Lecture 24: Gyroscopes & Sensing Circuits

- Announcements:
- HW#6 online and due Friday, April 22
- Module 14 on Sensing Circuits online
- Module 15 on Gyros, Noise, & MDS online
- Project slide #3 due Friday, April 29
- I will be at the EECS Retreat this coming Thursday, so the lecture will be recorded and on video
  - Please watch the Thursday Lecture video before class on Tuesday, next week
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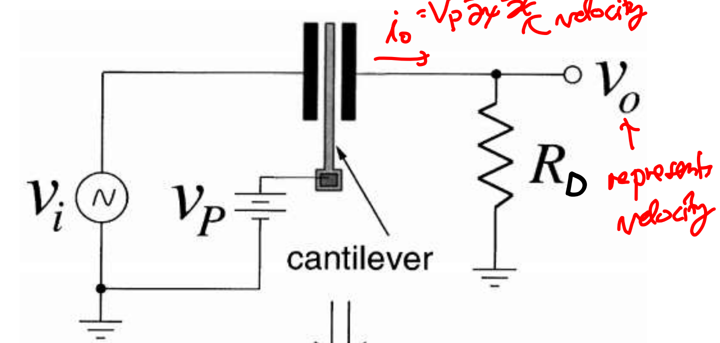
- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
- ------------------------------------
- Last Time:
- Started gyroscopes by going through slides 1-6 in Module 15
- Now, continue with this, going through slide 16

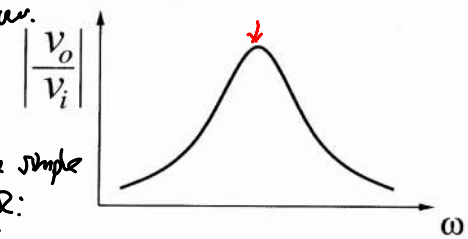
Velocity-to-Voltage Conversion

$i_o = V_P \frac{\partial C}{\partial x}\frac{\partial x}{\partial t}$ ← velocity

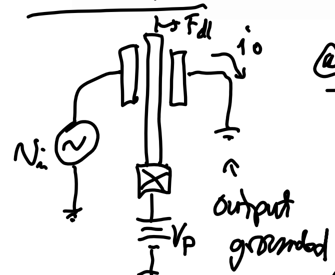


V_o ↑ represents velocity

Neglect C_o for now.



First, review the simple case w/ $R_D = 0\Omega$:



@ resonance: $\frac{\dot{x}}{F_{d1}}(j\omega_o) = \frac{j\omega_o Q}{jk} = \frac{\omega_o Q}{k}$

$\frac{\dot{x}}{F_{d1}}(s) = \frac{\omega_o Q}{k} H(s)$ ← at all freqs.

$$\left[H(s) = \frac{s(\omega_o/Q)}{s^2 + s(\omega_o/Q) + \omega_o^2} \right]$$

$\{F_{d1} = \eta_{e1} v_i\}$

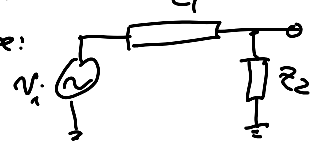
$$\frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_o Q}{k} H(s)$$

$$(i_o = \eta_{e2}\dot{x}) \Rightarrow \frac{i_o}{v_i}(s) = \eta_{e1}\eta_{e2}\frac{\omega_o Q}{k} H(s) = \frac{\eta_{e1}\eta_{e2} Q}{m\omega_o} H(s)$$

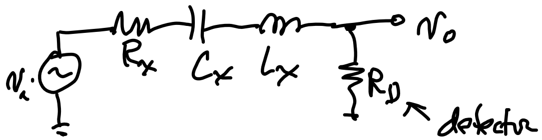
$\frac{1}{R_{x12}}$

Now, include finite R_D:

⇒ Really have:



$$\frac{v_o}{v_i} = \frac{Z_2}{Z_1+Z_2}$$ (voltage divider)



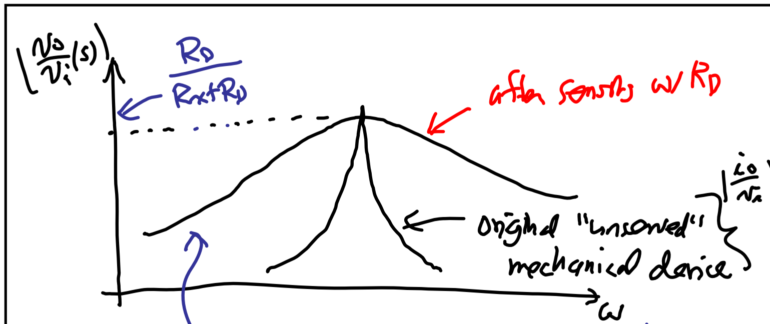
$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \ldots \text{math} \ldots$$

$$= \underbrace{\frac{R_D}{R_x+R_D}}_{\text{Gain Term}} \underbrace{\frac{s\left(\frac{R_x+R_D}{L_x}\right)}{s^2 + s\left(\frac{R_x+R_D}{L_x}\right) + \frac{1}{L_x C_x}}}_{\text{Freq. Shaping Term}}$$

$$\left[Q = \frac{\omega_o L_x}{R_x} \rightarrow Q' = \frac{\omega_o L_x}{R_x+R_D} \rightarrow \frac{R_x+R_D}{L_x} = \frac{\omega_o}{Q'} \right]$$

$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D} \frac{s(\omega_o/Q')}{s^2 + s(\omega_o/Q') + \omega_o^2}$$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D} \cdot H(s, Q'), \quad Q' = Q\left(\frac{R_x}{R_x+R_D}\right)}$$



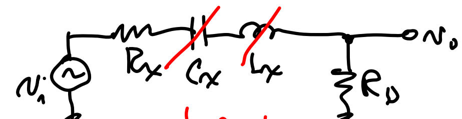
$\left|\frac{v_o}{v_i}(s)\right| \Rightarrow Q'$ lower than Q ← large

$Q\left(\frac{R_x}{R_x+R_D}\right) = Q'$ ← small

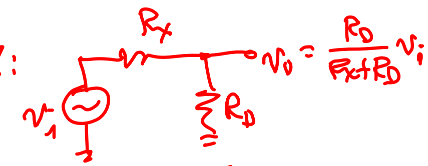
$\frac{R_x}{R_x+R_D} < 1$

Big problem!

Analysis @ Resonance:



$\frac{1}{sC_x} = sL_x$ cancel:



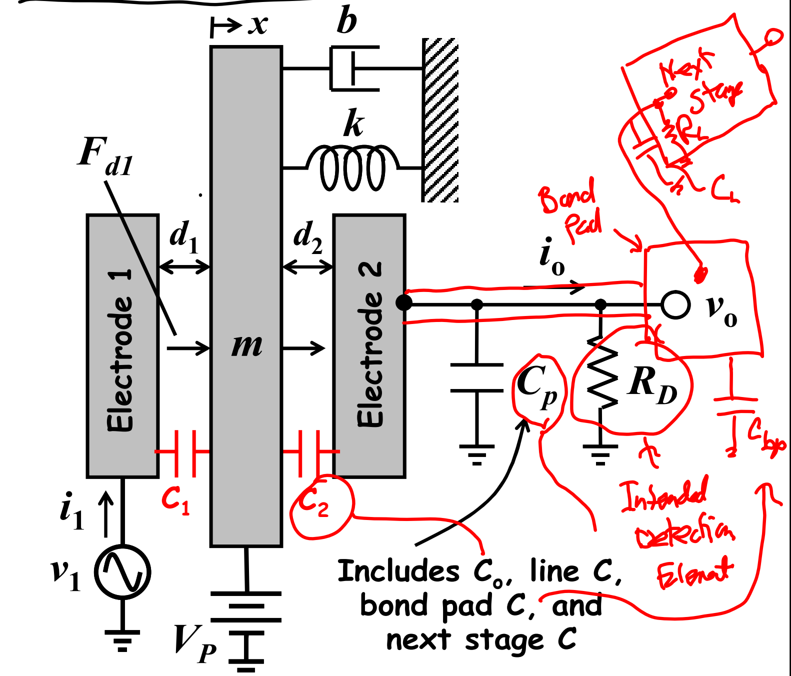
@ resonance

Convert from resonance to general freqs.: $\times\, H(s, Q')$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D} H(s, Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)}$$

very fast, by inspection!

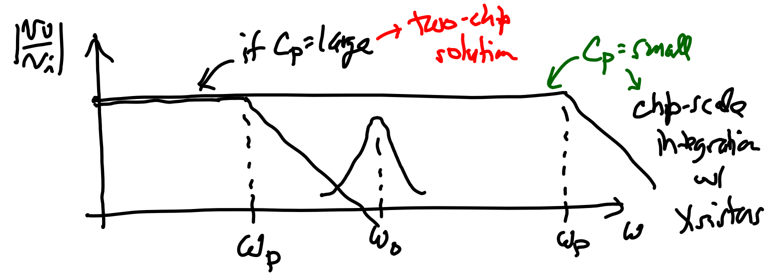
The Problem is Actually Bigger!



Now, we get:

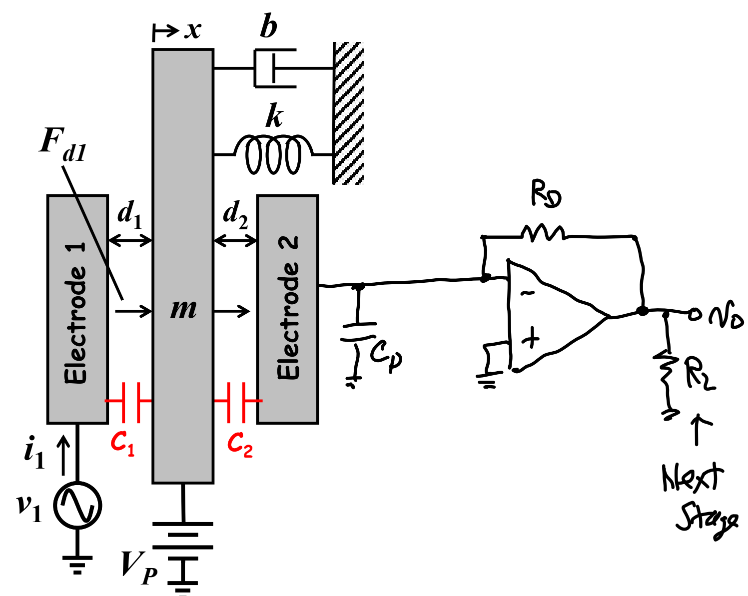
$$\frac{v_o}{v_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1+\frac{s}{\omega_p}} \cdot H(s, \omega_o', Q')$$

$$\omega_p = \frac{1}{(R_x \| R_D) C_p}$$



Problems w/ Purely Resistive Detection

1. Need large R_D for large gain... but...
2. $R_D \uparrow \rightarrow Q \downarrow$
3. $R_D \uparrow \rightarrow \omega_p = \frac{1}{R_D C_p} \downarrow \rightarrow$ get undesirable LPF cut-off
4. Load $R_L \rightarrow$ affects gain!



Ideal Op Amp Laws

1. $R_i = \infty \rightarrow i_- = i_+ = 0$
2. $R_o = 0$
3. Gain $= A_o = \infty$

neg. FB $\rightarrow v_+ = v_-$