Lecture 18: Resonance Frequency II

- Announcements:
  - Module 10 on Resonance Frequency online
  - Module 11 on Equivalent Circuits online
  - Graded midterms coming back today with solutions
  - Also, will show Z-scores
  - HW#5 online soon
  - Project described today (info online)

- Reading: Senturia, Chpt. 10: §10.5, Chpt. 19

- Lecture Topics:
  - Estimating Resonance Frequency
  - Lumped Mass-Spring Approximation
  - ADXL-50 Resonance Frequency
  - Distributed Mass & Stiffness
  - Folded-Beam Resonator
  - Resonance Frequency Via Differential Equations

- Reading: Senturia, Chpt. 5

- Lecture Topics:
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies

- Last Time:
  - Determined resonance frequency for a lumped mass-spring system
  - Now, look at a distributed system

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Find the Resonance Frequency When Mass & Stiffness Are Distributed

* Vibrating structure displacement function:
  \[ y'(x, t) = \hat{y}(x) \cos(\omega t) \]

Maximum displacement function (i.e., mode shape function)

* Procedure for determining resonance frequency:
  - Use the static displacement of the structure as a trial function and find the strain energy \( W_{\text{max}} \) at the point of maximum displacement (e.g., when \( t=0, \pi/\omega, \ldots \))
  - Determine the maximum kinetic energy when the beam is at zero displacement (e.g., when it experiences its maximum velocity)
  - Equate energies and solve for frequency

Get Maximum Kinetic Energy

\[
\text{velocity: } N(x,t) = \frac{\partial y(x,t)}{\partial t} = -\omega \hat{y}(x) \sin(\omega t)
\]

\[
N(x, \frac{2\pi n + \pi}{2\omega}) = -\omega \hat{y}(x)
\]

\[
t = \frac{n\pi}{\omega}, \frac{3\pi}{2\omega}, \ldots
\]

Velocity topographical mapping

\(\text{Vibraing} = 0\), all the energy in the structure is kinetic. \(N=0, K = \text{max}\)
Derive an expression for the resonance frequency of the above structure:

\[ \omega = \sqrt{\frac{W_{\text{max}}}{\int_0^L \rho \bar{w} h \left[ \eta(x) \right]^2 \, dx}} \]

- \( \omega \): radian resonant frequency
- \( W_{\text{max}} \): maximum potential energy
- \( \rho \): density of the structural material
- \( W \): beam width
- \( h \): thickness
- \( \eta(x) \): resonance mode shape

To get frequency:

\[ \omega_{\text{max}} = \frac{W_{\text{max}}}{\int_0^L \rho \bar{w} h \left[ \eta(x) \right]^2 \, dx} \]

- \( \omega_{\text{max}} \): maximum resonant frequency

Use the Rayleigh–Ritz Method: (energy method)

Derive an expression for the resonance frequency of the above structure.

\[ W_{\text{max}} = \int_0^L \rho \bar{w} h \left[ \eta(x) \right]^2 \, dx \]

Find the kinetic energy → one place at a time:

\[ W_{\text{max}} = K_s + K_t + K_b \]

- \( K_s \): shuttle truss beams
- \( K_t \): truss beams
\[ k_{max} = \frac{1}{2} N_e^2 M_e + \frac{1}{2} N_b^2 M_b + \frac{1}{2} \int N_b' dM_b \]

**Velocity of Shuttle:**  
\[ N_e = \frac{\omega_0 X_0}{\omega} \]

\( \omega \) = resonant frequency  
\( X_0 \) = max. displacement

\[ K_s = \frac{1}{2} N_e^2 M_e = \frac{1}{2} \omega_0^2 X_0^2 M_e \]

**Velocity of Truss:**  
\[ N_b = \frac{1}{2} N_b^2 = \frac{1}{2} \omega_0 X_0 \]

\[ K_b = \frac{1}{2} \left( \frac{1}{2} \omega_0 X_0 \right)^2 M_b = \frac{1}{8} \omega_0^2 X_0^2 \frac{M_b}{\pi} \]

mass of both truss combined

**Velocity of the Beam Segments:**

\[ \frac{X_0}{2} \]

\[ \frac{X_0}{2} \]

Assume the mode shape is the same as the static displacement mode shape.

guided

fixed