Parallel-Plate Capacitive Nonlinearity

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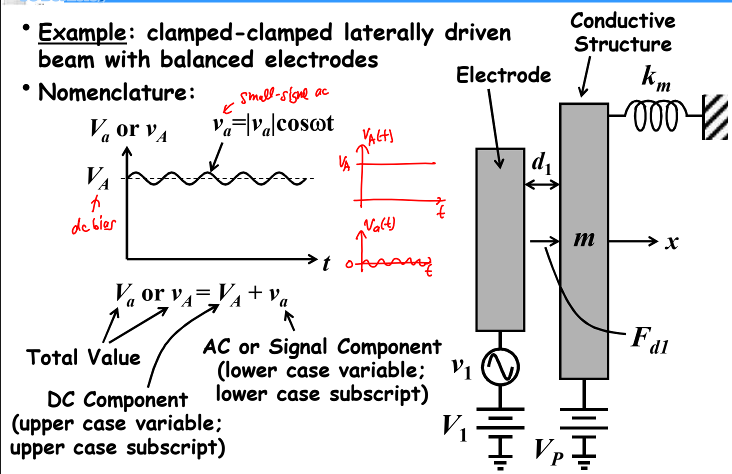
- Example: clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:



Voltage-Controllable Center Frequency

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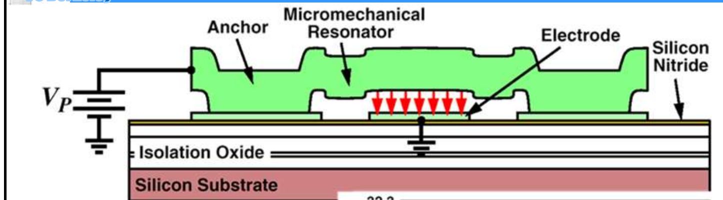
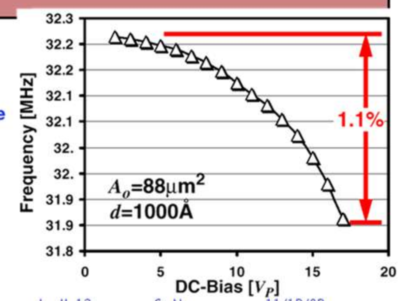


- Quadrature force ➠ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

(Electrode Overlap Area; Gap)

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



Microresonator Thermal Stability

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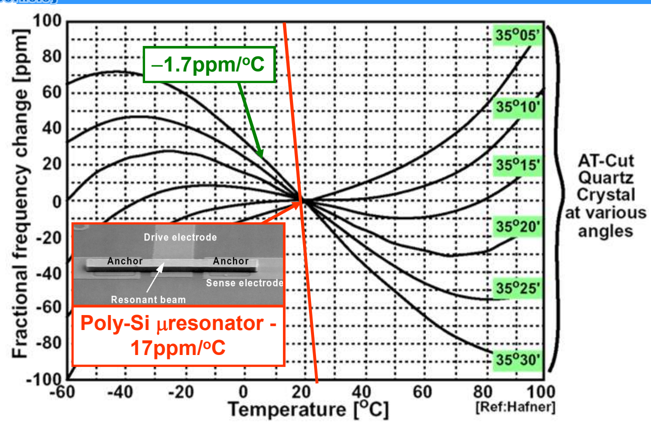

- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

Geometric-Stress Compensation

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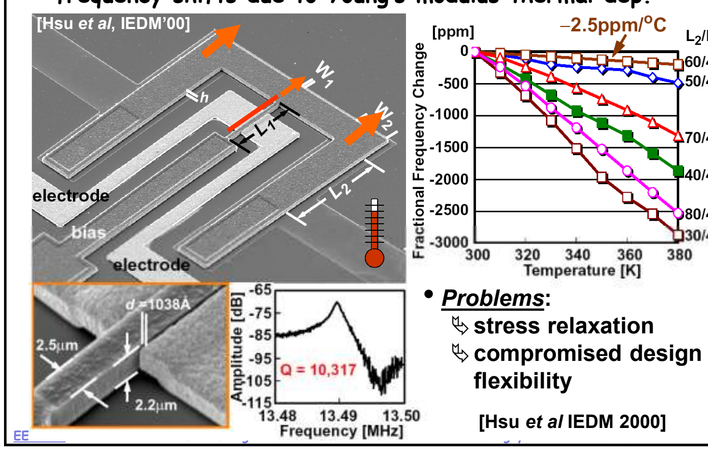
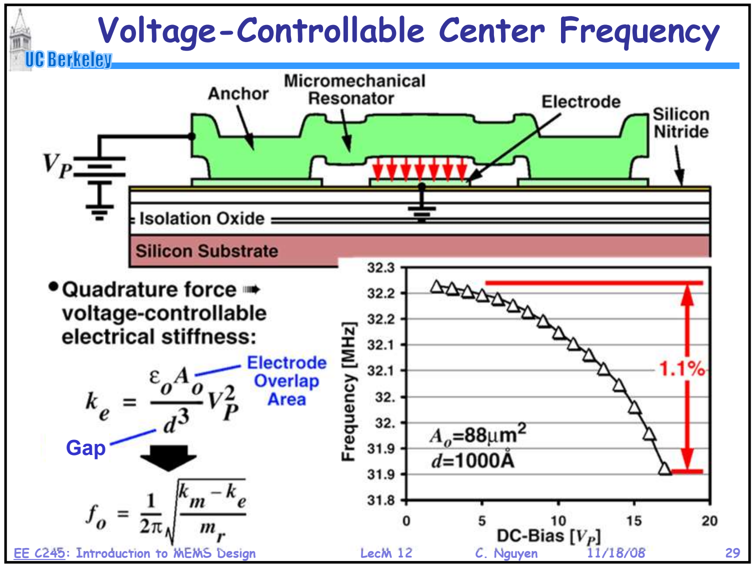
- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.



- Problems:
  - stress relaxation
  - compromised design flexibility

[Hsu et al IEDM 2000]


Voltage-Controllable Center Frequency
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Anchor
Micromechanical Resonator
Electrode
Silicon Nitride
V_P
Isolation Oxide
Silicon Substrate
• Quadrature force ➠ voltage-controllable electrical stiffness:
$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$
Electrode Overlap Area
Gap
$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$
Frequency [MHz]
DC-Bias [V_P]
1.1%
A_o=88μm²
d=1000Å
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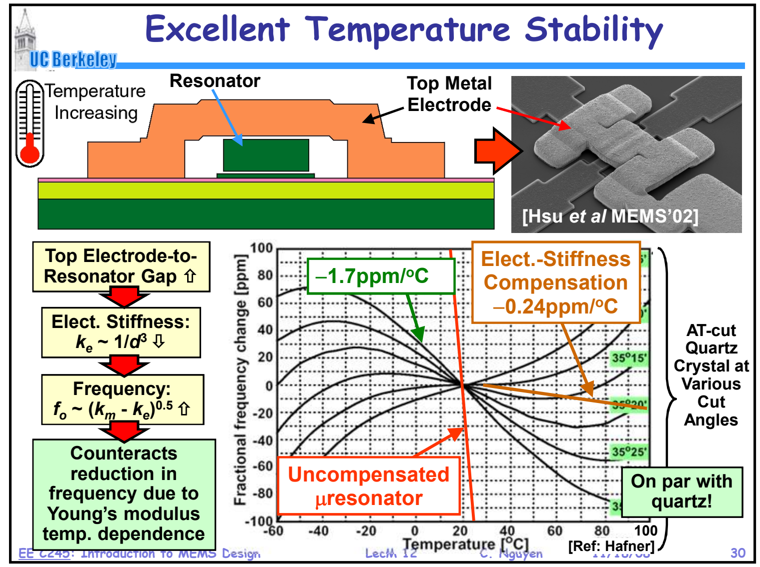
Excellent Temperature Stability
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Temperature Increasing
Resonator
Top Metal Electrode
[Hsu et al MEMS'02]
Top Electrode-to-Resonator Gap ⇧
Elect. Stiffness: $k_e \sim 1/d^3$ ⇩
Frequency: $f_o \sim (k_m - k_e)^{0.5}$ ⇧
Counteracts reduction in frequency due to Young's modulus temp. dependence
–1.7ppm/°C
Elect.-Stiffness Compensation –0.24ppm/°C
Uncompensated μresonator
AT-cut Quartz Crystal at Various Cut Angles
On par with quartz!
Fractional frequency change [ppm]
Temperature [°C]
[Ref: Hafner]
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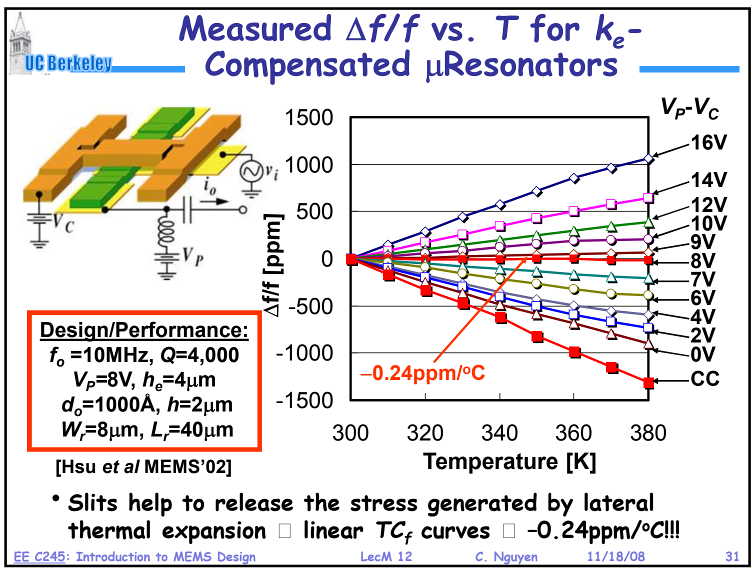
Measured Δf/f vs. T for k_e-Compensated μResonators
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V_P-V_C
16V
14V
12V
10V
9V
8V
7V
6V
4V
2V
0V
CC
Δf/f [ppm]
Temperature [K]
–0.24ppm/°C
Design/Performance:
f_o=10MHz, Q=4,000
V_P=8V, h_e=4μm
d_o=1000Å, h=2μm
W_r=8μm, L_r=40μm
[Hsu et al MEMS'02]
• Slits help to release the stress generated by lateral thermal expansion → linear TC_f curves → -0.24ppm/°C!!!
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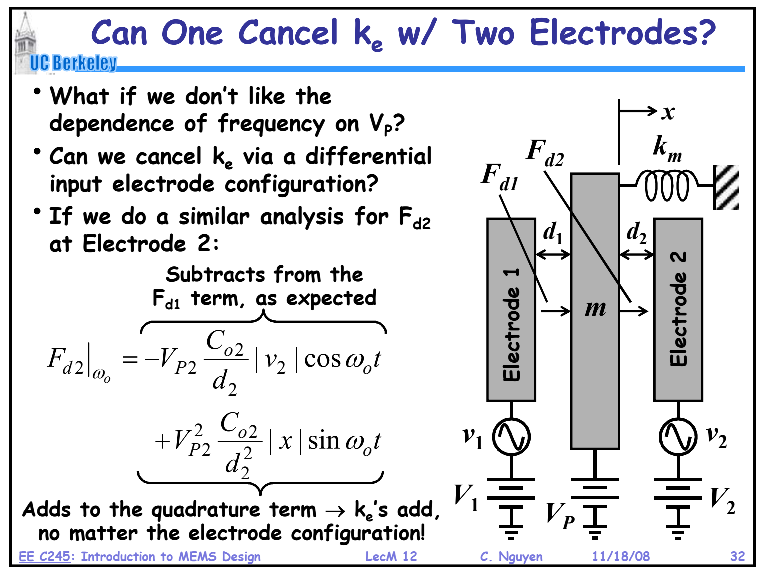
Can One Cancel k_e w/ Two Electrodes?
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• What if we don't like the dependence of frequency on V_P?
• Can we cancel k_e via a differential input electrode configuration?
• If we do a similar analysis for F_{d2} at Electrode 2:
Subtracts from the F_{d1} term, as expected
$F_{d2}\big|_{\omega_o} = -V_{P2}\frac{C_{o2}}{d_2}|v_2|\cos\omega_o t + V_{P2}^2\frac{C_{o2}}{d_2^2}|x|\sin\omega_o t$
Adds to the quadrature term → k_e's add, no matter the electrode configuration!
F_{d1} F_{d2} k_m x d_1 d_2 m
Electrode 1
Electrode 2
v_1 v_2 V_1 V_P V_2
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Problems With Parallel-Plate C Drive

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- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_o)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap … but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)