Lecture 22: Comb Drive & Equivalent Circuits II

- Announcements:
- Module 13 on Equivalent Circuits II online
- HW#6 online soon
- Project slide #2 due Friday, April 15
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- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- -------------------------------------
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
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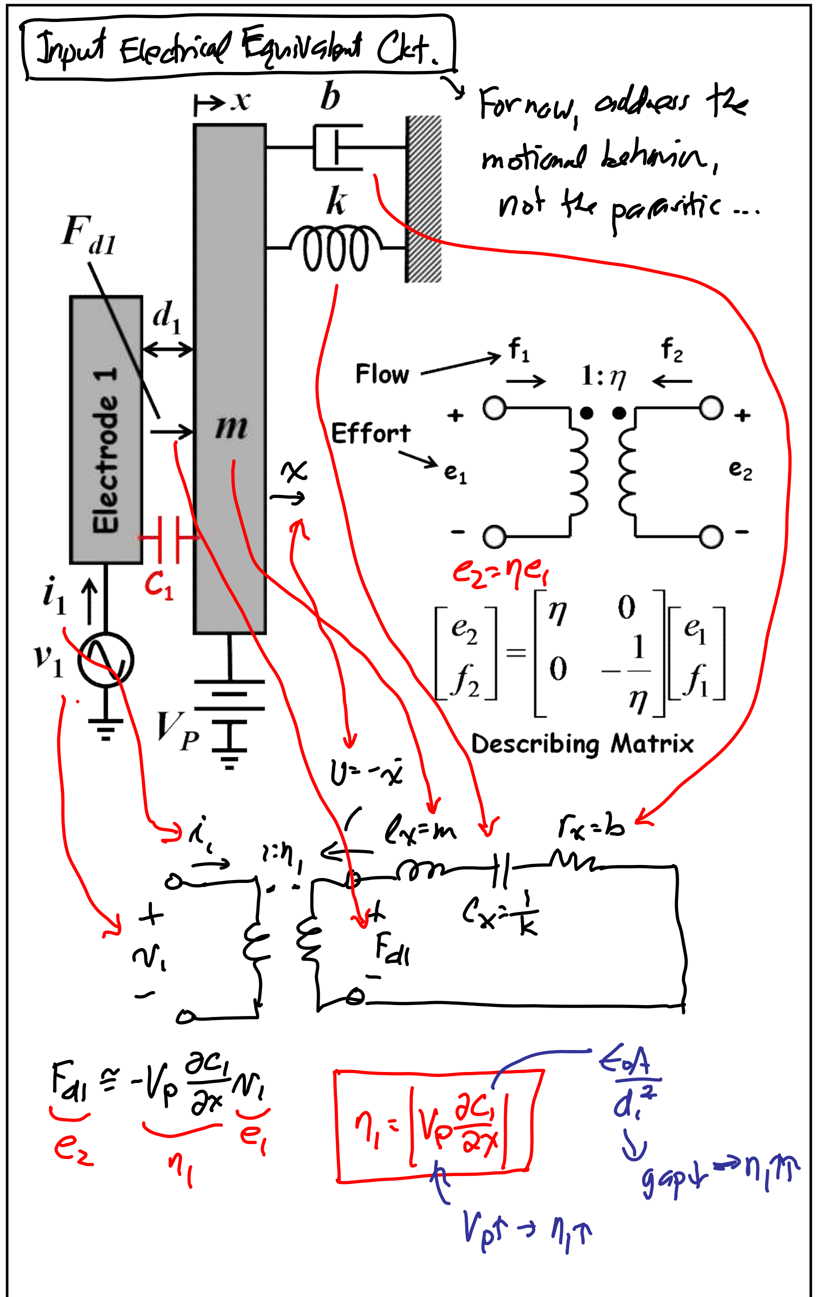
- Last Time: Going through Module 12 slides 26-35



$e_2 = \eta e_1$

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

$v = -\dot{x}$, $l_x = m$, $r_x = b$, $c_x = \frac{1}{k}$

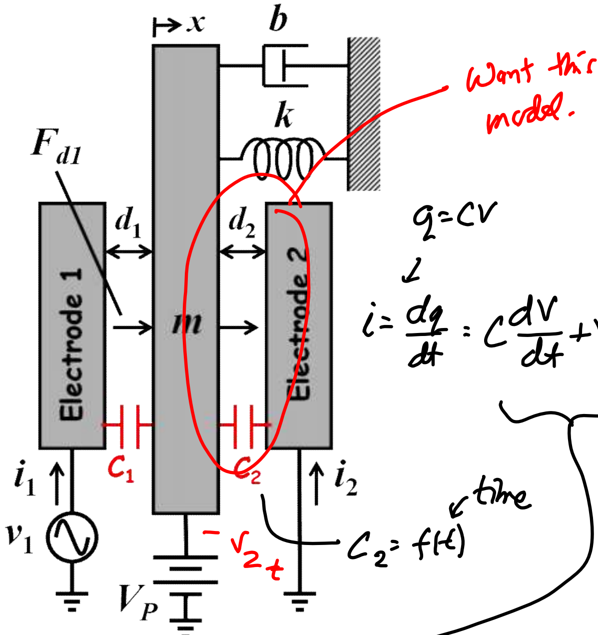
$$\underbrace{F_{d1}}_{e_2} \cong \underbrace{-V_P \frac{\partial C_1}{\partial x}}_{\eta_1} \underbrace{v_1}_{e_1}$$

$$\boxed{\eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|}$$

$\frac{\partial C_1}{\partial x} = \frac{\varepsilon_o A}{d_1^2}$, gap$\downarrow \Rightarrow \eta_1 \Uparrow$

$V_P \uparrow \rightarrow \eta_1 \uparrow$

Output Current Into Ground



$q = CV$

$i = \frac{dq}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$

$C_2 = f(t)$ ← time

$i_2 = \cancel{C_2(x,t)\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$

$[V_2(t) = -V_P] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$

In phasor form: $I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$

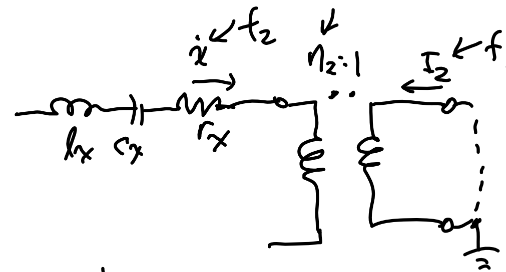
$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

↳ motional current

$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C_2}{\partial x}\dot{x}$

90° phase lag (+) (+) → $I_2 = (-)$ when $x > (+)$

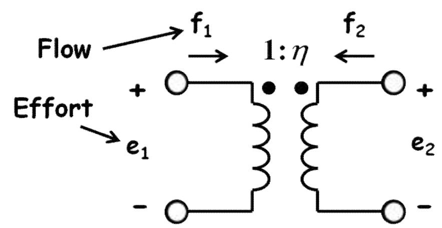
velocity



$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$

$[f_1 = I_2,\ f_2 = \dot{x}] \Rightarrow I_2 = -\eta_2\dot{x}$

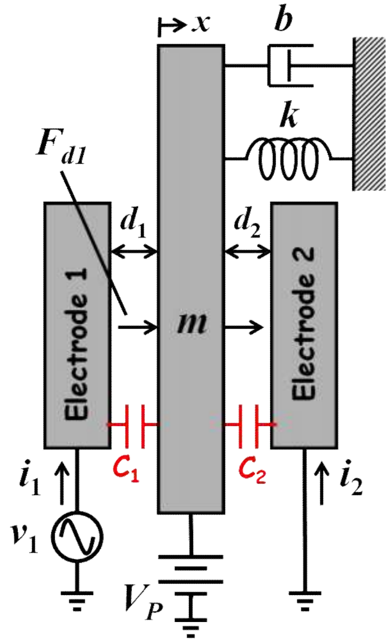
$$\therefore \boxed{\eta_2 = \left|V_P\frac{\partial C_2}{\partial x}\right|}$$



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

Input Current Expression



Get $I_1(j\omega)$:

$$i_1(t) = C_1(x,t)\frac{dV_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

$$\left[V_1(t) = \underset{\substack{\uparrow \\ f(t)}}{v_1} - V_P\right] \Rightarrow i_1 = C_1\frac{dv_1}{dt} + [v_1 - V_P]\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

due to mass motion

$$\therefore I_1(j\omega) = \underbrace{j\omega C_1 V_1}_{\text{Feedthrough Current}} + \cancel{j\omega v_1}\frac{\partial C_1}{\partial x}X - \underbrace{j\omega V_P \frac{\partial C_1}{\partial x}X}_{\text{Motional Current}}$$