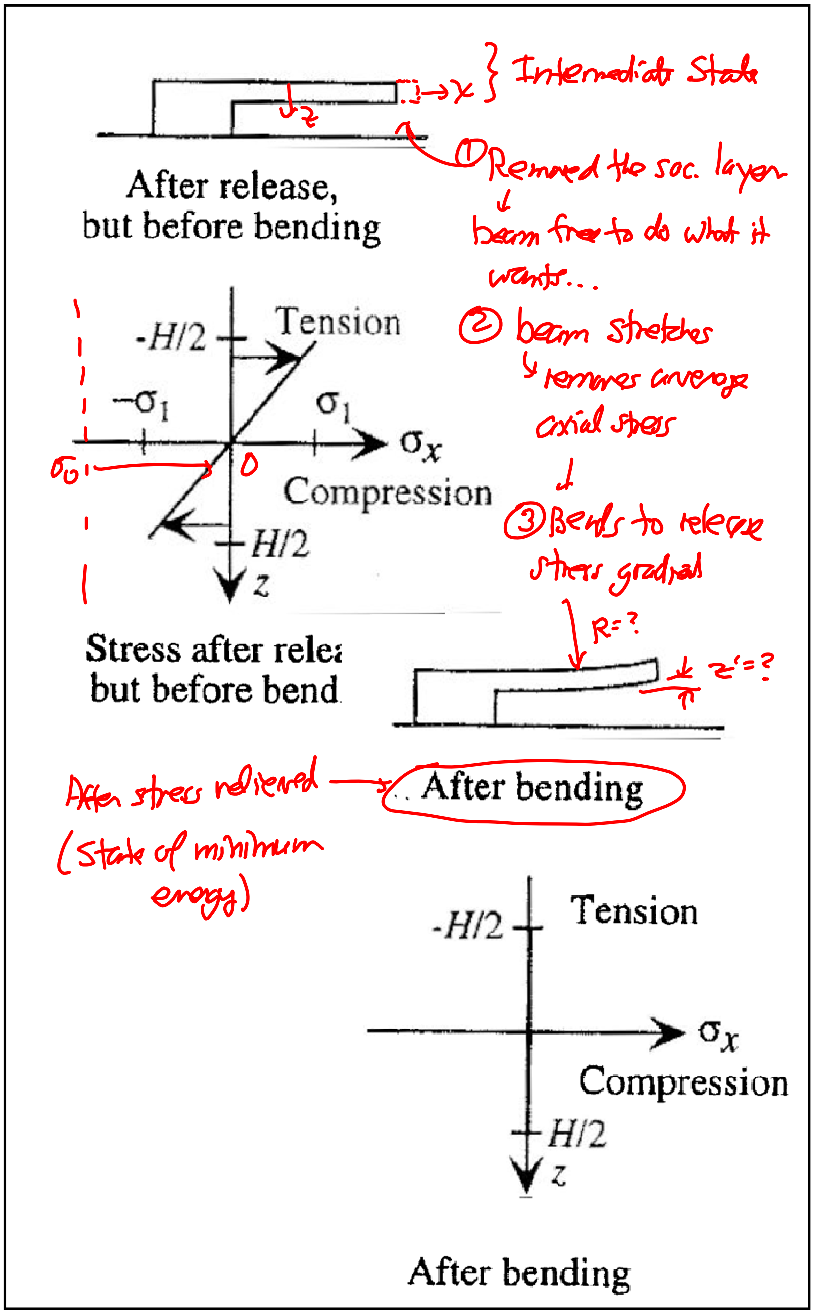
Stress Gradients in Cantilevers

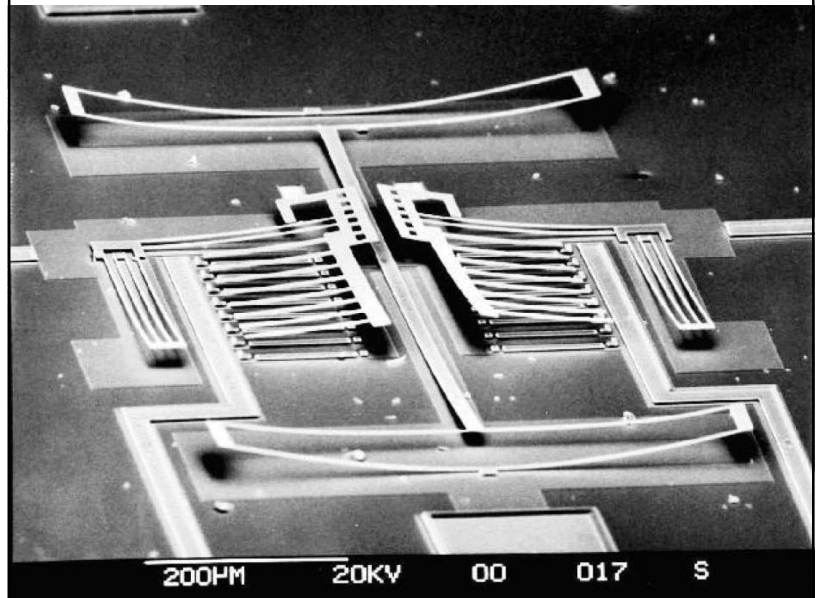
Process Flow
1. Deposit film @ high temp
2. Cool it down
⋮
3. Release.











Bending Due to Stress Gradients

Find the radius of curvature.

Prior to release, axial stress: $\sigma = \sigma_0 - \frac{\sigma_1}{(H/2)} z$

The internal moment:

$$M_x = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left[(W dz)\cdot\sigma\right]\cdot z = \int_{-\frac{H}{2}}^{\frac{H}{2}} W\left(z\sigma_0 - \frac{\sigma_1 z^2}{(H/2)}\right)dz$$

$$= W\left(\frac{1}{2}\sigma_0 z^2 - \frac{2\sigma_1 z^3}{3H}\right)\Bigg|_{-H/2}^{H/2}$$

$$= W\left(\cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\sigma_1\frac{H^2}{8} - \cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\sigma_1\frac{H^2}{8}\right)$$

$$\boxed{M_x = -\frac{1}{6}\sigma_1 W H^2}$$

Thus, the radius of curvature:

$$\frac{1}{R} = -\frac{M_x}{E'I} \rightarrow R = \frac{E'I}{M_x} = \frac{1}{2}\frac{E'H}{\sigma_1}$$

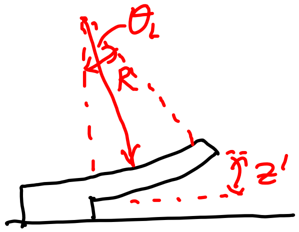
Biaxial Modulus

$$\left[I = \frac{1}{12}WH^3\right]$$

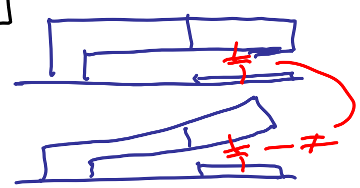
$$\boxed{R = \frac{1}{2}\frac{E}{(1-\nu)}\frac{H}{\sigma_1}}$$

[Radius of Curvature for a Cantilever w/ a Stress Gradient]

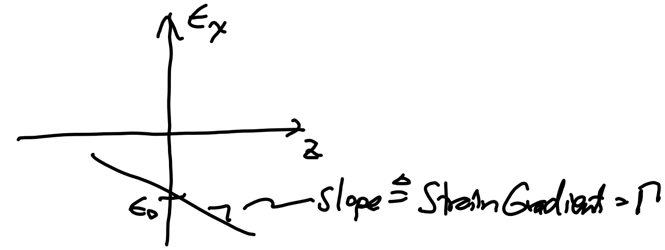
Radius of Curvature $\rightarrow z'$]



$\rightarrow$ Integrate over θ_L to get z' $\rightarrow$ Do this in homework

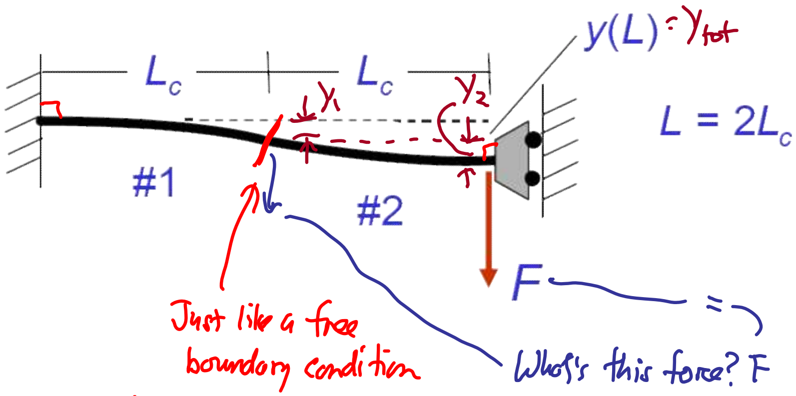


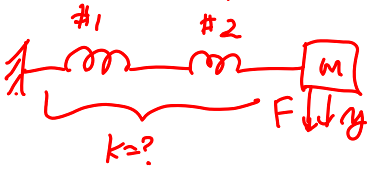
Definition. Strain Gradient



$$R = \frac{1}{2}\frac{E}{(1-\nu)}\frac{H}{\sigma_1} = \frac{1}{2}\frac{E'}{\sigma_1} = \frac{(H/2)}{\epsilon_1} = \frac{1}{\Gamma} \rightarrow \boxed{\Gamma = \frac{1}{R}} \checkmark$$

Series Combination of Springs





Series: $y_{tot} = y_1 + y_2$ } To tell if springs are in series: $y_{tot} > y_1$ ✓ $y_{tot} > y_2$ ✓

$$y_{tot} = y(L) = 2y(L_c) = 2\left(\frac{F}{k_c}\right) = F\left(\frac{1}{k_c} + \frac{1}{k_c}\right) = \frac{F}{k}$$

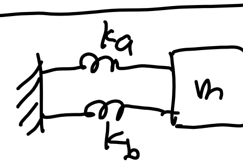
stiffness of a cantilever

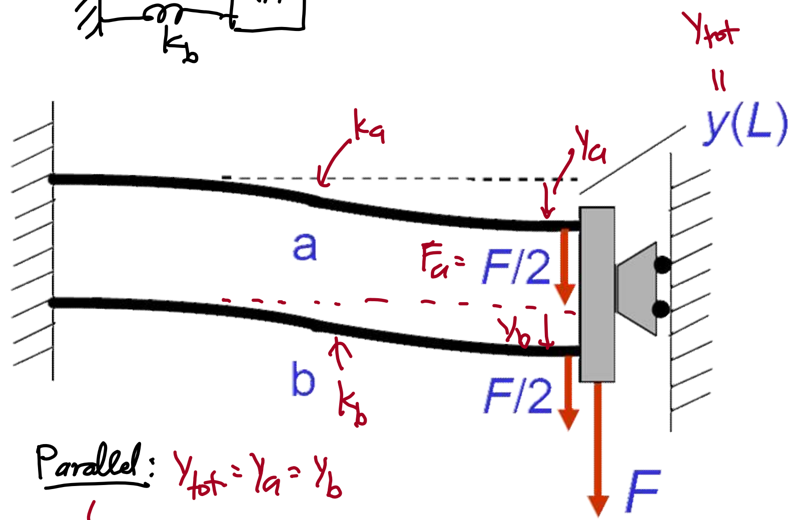
stiffness of whole thing

$$\therefore \boxed{\frac{1}{k} = \frac{1}{k_c} + \frac{1}{k_c}} \rightarrow \boxed{k = k_c \| k_c}$$

Definition for "||" $\left\{ A\|B = \frac{1}{\frac{1}{A} + \frac{1}{B}} = \frac{AB}{A+B} \right.$

Parallel Combination of Springs





Parallel: $y_{tot} = y_a = y_b$

Same displacement of each constituent spring as the total displacement

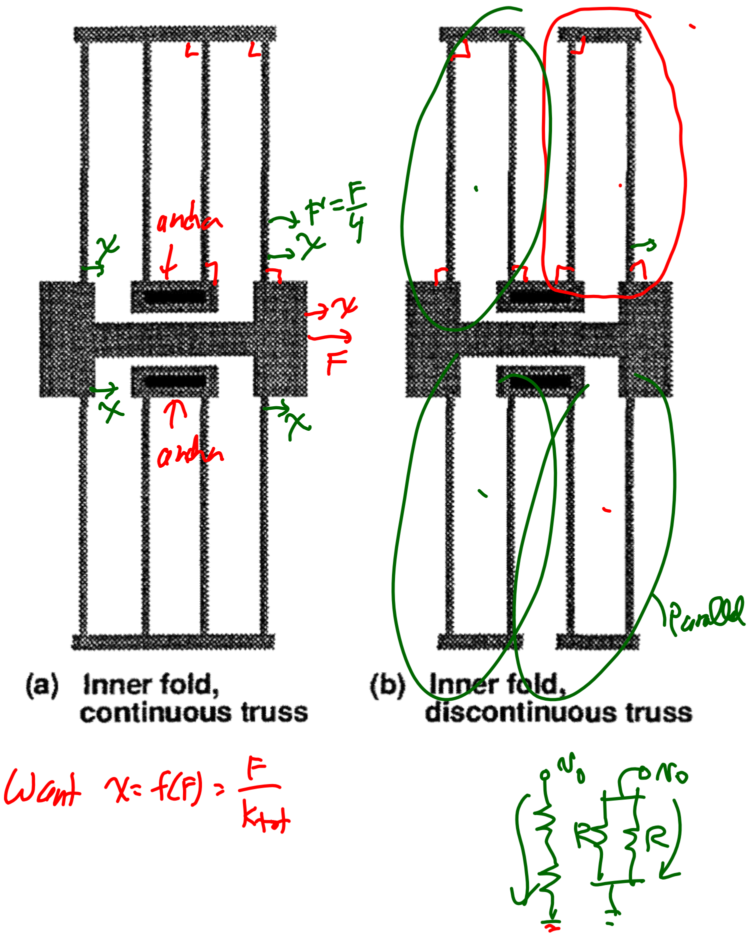
$$y(L) = y_{tot} = \frac{F}{k} = \frac{F_a}{k_a} = \frac{F_b}{k_b} = \left(\frac{F}{2}\right)\left(\frac{1}{k_a}\right), \quad F_a = \frac{F}{2},\ F_b = \frac{F}{2}$$

of the whole thing

$$\therefore \boxed{k = 2k_a}$$

In general: $\boxed{k_{tot} = k_a + k_b}$

For EE's: springs combine like capacitors



(a) Inner fold, continuous truss

(b) Inner fold, discontinuous truss

Want $x = f(F) = \frac{F}{k_{tot}}$