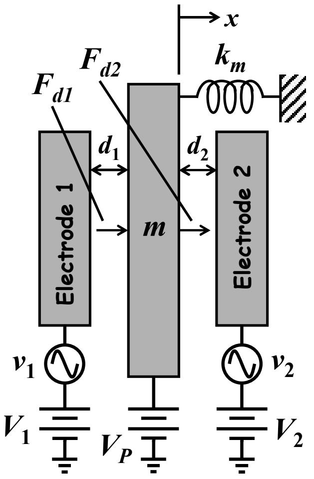
## Can One Cancel $k_e$ w/ Two Electrodes?

- What if we don't like the dependence of frequency on $V_P$?
- Can we cancel $k_e$ via a differential input electrode configuration?
- If we do a similar analysis for $F_{d2}$ at Electrode 2:

$$F_{d2}\big|_{\omega_o} = \overbrace{-V_{P2}\frac{C_{o2}}{d_2}|v_2|\cos\omega_o t}^{\text{Subtracts from the } F_{d1} \text{ term, as expected}} \quad \underbrace{+V_{P2}^2\frac{C_{o2}}{d_2^2}|x|\sin\omega_o t}_{\text{Adds to the quadrature term} \rightarrow k_e\text{'s add, no matter the electrode configuration!}}$$



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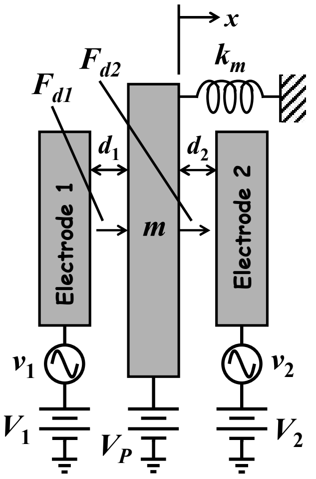
## Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage $V_P$)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches $d_o$)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap … but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)



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# Electrostatic Comb Drive

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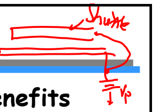
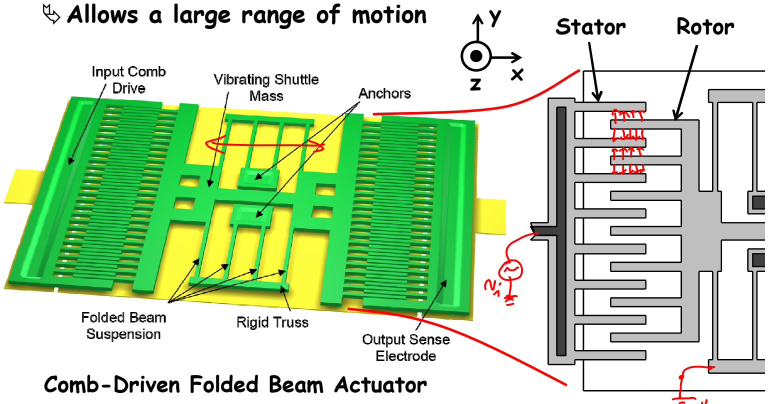
## Electrostatic Comb Drive



- Use of comb-capacitive tranducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion



Comb-Driven Folded Beam Actuator

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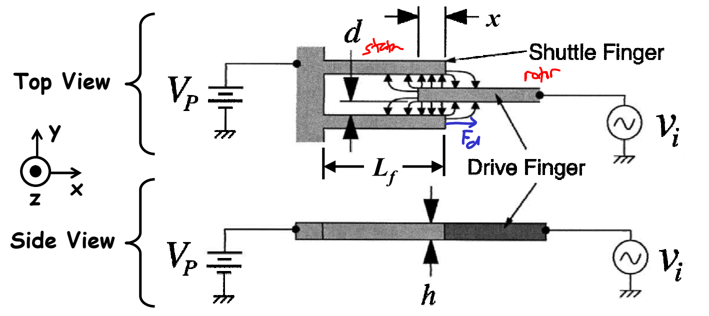
## Comb-Drive Force Equation (1st Pass)



$C(x) = \frac{2\epsilon_o x h}{d} \rightarrow \left[\frac{\partial C}{\partial x} = \frac{2\epsilon_o h}{d}\right]$

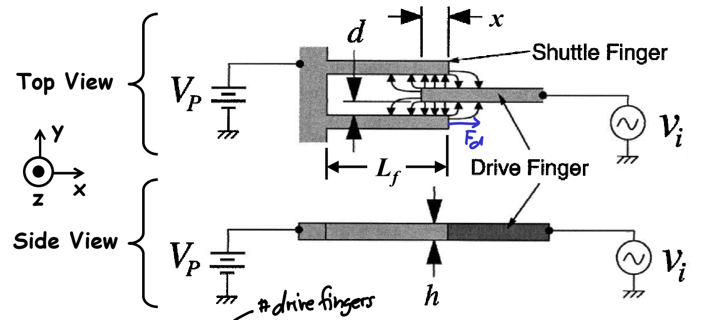
When $v_i = (+) \rightarrow F_d = (-)$ ✓

$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2 = \frac{2}{2}\frac{\epsilon_o h}{d}(V_P^2 - 2V_P v_i + v_i^2) \approx \boxed{-2V_P \frac{\epsilon_o h}{d} v_i = F_d}$

But wait! This ignores other practical effects!

(No dependence on $x$! LINEAR!)

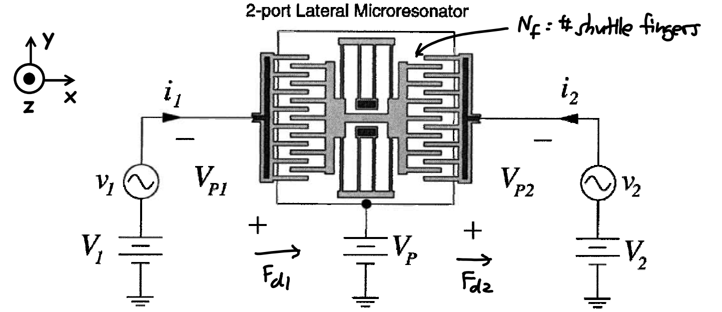
## Lateral Comb-Drive Electrical Stiffness



# drive fingers

- Again: $C(x) = \frac{2N\varepsilon h x}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2N\varepsilon h}{d}$
- No ($\partial C/\partial x$) x-dependence → no electrical stiffness: $k_e = 0$!
- Frequency immune to changes in $V_P$ or gap spacing!

## Typical Drive & Sense Configuration



Simple Analysis:

$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(v_1 - V_{P1})^2 = \frac{1}{2}\left(-\frac{\epsilon_o h}{d_1}\right)(v_1^2 - 2V_{P1}v_1 + V_{P1}^2)(2N_f)$

$F_{d2} = \frac{1}{2}\frac{\partial C_2}{\partial x}(v_2 - V_{P2})^2 = \frac{1}{2}\left(\frac{\epsilon_o h}{d_2}\right)(v_2^2 - 2V_{P2}v_2 + V_{P2}^2)(2N_f)$

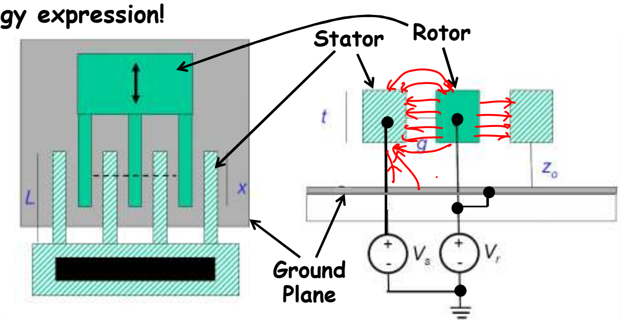
$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2}\left(\frac{\epsilon_o h}{d}\right)(v_2^2 - v_1^2 - 2(V_{P2}v_2 - V_{P1}v_1) + V_{P2}^2 - V_{P1}^2)(2N_f)$

For $V_1 = V_2$, $v_1 = -v_2$

$\boxed{F_{net} = 2(2N_f)\left(\frac{\epsilon_o h}{d}\right)V_{P1}v_1}$

## Comb-Drive Force Equation (2nd Pass)

- In our 1st pass, we accounted for
  - Parallel-plate capacitance between stator and rotor
- … but neglected:
  - Fringing fields
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!



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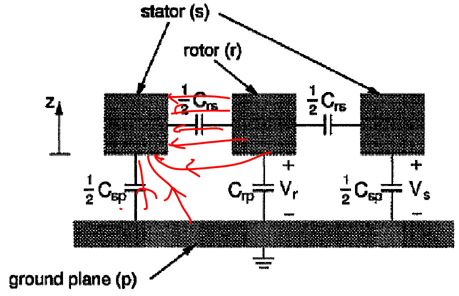
## Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$



[Gary Fedder, Ph.D., UC Berkeley, 1994]

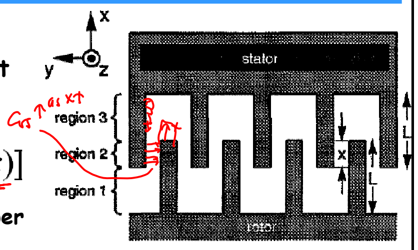
## Capacitance Expressions

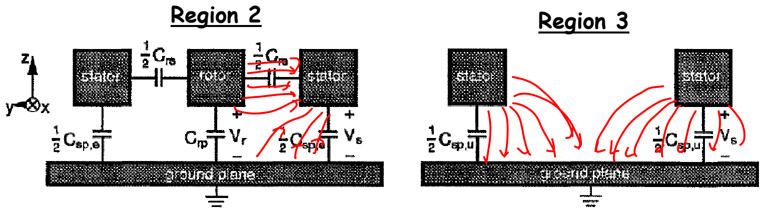
- Case: $V_r = V_P = 0V$
- $C_{sp}$ depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC''_{rs}x$$

Capacitance per unit length





[Gary Fedder, Ph.D., UC Berkeley, 1994]

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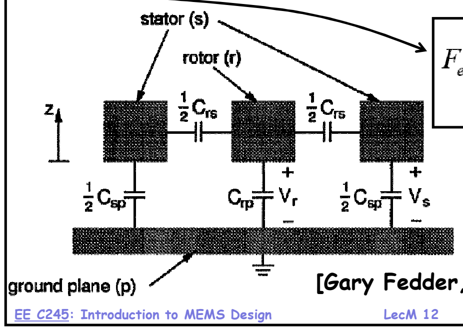


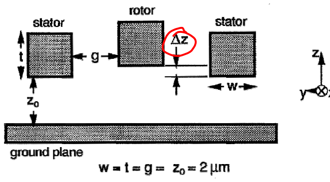
$$\boxed{F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2}$$

(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]

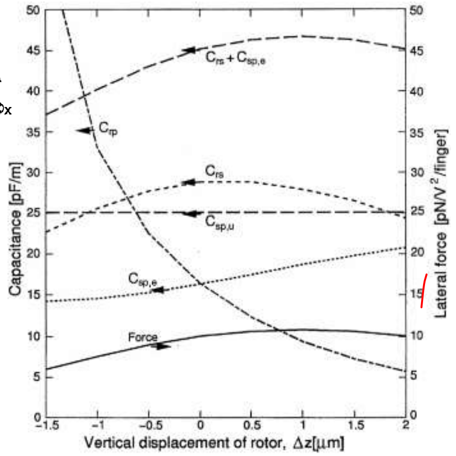
## Simulate to Get Capacitors → Force

- **Below**: 2D finite element simulation



$$\boxed{F_{e,x} = \frac{N}{2}(C'_{rs} + \underbrace{C'_{sp,e} - C'_{sp,u}}_{})V_s^2}$$

20-40% reduction of $F_{e,x}$



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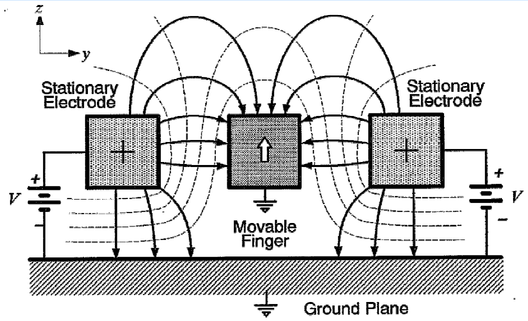
## Vertical Force (Levitation)

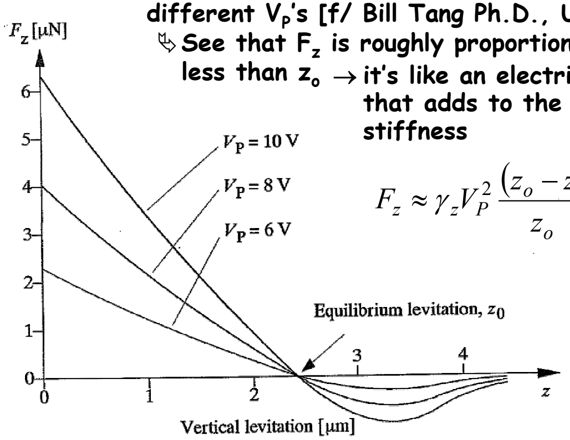


$$F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2}\frac{dC_{sp}}{dz}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dz}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dz}\left(V_s - V_r\right)^2$$

- For $V_r$ = 0V (as shown): $F_{e,z} = \frac{1}{2}Nx\left[\frac{d\left(C'_{sp,e} + C'_{rs}\right)}{dz}\right]V_s^2$

## Simulated Levitation Force

- Below: simulated vertical force $F_z$ vs. z at different $V_P$'s [f/ Bill Tang Ph.D., UCB, 1990]
  - See that $F_z$ is roughly proportional to −z for z less than $z_o$ → it's like an electrical stiffness that adds to the mechanical stiffness



$$F_z \approx \gamma_z V_P^2 \frac{\left(z_o - z\right)}{z_o} = k_e\left(z_o - z\right)$$

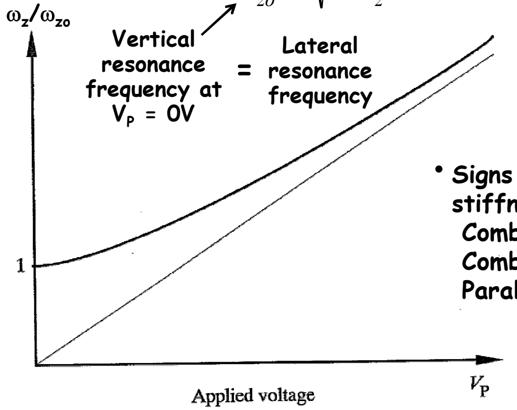
Electrical Stiffness

## Vertical Resonance Frequency

Vertical resonance frequency

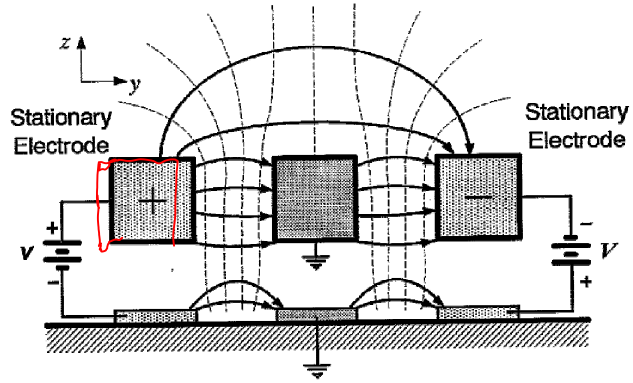
$$\frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}} \quad \text{where} \quad k_e = \left(\frac{\gamma_z}{z_o}\right)V^2$$

Vertical resonance frequency at $V_P$ = 0V = Lateral resonance frequency



- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → $k_e$ = 0
  - Comb (z-axis) → $k_e$ > 0
  - Parallel Plate → $k_e$ < 0

## Suppressing Levitation



- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

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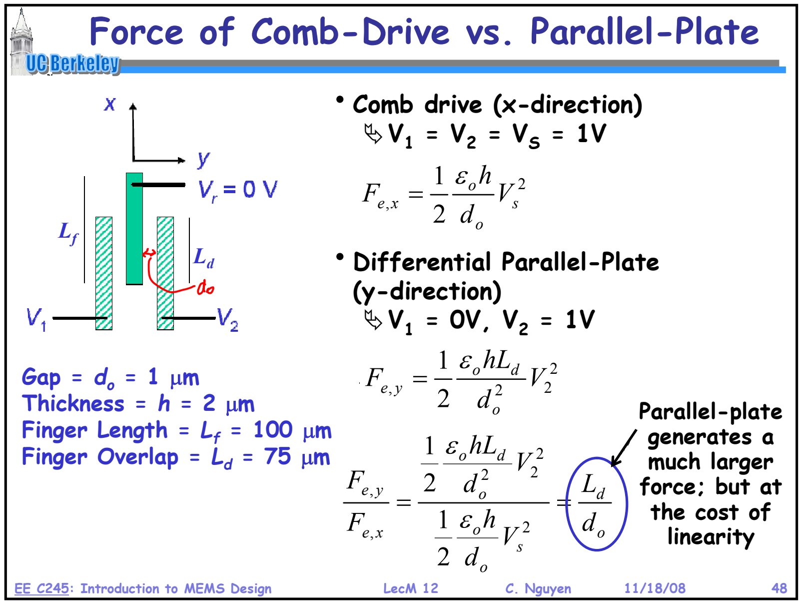

## Force of Comb-Drive vs. Parallel-Plate

**Gap = $d_o$ = 1 μm**
**Thickness = $h$ = 2 μm**
**Finger Length = $L_f$ = 100 μm**
**Finger Overlap = $L_d$ = 75 μm**

- Comb drive (x-direction)
  - ↳ $V_1 = V_2 = V_s = 1$V

$$F_{e,x} = \frac{1}{2}\frac{\varepsilon_o h}{d_o}V_s^2$$

- Differential Parallel-Plate (y-direction)
  - ↳ $V_1 = 0$V, $V_2 = 1$V

$$F_{e,y} = \frac{1}{2}\frac{\varepsilon_o h L_d}{d_o^2}V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{1}{2}\frac{\varepsilon_o h L_d}{d_o^2}V_2^2}{\frac{1}{2}\frac{\varepsilon_o h}{d_o}V_s^2} = \boxed{\frac{L_d}{d_o}}$$

Parallel-plate generates a much larger force; but at the cost of linearity

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