Lecture 23: Mechanical Circuit Analysis

- Announcements:
- Module 13 on Equivalent Circuits II online
- HW#6 online and due Friday, April 22
- Module 14 on Sensing Circuits online
- Module 15 on Gyros, Noise, & MDS online
- Project slide #2 due Friday, April 15
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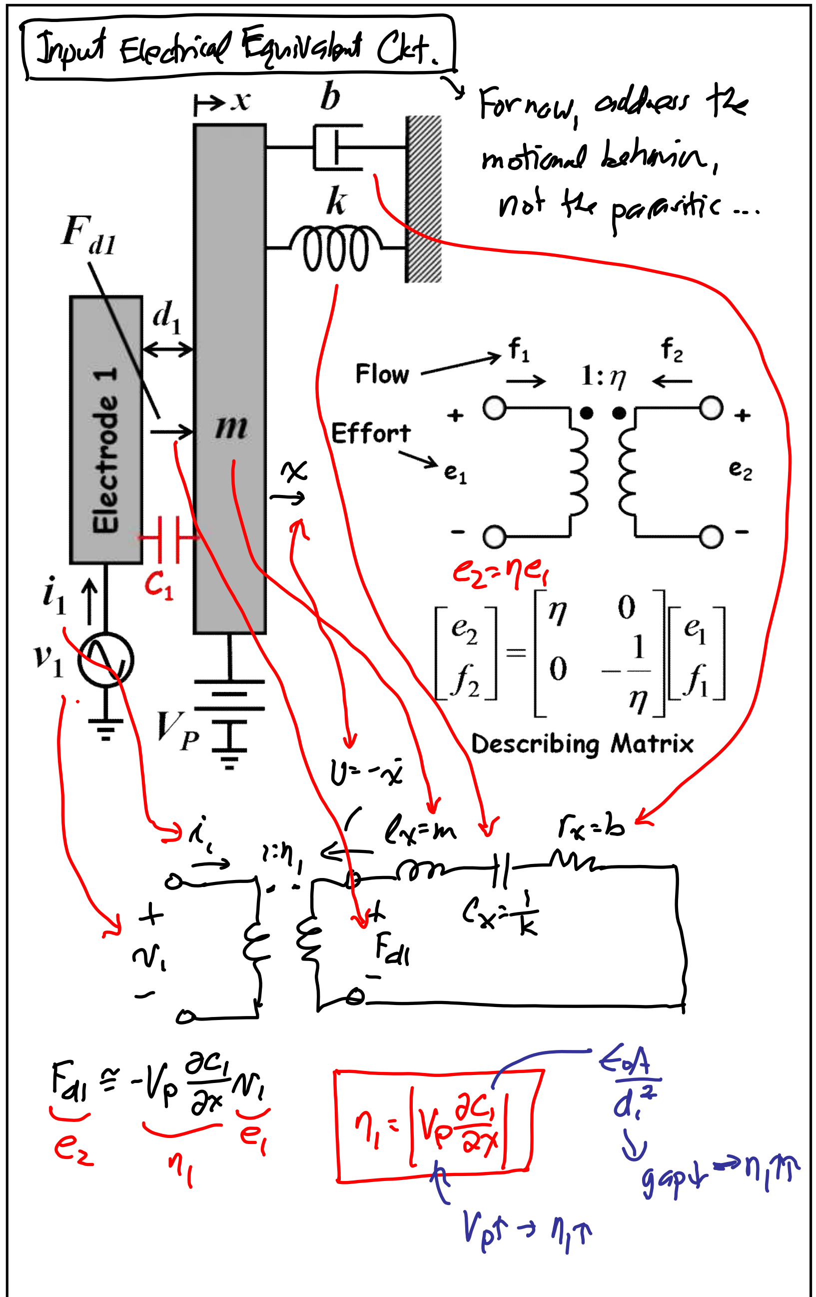
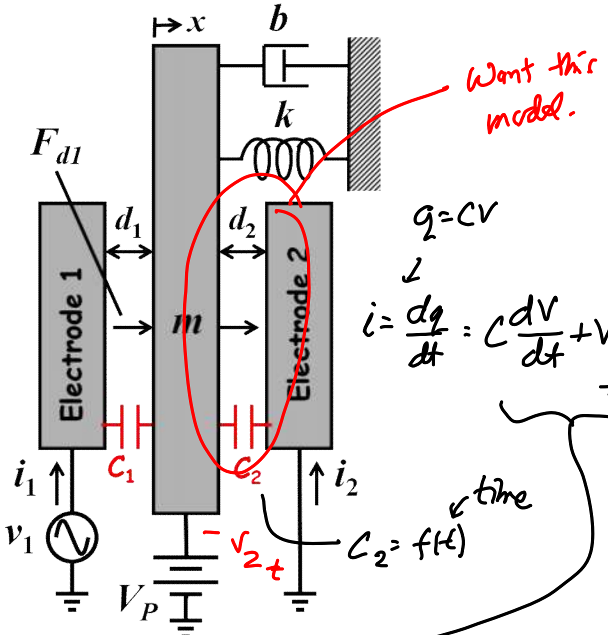
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- ------------------------------------
- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
- ------------------------------------
- Last Time: Determining currents into electrodes



Output Current Into Ground



$q = CV$

$$i = \frac{dq}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

$$i_2 = \cancel{C_2(x,t)\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$$

$$[V_2(t) = -V_P] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

In phasor form:

$$I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$$

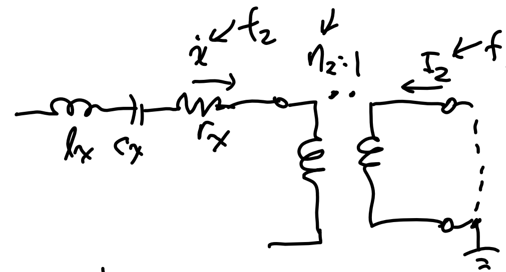
$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

↳ motional current

$$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C_2}{\partial x}\dot{x}$$

90° phase lag (+) (+) → $I_2 = (-)$ when $x = (+)$

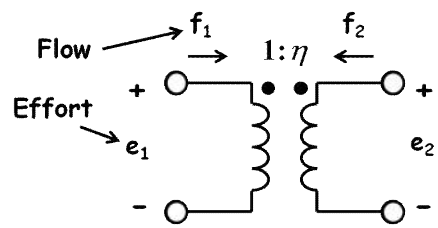
velocity



$$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$$

$$[f_1 = I_2, f_2 = \dot{x}] \Rightarrow I_2 = -\eta_2\dot{x}$$

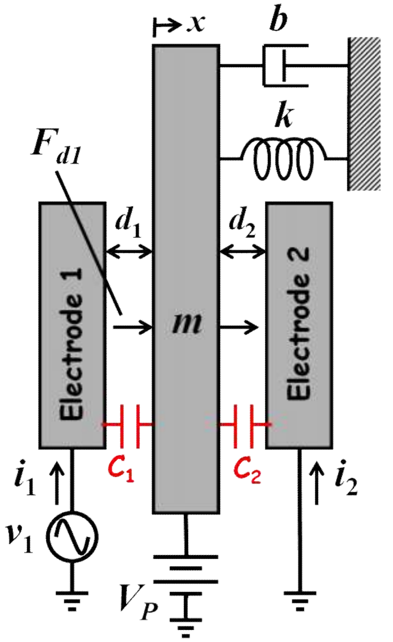
$$\therefore \boxed{\eta_2 = \left|V_P\frac{\partial C_2}{\partial x}\right|}$$



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

Input Current Expression



Get $I_1(j\omega)$:

$$i_1(t) = C_1(x,t)\frac{dV_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

$$[V_1(t) = \underset{f(t)}{v_1} - V_P] \Rightarrow i_1 = C_1\frac{dv_1}{dt} + [v_1 - V_P]\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

due to mass motion

$$\therefore I_1(j\omega) = \underbrace{j\omega C_1 V_1}_{\text{Feedthrough Current}} + \cancel{j\omega v_1 \frac{\partial C_1}{\partial x} X}^{\,0} - \underbrace{j\omega V_P \frac{\partial C_1}{\partial x} X}_{\text{Motional Current}}$$

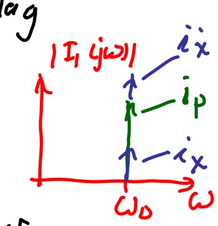
@DC: $x = \frac{F_{d1}}{k} = -\frac{1}{k}V_P\frac{\partial C_1}{\partial x}v_1$

@resonance: $x = \frac{QF_{d1}}{jk} = -\frac{Q}{jk}V_P\frac{\partial C_1}{\partial x}v_1 = \underline{X}$

90° phase lag

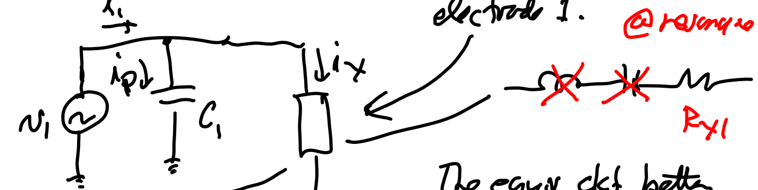
Thus: (@ resonance) $\rightarrow \omega_0$

$$I_1(j\omega_0) = \underbrace{j\omega_0 C_1 V_1}_{= i_p} + \underbrace{\omega_0\frac{Q}{k}\eta_{e1}^2 v_1}_{= i_x}$$



90° phase-shifted from v_1 → This is a capacitor in shunt w/ the input!

In phase w/ v_1. → This is an effective resistance @ ω_0 seen looking into electrode 1.



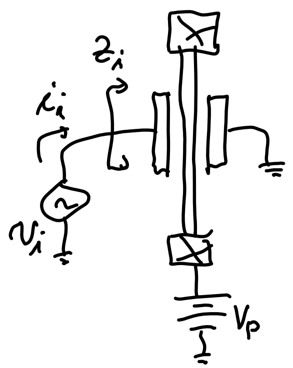
The equiv. ckt. better get this right!

Motional Resistance:

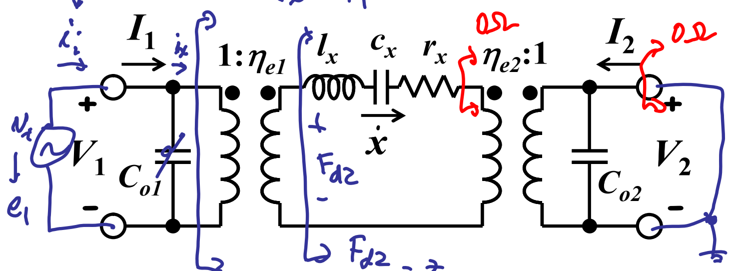
$$R_{x1} = \frac{v_1}{i_1} = \frac{k}{\omega_0 Q\eta_{e1}^2} = \frac{m\omega_0}{Q\eta_{e1}^2} = \boxed{\frac{b}{\eta_{e1}^2} = R_{x1}}$$

- Look at slide 16 in Module 13

Input Impedance Into Port 1



$z_i' = \frac{v_i}{i_x} = \frac{e_1}{f_1}$



$\frac{F_{d2}}{(-\dot{x}_2)} = z_x$

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{array}{l} e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \\ f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2 \end{array}$$

$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(-\frac{1}{\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \rightarrow \frac{v_i}{i_x} = z_i' = -\frac{1}{\eta_{e1}^2}\frac{F_{d2}}{(-\dot{x}_2)}$$

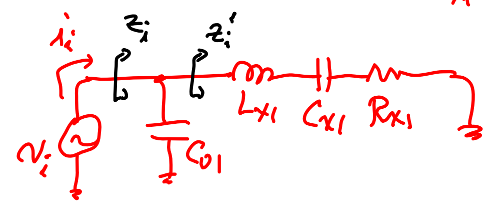
$$= \frac{1}{\eta_{e1}^2} z_x$$

$$z_x = j\omega l_x + \frac{1}{j\omega c_x} + r_x$$

$$z_i' = \frac{1}{\eta_{e1}^2} z_x = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

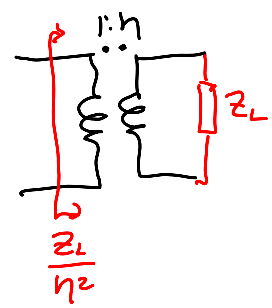
$$= j\omega \underbrace{\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \frac{1}{j\omega \underbrace{(\eta_{e1}^2 c_x)}_{C_{x1}}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$

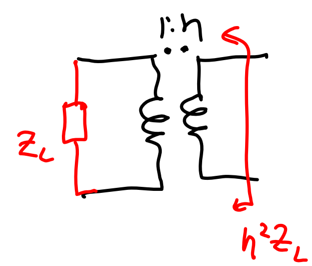
$r_x = b$



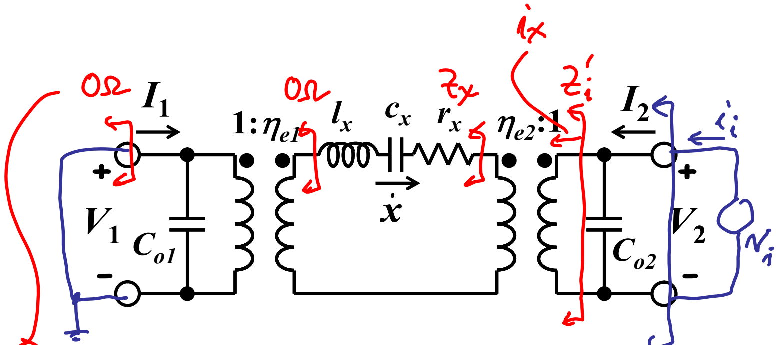
Purely Electrical Equiv. Ckt.

Xform Inspection Analysis

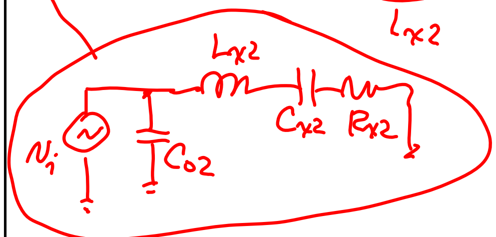




Input Impedance Into Port 2

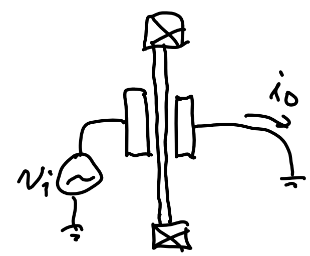


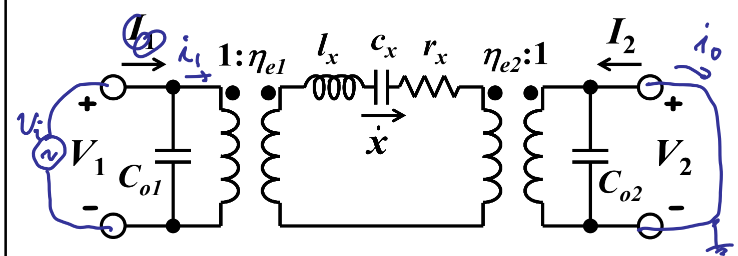
$$z_i' = \frac{v_i}{i_x} = \frac{z_x}{\eta_{e2}^2} = j\omega\underbrace{\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \frac{1}{j\omega\underbrace{(\eta_{e2}^2 c_x)}_{C_{x2}}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$



Port 1 to Port 2 Transconductance

$G = \frac{i_o}{v_i}$





$\dot{x} = \frac{1}{\eta_{e1}} i_1$

$i_o = \eta_{e2}\dot{x}$

$$\rightarrow i_o = \frac{\eta_{e2}}{\eta_{e1}} i_1 = \frac{\eta_{e2}}{\eta_{e1}}\left(\frac{v_i}{z_i}\right) = \frac{\eta_{e2}}{\eta_{e1}}\left[\eta_{e1}^2 \frac{1}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}\right]$$

$$\therefore G = \frac{i_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega c_x} + r_x} = \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}\right]^{-1}$$

*

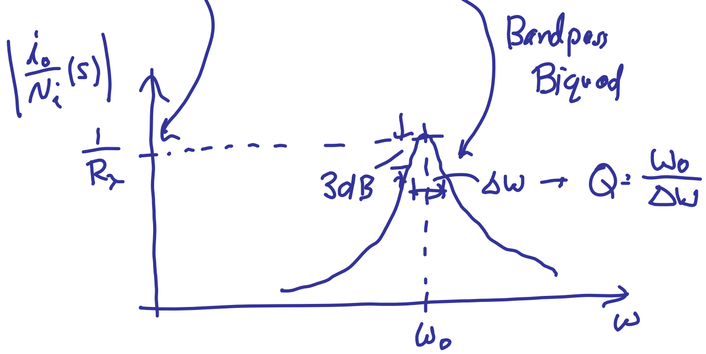
$L_{x12} = \frac{l_x}{\eta_{e1}\eta_{e2}}, \quad C_{x12} = \eta_{e1}\eta_{e2} C_x, \quad R_{x12} = \frac{r_x}{\eta_{e1}\eta_{e2}}$

$\Rightarrow$ separate freq. response & magnitude

$$* \quad \frac{i_o}{v_i}(s) = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{s\left(\frac{1}{L_x}\right)}{s^2 + \frac{1}{L_x C_x} + s\left(\frac{R_x}{L_x}\right)}$$

$$\left[\frac{1}{L_x C_x} = \omega_0^2, \quad Q = \frac{\omega_0 L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_0}{Q} \right]$$

$$\frac{i_o}{v_i}(s) = \underbrace{\frac{1}{R_x}}_{\text{Gain Term}} \; \underbrace{\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}}_{\text{Freq. Shaping Term}} = \frac{1}{R_x} H(s)$$



- Now, go through slides 21-22 in Module 13
- Then, start gyroscopes by going through slides 1-6 in Module 15