**Position-to-Voltage Conversion**

To sense position (i.e., displacement), use a capacitive load. Again, have port input I/O symmetry:

\[
\frac{V_p}{V_o} = \frac{C_D}{C_D + C_L} = \frac{V_p}{V_o} = \frac{C_D}{C_D + C_L}
\]

\[
\frac{C_D}{C_D + C_L} = \frac{1}{1 + \frac{C_L}{C_D}}
\]

\[
\frac{1}{1 + \frac{C_L}{C_D}} = \frac{C_D}{C_D + C_L}
\]

\[
\frac{C_D}{C_D + C_L} = \frac{1}{1 + \frac{C_L}{C_D}}
\]

- DC Gain: \(\frac{V_p}{V_o} = \frac{C_D}{C_D + C_L}\)
- AC Gain: \(\frac{V_p}{V_o} = \frac{C_D}{C_D + C_L}\)

\[
\text{To maximize gain, } C_L = 0.\text{ (raw mininum C_D)}
\]

**Velocity Sensing Circuits**

- To convert velocity to a voltage, use a resistive load.

\[
\text{Voltage-to-Voltage Conversion}
\]

\[
V_p = \frac{R_o}{R_p + R_o} \times V_o
\]

Where \(V_p\) represents the velocity.
Problems With Purely Resistive Sensing

In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input $R_i$ of these circuits can load $R_D$.

Problem: need a sensing circuit that is immune to parasitics or loading.

Solution: use op amps.

The TransR Amplifier Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance $C_p$ and $R_i$.
- The zero output resistance of the (ideal) op amp can drive virtually anything $\Rightarrow V_{out} = \frac{R_s}{R_s + R_D} N_0$.

Position Sensing Circuits
Problems With Pure-C Position Sensing

To sense position (i.e., displacement), use a capacitive load

\[ V_0 = \frac{C_0V_i}{1 + sC_0C_p} \]

Integration yields displacement:

\[ \Delta x = \frac{1}{C_0} \int V_0 \, ds \]

To maximize gain, minimize \( C_0 \).

Problem: parasitic capacitance

\[ C_B \rightarrow C_1 + C_2 + C_p \]

\[ \Delta G \triangleq \frac{C_0(C_1C_2 + C_p)}{1 + sC_0(C_1 + C_2 + C_p)} \]

Remedy: Suppress \( C_B \) via use of op amps.

Differential Position Sensing

- Tethers with fixed ends
- Sensing beam in tension

The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance \( C_p \)

The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance \( C_p \)

Differential Position Sensing

Example: ADXL-50

- Tethers with fixed ends
- Proof mass
- Sense finger
- Capacitive divider

\[ V_o = \frac{V_p - V_b}{2} \]

\[ V_o = \frac{V_p - 2V_b}{2} \]

Parasitic capacitance

\[ V_o = \frac{V_p - C_{ch2}V_b}{C_{ch1} + C_{ch2}} \]

\[ V_o = \frac{V_p - C_{ch2}V_b}{C_{ch1} + C_{ch2}} \]

Air bearing, \( C_p \) reduces gain.

\[ C_{ch1} \text{ use op amps!} \]
Buffer-Bootstrapped Position Sensing

- Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across $C_p$
  - It's effectively not there!

$$V_0 = -V_P$$

Unity Gain Buffer

Includes capacitance from interconnects, bond pads, and $C_{gs}$ of the op amp

$C_{gd} = $ gate-to-drain capacitance of the input MOS transistor

$C_p$ includes capacitance from interconnects, bond pads, and $C_{gs}$ of the op amp

Effect of Finite Op Amp Gain

Total ADXL-50 Sense $C \sim 100fF$

Integrator-Based Diff. Position Sensing

$\int I_0 = \int i_x + \int i_z = \int i_d = \int (C_p - C_g)\, ds$

$= V_P \cdot s(C_f + C)$

$\Rightarrow \frac{V_0}{V_P} = \frac{C_f}{C_p}$

$R_2 \gg \frac{1}{sC_f}$

$R_2 \gg \frac{1}{sC_f}$

$\Rightarrow \text{A seemingly perfect differential sensor/amplifier output!... but only when the op amp is ideal...}$