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## EE C247B - ME C218 Introduction to MEMS Design Spring 2016

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**Lecture Module 13: Equivalent Circuits II**

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## Lecture Outline

- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - ↪ Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - ↪ Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - ↪ Impedance & Transfer Functions

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## Input Modeling

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## Electromechanical Analogies

$F(t) = F \cos(\omega t) \rightarrow x(t) = X \cos \omega t$   
 Equation of Motion:  
 $m_{eq} \ddot{x} + c_{eq} \dot{x} + k_{eq} x = F(t)$   
 $\Rightarrow$  using phasor concepts:  
 $F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} x + c_{eq} \dot{x}$   
 $\Rightarrow$  by analogy:

$F \rightarrow N$	$m_{eq} \rightarrow l_x$	$c_{eq} \rightarrow r_x$
$\dot{x} \rightarrow \dot{i}$	$k_{eq} \rightarrow \frac{1}{c_x}$	

[Parameter Relationships in the Current Analogy]

$N(t) = V \cos \omega t \rightarrow i(t) = I \cos \omega t$   
 Impedance looking in:  
 $\frac{N}{\dot{i}} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$   
 $N = j\omega l_x \dot{i} + \frac{(V C_x)}{j\omega} \dot{i} + r_x \dot{i}$

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### Bandpass Biquad Transfer Function

$F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} x + c_{eq} \dot{x}$   
 $\Rightarrow$  Converting to full phasor form:  
 $F = (j\omega)(j\omega X) m_{eq} + \frac{k_{eq}}{j\omega} (j\omega X) + c_{eq} (j\omega X)$   
 $\frac{X}{F}(j\omega) = \frac{1}{k_{eq}} \left[ -\omega^2 \frac{m_{eq}}{k_{eq}} + 1 + j \frac{c_{eq}\omega}{k_{eq}} \right]^{-1} = \frac{1}{k_{eq}} \left[ -\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j \frac{\omega}{Q\omega_0} \right]^{-1}$   
 $\left[ \frac{k_{eq}}{m_{eq}} = \omega_0^2, Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0 \right]$

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### Force-to-Velocity Relationship

- The relationship between input voltage  $v_1$  and force  $F_{d1}$ :  

$$F_{d1} \approx -V_P \frac{\partial C_1}{\partial x} v_1$$
- When displacement  $x$  is the mechanical output variable:  

$$\frac{X(s)}{F_{d1}(s)} = \frac{1}{k} \frac{\omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$
- When velocity  $v$  is the mechanical output variable:  

$$\frac{v(s)}{F_{d1}(s)} = \frac{sX(s)}{F_{d1}(s)} = \frac{1}{k} \frac{\omega_0^2 s}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

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### Force-to-Velocity Equiv. Ckt.

- Combine the previous lumped LCR mechanical equivalent circuit with a circuit modeling the capacitive transducer  $\rightarrow$  circuit model for voltage-to-velocity

Electrical:  $I_1$ ,  $V_1$   
 Mechanical:  $U = -\dot{x}$ ,  $F_{d1}$   
 Parameters:  $r_x = b$ ,  $c_x = 1/k$

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### Equiv. Circuit for a Linear Transducer

- A transducer ...
  - converts energy from one domain (e.g., electrical) to another (e.g., mechanical)
  - has at least two ports
  - is not generally linear, but is virtually linear when operated with small signals (i.e., small displacements)

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**Equiv. Circuit for a Linear Transducer**

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Current  $\rightarrow I$   
 Voltage  $\rightarrow V$   
 Linear Two-Port Element  
 $U = -\dot{x}$  Velocity  
 Force  $\leftarrow F$

Electrical | Mechanical

- For physical consistency, use a transformer equivalent circuit to model the energy conversion from the electrical domain to mechanical domain

Flow  $\rightarrow f_1$   
 Effort  $\rightarrow e_1$   
 $1:\eta$   
 Flow  $\leftarrow f_2$   
 Effort  $\rightarrow e_2$

Describing Matrix

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

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**Electromechanical Equivalent Circuit**

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- $e_2 = F_{d1}$ ,  $e_1 = v_1$ , just need  $\eta_1$ :
- From the matrix:  $e_2 = \eta_1 e_1$

$$F_{d1} \approx -V_P \frac{\partial C_1}{\partial x} v_1 \rightarrow \eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|$$

Electrode 1  
 $d_1$   
 $F_{d1}$   
 $C_1$   
 $V_P$   
 $i_1$   
 $v_1$   
 $x$   
 $b$   
 $k$   
 $m$   
 Current  $\rightarrow I_1$   
 Voltage  $\rightarrow V_1$   
 $1:\eta_1$   
 Velocity  $\leftarrow U = -\dot{x}$   
 $I_x = m$   
 $r_x = b$   
 $c_x = 1/k$   
 Force  $\leftarrow F_{d1}$

Electrical | Mechanical

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