Parallel-Plate Capacitive Nonlinearity

- Example: clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:
  - $V_1$ or $v_1$
  - $V_d$
  - $v_d$
  - $V_{a}$ or $v_{a}$
  - $V_{c}$
  - $V_{p}$

Voltage-Controllable Center Frequency

- Quadrature force $\rightarrow$ voltage-controllable electrical stiffness:
  - $k_e = \frac{\varepsilon_0 A_o V_p^2}{d^3}$

Microresonator Thermal Stability

- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

Geometric-Stress Compensation

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.

Problems:
- $\sigma_0$ stress relaxation
- $\sigma_0$ compromised design flexibility
Voltage-Controllable Center Frequency

Excellent Temperature Stability

Can One Cancel $k_e$ w/ Two Electrodes?

- What if we don’t like the dependence of frequency on $V_p$?
- Can we cancel $k_e$ via a differential input electrode configuration?

If we do a similar analysis for $F_{d2}$ at Electrode 2:

$F_{d2}\approx -V_{p2}\frac{C_{\perp}}{d_2}x\cos{\omega_0t}$

$+V_{p2}\frac{C_{\perp}}{d_2}x\sin{\omega_0t}$

Add to the quadrature term → $k_e$’s add, no matter the electrode configuration!
Problems With Parallel-Plate C Drive

* Nonlinear voltage-to-force transfer function
  % Resonance frequency becomes dependent on parameters (e.g., bias voltage $V_p$)
  % Output current will also take on nonlinear characteristics as amplitude grows (i.e., as $x$ approaches $d_o$)
  % Noise can alias due to nonlinearity

* Range of motion is small
  % For larger motion, need larger gap ... but larger gap weakens the electrostatic force
  % Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

![Diagram of parallel-plate capacitor with forces and motion diagram]