Lecture 18: Resonance Frequency II

- Announcements:
- Module 10 on Resonance Frequency online
- Module 11 on Equivalent Circuits online
- Graded midterms coming back today with solutions
- Also, will show Z-scores
- HW#5 online soon
- Project described today (info online)
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- Reading: Senturia, Chpt. 10: §10.5, Chpt. 19
- Lecture Topics:
  - Estimating Resonance Frequency
  - Lumped Mass-Spring Approximation
  - ADXL-50 Resonance Frequency
  - Distributed Mass & Stiffness
  - Folded-Beam Resonator
  - Resonance Frequency Via Differential Equations
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- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies
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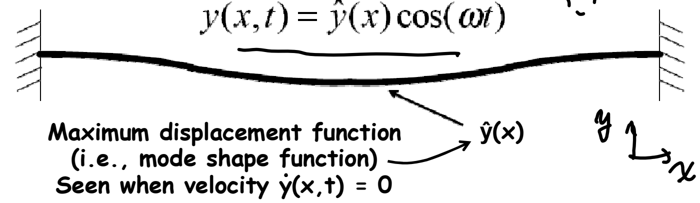
- Last Time:
- Determined resonance frequency for a lumped mass-spring system
- Now, look at a distributed system

Find the Resonance Frequency When Mass & Stiffness are Distributed

- Vibrating structure displacement function:

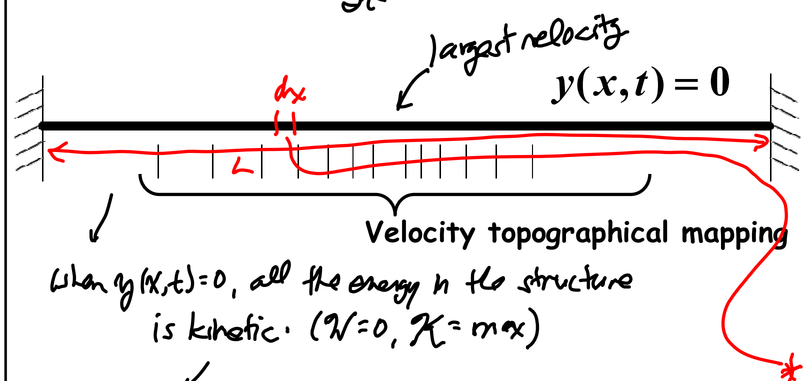
$$y(x,t) = \hat{y}(x)\cos(\omega t)$$



- Procedure for determining resonance frequency:
  - Use the static displacement of the structure as a trial function and find the strain energy W_{max} at the point of maximum displacement (e.g., when t=0, π/ω, …)
  - Determine the maximum kinetic energy when the beam is at zero displacement (e.g., when it experiences its maximum velocity)
  - Equate energies and solve for frequency

Get Maximum Kinetic Energy

velocity: $v(x,t) = \frac{\partial y(x,t)}{\partial t} = -\omega \hat{y}(x)\sin(\omega t)$



When $y(x,t)=0$, all the energy in the structure is kinetic. ($W=0$, $K=max$)

$$v\left(x, \frac{(2m+1)\pi}{2\omega}\right) = -\omega\hat{y}(x)$$

$t = \frac{\pi}{2\omega}, \frac{3\pi}{2\omega}, \ldots$

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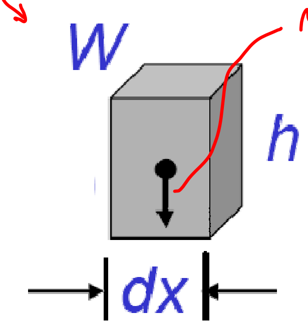


velocity: ← mode shape

$v = -\omega \hat{y}(x)$ ← $\frac{(2m-1)\pi}{2\omega}$

$d\mathcal{K} = \frac{1}{2} \cdot dm \cdot [v(x,t)]^2$

$dm = \rho(Whdx)$

↑ density

Maximum $\mathcal{K}$:

$$\mathcal{K}_{max} = \int_0^L \frac{1}{2}\rho W h\, dx\, v^2(x,t) = \int_0^L \frac{1}{2}\rho W h \omega^2 [\hat{y}(x)]^2 dx$$

To get frequency:

$$\mathcal{K}_{max} = W_{max}$$

$$\therefore \quad \omega = \sqrt{\frac{W_{max}}{\int_0^L \frac{1}{2}\rho W h [\hat{y}(x)]^2 dx}}$$

ω = radian resonance frequency

W_{max} = maximum potential energy

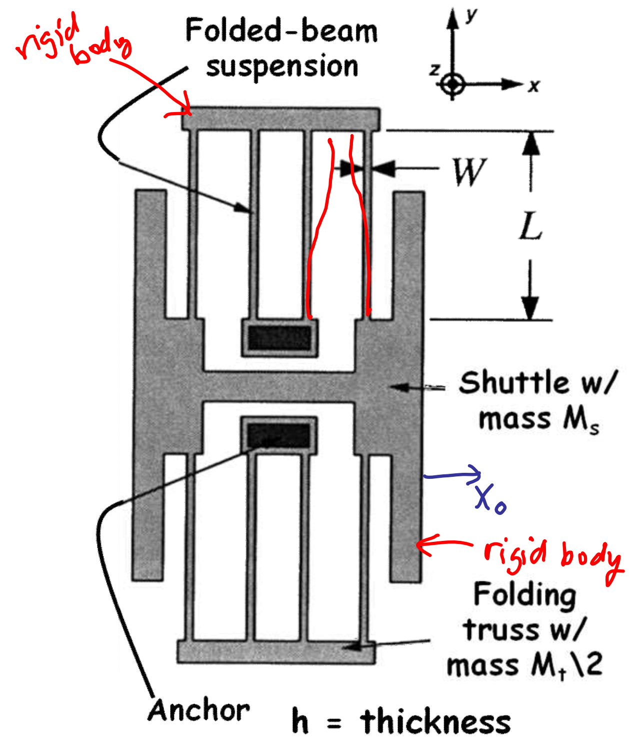
ρ = density of the structural material

W = beam width

h = " thickness

$\hat{y}(x)$ = resonance mode shape

Resonance Freq. of a Folded-Beam Resonator



- Derive an expression for the resonance frequency of the above structure

Use the Rayleigh-Ritz Method: (energy method)

$$\mathcal{K}_{max} = W_{max}$$

Find the kinetic energy → one piece @ a time:

$$\mathcal{K}_{max} = \underbrace{\mathcal{K}_s}_{shuttle} + \underbrace{\mathcal{K}_t}_{truss} + \underbrace{\mathcal{K}_b}_{beams}$$

$$K_{max} = \frac{1}{2}v_s^2 M_s + \frac{1}{2}v_t^2 M_t + \frac{1}{2}\int v_b^2 \, dM_b$$

Velocity of Shuttle: $v_s = \omega_o X_o$

ω_o: resonance freq.; X_o: max. displacement of the shuttle

$$\therefore K_s = \frac{1}{2}v_s^2 M_s = \frac{1}{2}\omega_o^2 X_o^2 M_s$$

Velocity of Truss: $v_t = \frac{1}{2}v_s = \frac{1}{2}\omega_o X_o$

$$\therefore K_t = \frac{1}{2}\left(\frac{1}{2}\omega_o X_o\right)^2 M_t = \frac{1}{8}\omega_o^2 X_o^2 M_t$$

M_t: mass of both trusses combined

Velocity of the Beam Segments: