Lecture 14: Beam Combos

• Announcements:
  - HW#4 online and due Friday, March 11, 8 a.m.
  - Midterm Exam two weeks away, Thursday, March 17, 5:10-7 p.m. in 3107 Etcheverry
• Reading: Senturia, Chpt. 9
• Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
• Last Time:

Problem: Bending a Cantilever Beam

Objective: Find relation between tip deflection \( y(x=L_c) \) and applied load \( F \)

Assumptions:
1. Tip deflection is small compared with beam length
2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
3. Shear stresses are negligible
After release, but before bending:

Intermediate State:
1. Removed the soc. layer and beam free to do what it wants.
2. Beam stretches by removing average axial stress.
3. Bends to release from gradual.

Stress after rele; but before bend:

After bending:

Bending Due to Stress Gradients:

Find the radius of curvature.

Prior to release, axial stress: $\sigma = \sigma_0 - \frac{\sigma_1}{(H/2)^2}$

The internal moment:

$$N_x = \int \frac{1}{2} \left[ (Wd\theta) \cdot \sigma \right] z = \int \frac{1}{2} w(z \sigma_o - \frac{\sigma_1}{(H/2)^2}) dz$$

$$= W \left( \frac{1}{2} \sigma_o^2 - \frac{2}{3} \sigma_1 \frac{H^2}{3H} \right) \left( \frac{H}{2} \right)$$

$$= W \left( \frac{1}{2} \sigma_o^2 - \frac{2}{3} \sigma_1 \frac{H}{6} - \frac{2}{3} \sigma_1 \frac{H^2}{3H} \right)$$
\[ M_x = -\frac{1}{6} \sigma_y W H^2 \]

This gives the radius of curvature:

\[ \frac{1}{R} = \frac{M_x}{E I} \Rightarrow R = \frac{E I}{M_x} = \frac{1}{2} \frac{E H}{\sigma_y} \]

Biased Modulus

\[ [I : \frac{1}{2} WH^3] \]

Radius of Curvature

For a Cantilever with a Shear Gradient

\[ R = \frac{1}{2} \frac{E I}{(1-v) \sigma_y} = \frac{1}{2} \frac{E I}{\sigma_y} \frac{H}{E} = \frac{1}{2} \frac{E}{\sigma_y} \Rightarrow R = \frac{1}{2} \frac{E}{\sigma_y} \]

**Definition: Strain Gradient**

Slope = Strain Gradient = \( \frac{1}{R} \)

Radius of Curvature \( R \)
**Series Combination of Springs**

\[ Y(L) = Y_{\text{tot}} \]

1. \( L = 2L_c \)
2. \( \text{Just like a free boundary condition} \)
3. \( \text{What's this force? F} \)
4. \( y_1 + y_2 = \left( \frac{L}{L_c} \right) y_2 \)
5. \( y_{\text{tot}} = y(L) = 2y(L_c) = 2\left( \frac{F}{k_c} \right) = \frac{F}{k_c} \)
6. \( k_{\text{tot}} = \frac{k_1 + k_2}{k_1 k_2} \rightarrow k = k_c k_1 \)
7. \( \text{Mechanics of a Cantilever} \)

Definition for \( \mathbf{\text{strain}} \):
\[ \varepsilon = \frac{1}{\mathbf{A}} \mathbf{B} = \frac{AB}{A + B} \]

**Parallel Combination of Springs**

1. \( Y_{\text{tot}} = Y_a = Y_b \)
2. \( \text{same displacement of each constituent} \)
3. \( \text{spring as the total displacement} \)
4. \( Y(L) = Y_{\text{tot}} = \frac{F}{k} = \frac{F_a}{k_a} + \frac{F_b}{k_b} = \frac{F_a}{k_a} + \frac{F_a}{k_b} = \left( \frac{F}{2} \right) \left( \frac{1}{k_a} + \frac{1}{k_b} \right) \)
5. \( k_{\text{tot}} = \frac{k_a k_b}{k_a + k_b} \)

In general: \( k_{\text{tot}} = k_a + k_b \)

For EE's: springs combine like capacitors
(a) Inner fold, continuous truss

(b) Inner fold, discontinuous truss

Want $x = f(F) = \frac{F}{k_h}$

$\frac{F}{4} = F_c$

$L_c = \frac{L}{2}$

$Free$  $C\ F_{by}$

Cantilever $K$