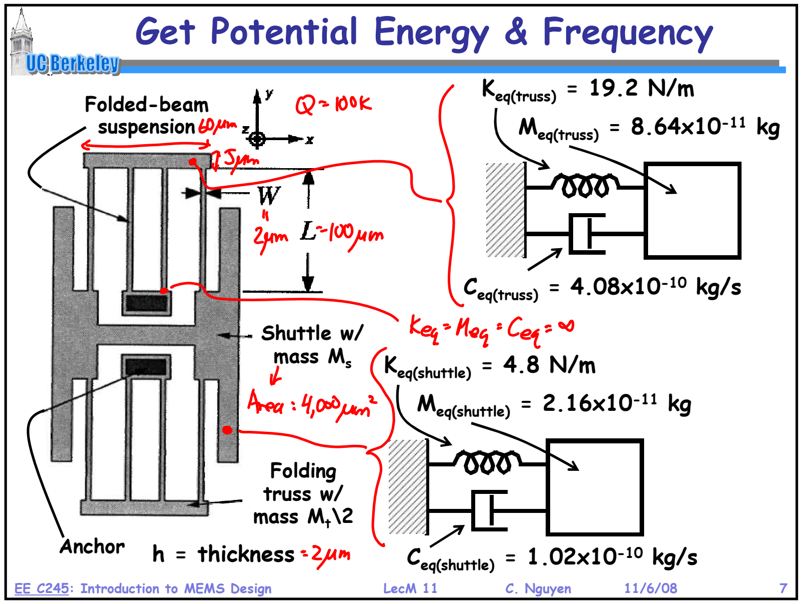
Get Potential Energy & Frequency
UC Berkeley
Folded-beam suspension
Q = 100K
W
L = 100 μm
Shuttle w/ mass M_s
Area: 4,000 μm²
Folding truss w/ mass M_t\2
Anchor
h = thickness = 2μm
$K_{eq(truss)}$ = 19.2 N/m
$M_{eq(truss)}$ = 8.64x10⁻¹¹ kg
$C_{eq(truss)}$ = 4.08x10⁻¹⁰ kg/s
$K_{eq(shuttle)}$ = 4.8 N/m
$M_{eq(shuttle)}$ = 2.16x10⁻¹¹ kg
$C_{eq(shuttle)}$ = 1.02x10⁻¹⁰ kg/s
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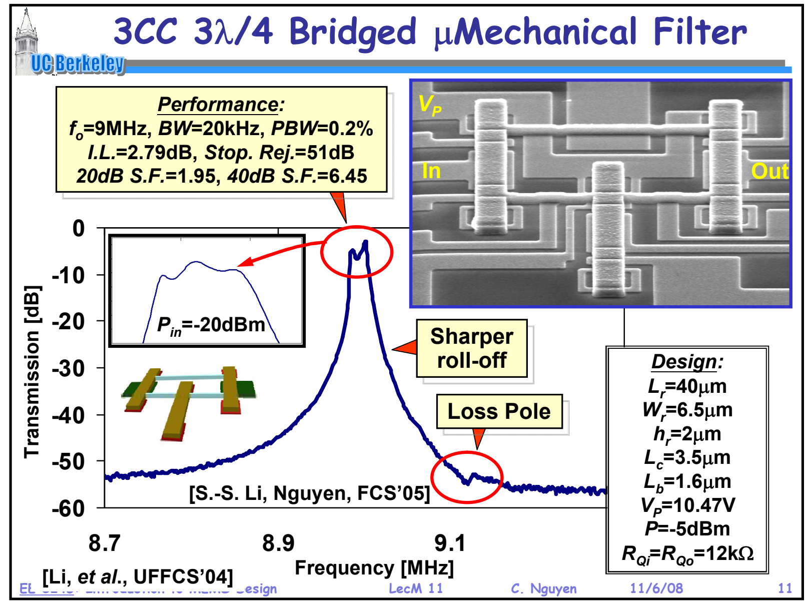
3CC 3λ/4 Bridged μMechanical Filter
UC Berkeley
Performance:
f_o=9MHz, BW=20kHz, PBW=0.2%
I.L.=2.79dB, Stop. Rej.=51dB
20dB S.F.=1.95, 40dB S.F.=6.45
V_P
In
Out
P_{in}=-20dBm
Sharper roll-off
Loss Pole
Design:
L_r=40μm
W_r=6.5μm
h_r=2μm
L_c=3.5μm
L_b=1.6μm
V_P=10.47V
P=-5dBm
R_{Qi}=R_{Qo}=12kΩ
Transmission [dB]
0
-10
-20
-30
-40
-50
-60
8.7
8.9
9.1
Frequency [MHz]
[S.-S. Li, Nguyen, FCS'05]
[Li, et al., UFFCS'04]
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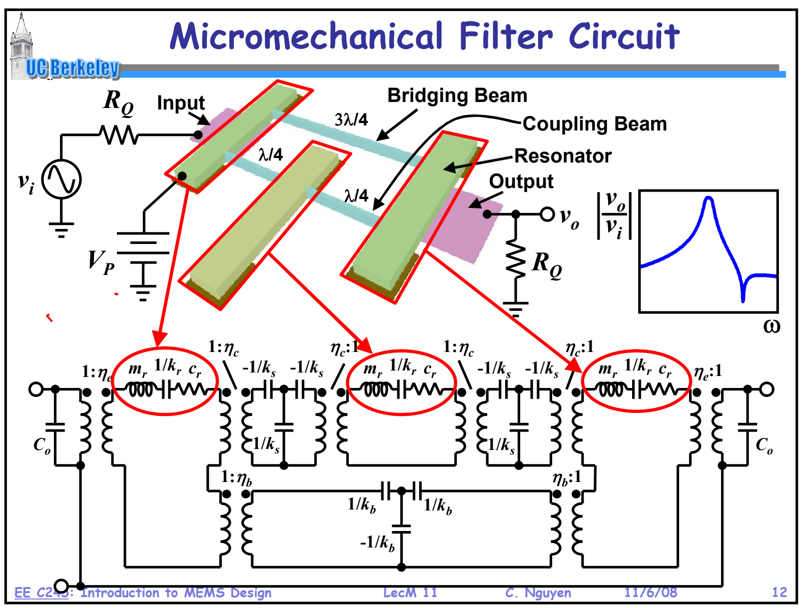
Micromechanical Filter Circuit
UC Berkeley
R_Q
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
Output
v_i
v_o
V_P
$|v_o/v_i|$
ω
1:η_e
m_r 1/k_r c_r
1:η_c
-1/k_s
η_c:1
1/k_s
η_e:1
C_o
1:η_b
η_b:1
1/k_b
-1/k_b
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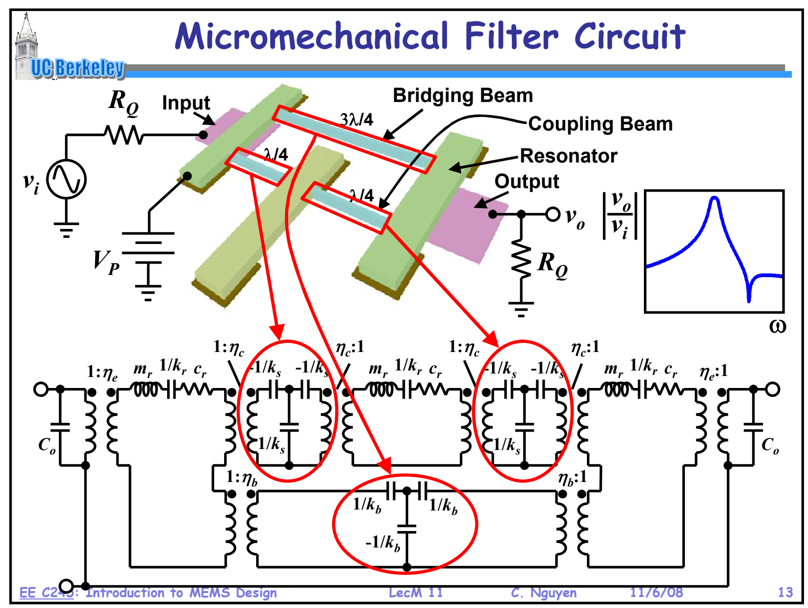
Micromechanical Filter Circuit
UC Berkeley
R_Q
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
Output
v_i
v_o
V_P
$|v_o/v_i|$
ω
1:η_e
m_r 1/k_r c_r
1:η_c
-1/k_s
η_c:1
1/k_s
η_e:1
C_o
1:η_b
η_b:1
1/k_b
-1/k_b
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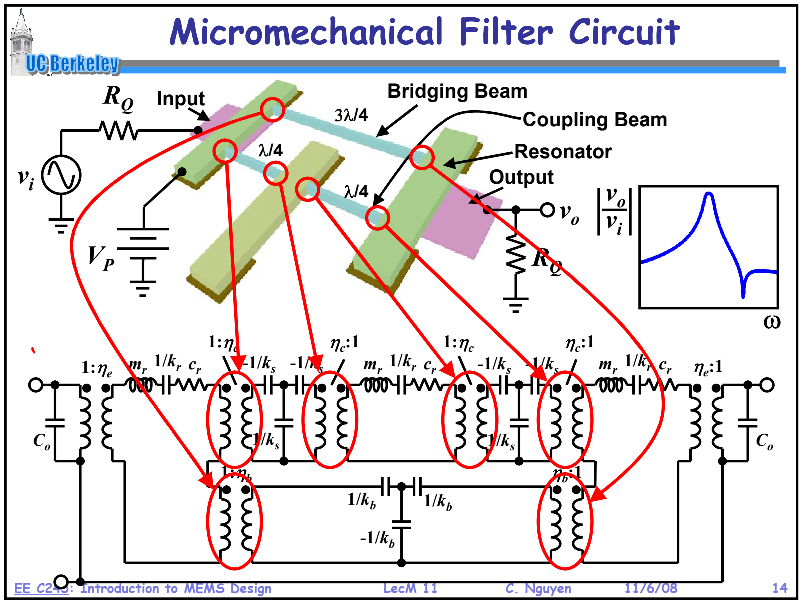
Micromechanical Filter Circuit
UC Berkeley
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
λ/4
Output
R_Q
v_i
V_P
v_o
$\left|\frac{v_o}{v_i}\right|$
ω
$1:\eta_e$
m_r $1/k_r$ c_r
$1:\eta_c$
$-1/k_s$
$\eta_c:1$
$1/k_s$
C_o
$1:\eta_b$
$\eta_b:1$
$1/k_b$
$-1/k_b$
$\eta_e:1$
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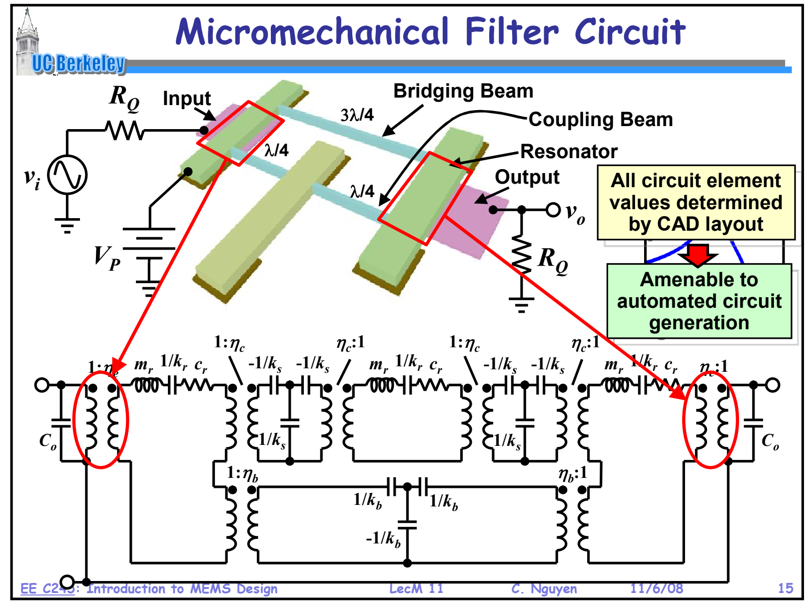
Micromechanical Filter Circuit
UC Berkeley
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
λ/4
Output
R_Q
v_i
V_P
v_o
All circuit element values determined by CAD layout
Amenable to automated circuit generation
$1:\eta_e$
m_r $1/k_r$ c_r
$1:\eta_c$
$-1/k_s$
$\eta_c:1$
$1/k_s$
C_o
$1:\eta_b$
$\eta_b:1$
$1/k_b$
$-1/k_b$
$\eta_e:1$
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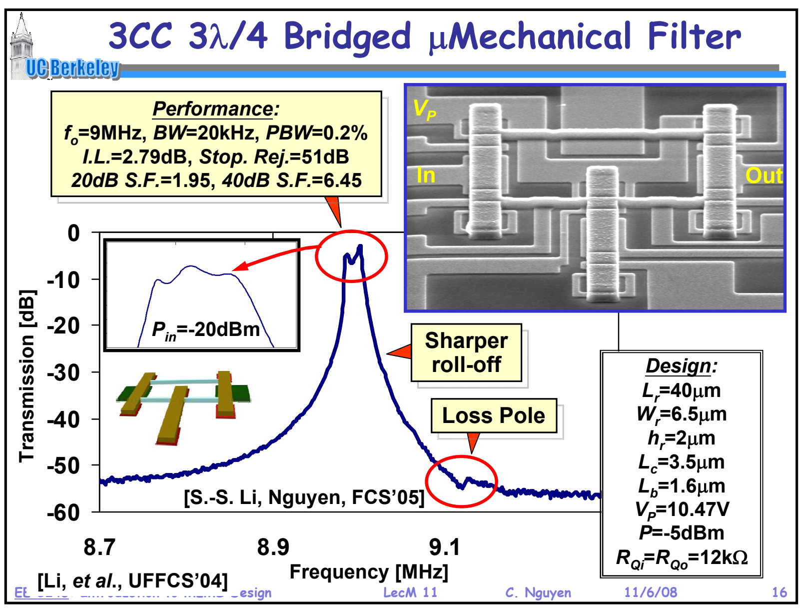
3CC 3λ/4 Bridged μMechanical Filter
UC Berkeley
Performance:
f_o=9MHz, BW=20kHz, PBW=0.2%
I.L.=2.79dB, Stop. Rej.=51dB
20dB S.F.=1.95, 40dB S.F.=6.45
V_P
In
Out
P_{in}=-20dBm
Sharper roll-off
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Design:
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L_c=3.5μm
L_b=1.6μm
V_P=10.47V
P=-5dBm
R_{Qi}=R_{Qo}=12kΩ
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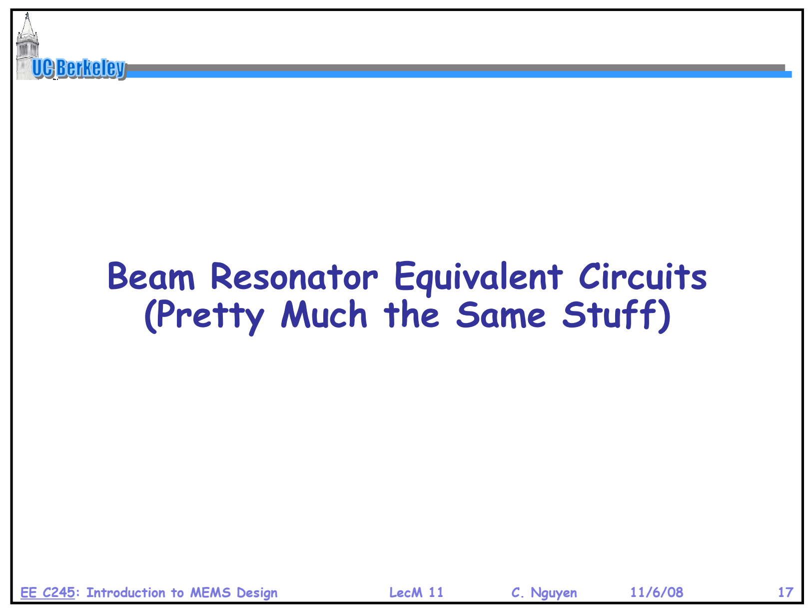
UC Berkeley
Beam Resonator Equivalent Circuits
(Pretty Much the Same Stuff)
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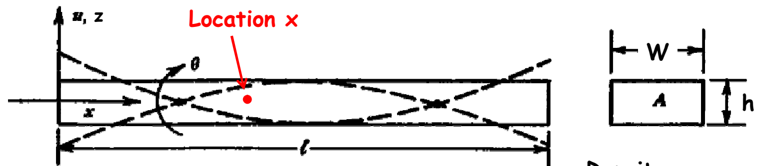
Equivalent Dynamic Mass

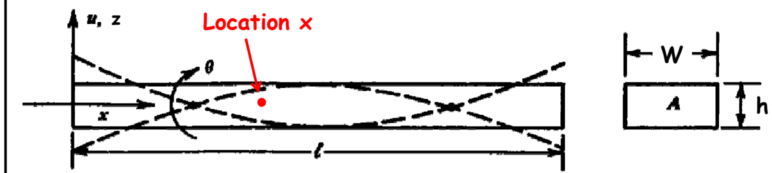
- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Equivalent Mass $= M_{eq\,x} = \dfrac{K.E.}{\frac{1}{2}V_x^2} = \dfrac{\frac{1}{2}\rho A \int_0^{\ell} V^2(x)\,dx}{\frac{1}{2}V_x^2}$

Maximum Kinetic Energy → K.E.; Density → ρ; Maximum Velocity @ location x → V_x; Maximum Velocity Function → $V(x)$

Equivalent Dynamic Mass



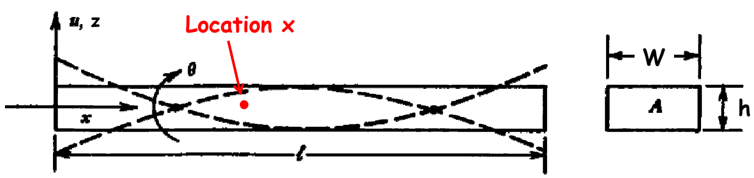
- We know the mode shape, so we can write expressions for displacement and velocity at resonance

Displacement: $u(x) = B[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]$, $\varsigma = \frac{A}{B}$

$[V(x) = \omega_0 u(x)] \Rightarrow M_{eq}(x) = \dfrac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \dfrac{\frac{1}{2}\rho A \int_0^{\ell} \omega_0^2 [u(x')]^2 dx'}{\frac{1}{2}\omega_0^2 [u(x)]^2}$

$$M_{eq}(x) = \frac{\rho A \int_0^{\ell} B^2[\varsigma(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')]^2 dx'}{B^2[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}$$

Equivalent Dynamic Stiffness & Damping



- Stiffness then follows directly from knowledge of mass and resonance frequency

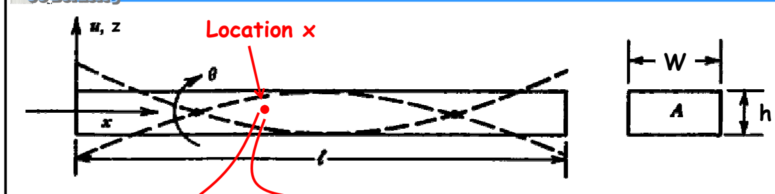
$\omega_0 = \sqrt{\dfrac{K_{eq}(x)}{M_{eq}(x)}} \longrightarrow \boxed{K_{eq}(x) = \omega_0^2 M_{eq}(x)}$

- And damping also follows readily

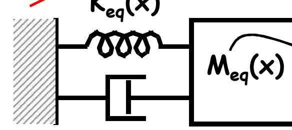
$Q = \dfrac{\omega_0 M_{eq}(x)}{C_{eq}(x)} \longrightarrow \boxed{C_{eq}(x) = \dfrac{\omega_0 M_{eq}(x)}{Q} = \dfrac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}}$

$C_{eq}(x)$ ← damping

Equivalent Lumped Mechanical Circuit

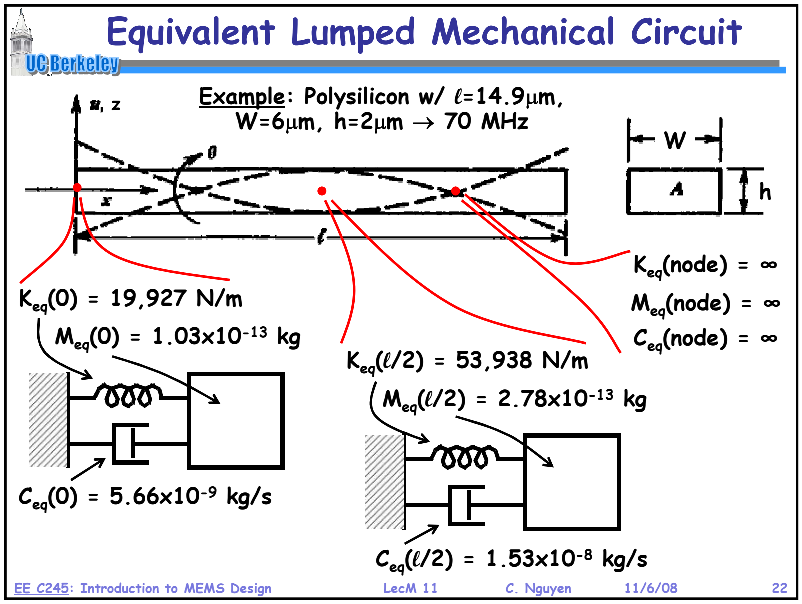


$$K_{eq}(x) = \omega_o^2 M_{eq}(x)$$



$$M_{eq}(x) = \frac{\rho A \int_o^l [u(x')]^2 dx'}{[u(x)]^2}$$

$$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$$


Equivalent Lumped Mechanical Circuit
UC Berkeley
Example: Polysilicon w/ ℓ=14.9μm, W=6μm, h=2μm → 70 MHz
W
h
A
ℓ
$K_{eq}(0)$ = 19,927 N/m
$M_{eq}(0)$ = 1.03x10^{-13} kg
$C_{eq}(0)$ = 5.66x10^{-9} kg/s
$K_{eq}(\ell/2)$ = 53,938 N/m
$M_{eq}(\ell/2)$ = 2.78x10^{-13} kg
$C_{eq}(\ell/2)$ = 1.53x10^{-8} kg/s
K_{eq}(node) = ∞
M_{eq}(node) = ∞
C_{eq}(node) = ∞
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