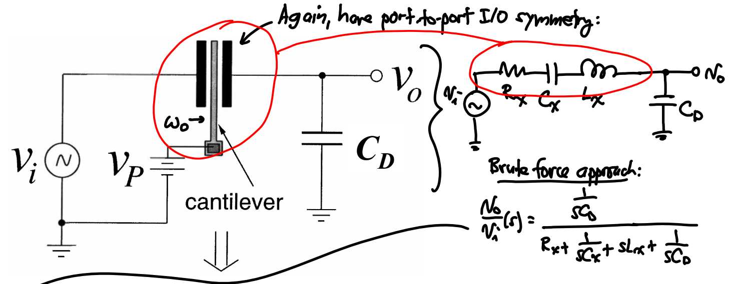
Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:



Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

$$\frac{v_o}{v_i}(s) = \frac{\frac{\cancel{s}C_x}{\cancel{s}C_D}}{sR_xC_x + 1 + s^2L_xC_x + \frac{\cancel{s}C_x}{\cancel{s}C_D}} = \frac{C_x/C_D}{1+C_x/C_D}\,\frac{1}{1 + \frac{sR_xC_x}{1+C_x/C_D} + s^2\frac{L_xC_x}{1+C_x/C_D}}$$

$$= \frac{C_x/C_D}{1+C_x/C_D}\,\frac{\frac{1+C_x/C_D}{L_xC_x}}{s^2 + s\left(\frac{R_x}{L_x}\right) + \frac{(1+C_x/C_D)}{L_xC_x}}$$

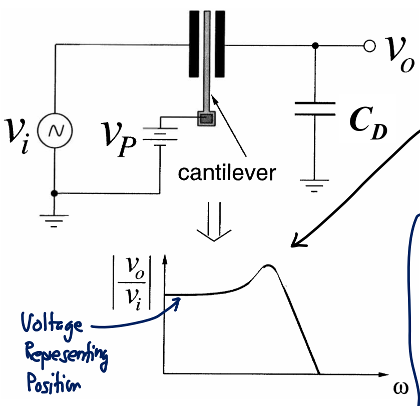
$$\left[\omega_o^2 = \frac{1}{L_xC_x} \rightarrow (\omega_o')^2 = \omega_o^2\left(1 + C_x/C_D\right)\right]$$

$$\left[Q' = \frac{\omega_o' L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o'}{Q'},\ Q' = Q\sqrt{1 + C_x/C_D}\right]$$

over ↷

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load



$$\frac{v_o}{v_i}(s) = \underbrace{\frac{C_x/C_D}{1+C_x/C_D}}_{\text{DC Gain Term}}\ \underbrace{\frac{(\omega_o')^2}{s^2 + \left(\frac{\omega_o'}{Q'}\right)s + (\omega_o')^2}}_{\text{Low-Pass Biquad}}$$

To maximize gain → 1, need $C_D \ll C_x$.
(must minimize C_D)

Note: Can use similar short-cut to the R case.

① Get DC response → C's dominate.

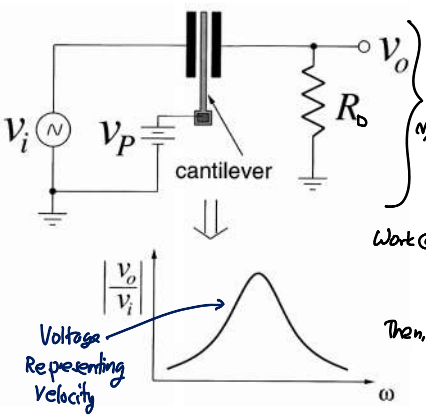
② Then:

$$\frac{v_o}{v_i}(s) = (\text{DC Gain})\cdot\frac{1}{s}\cdot Ⓗ(s, \omega_o', Q')\cdot\omega_o' Q'$$

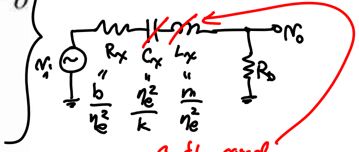
Velocity Sensing Circuits

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



Since this structure has completely symmetrical I/O ports:



$$R_x = \frac{b}{\eta_e^2},\quad C_x = \frac{\eta_e^2}{k},\quad L_x = \frac{m}{\eta_e^2}$$

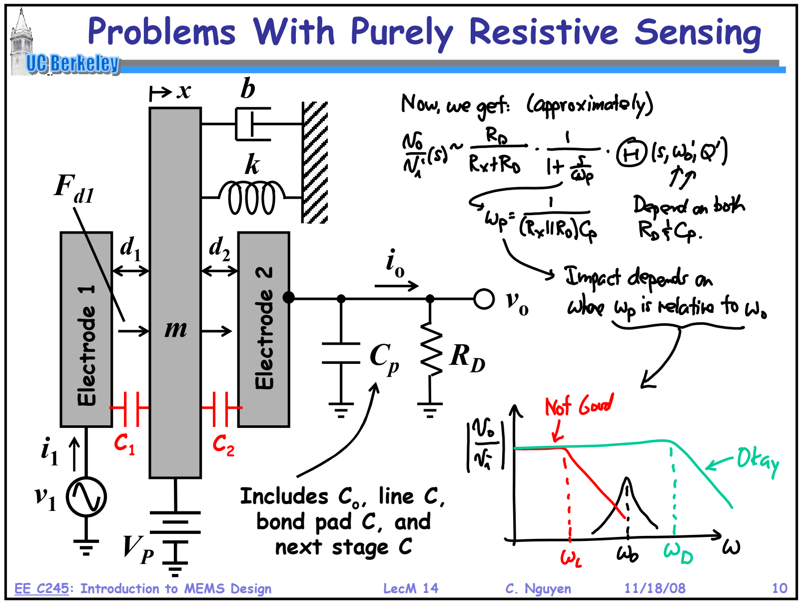
C_x & L_x cancel

Work @ resonance: (to simplify the analysis)

$$\frac{v_o}{v_i} = \frac{R_D}{R_x + R_D}\ (\text{@ resonance})$$

Then, generalize to off resonance:

$$\frac{v_o}{v_i} = \frac{R_D}{R_x + R_D}Ⓗ(s, Q'),\ \text{where } Q' = Q\left(\frac{R_x}{R_x + R_D}\right)$$



Problems With Purely Resistive Sensing

Now, we get: (approximately)

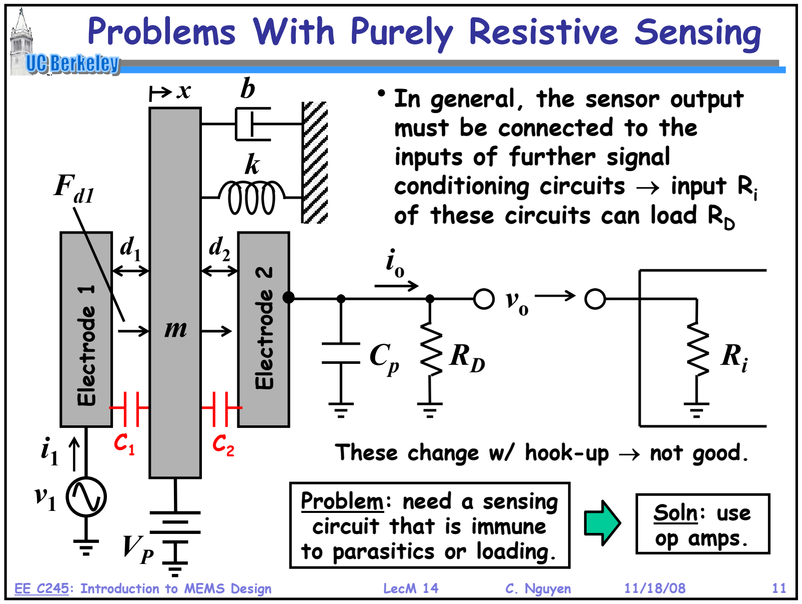
$$\frac{v_o}{v_i}(s) \sim \frac{R_D}{R_x+R_D}\cdot\frac{1}{1+\frac{s}{\omega_p}}\cdot \textcircled{H}(s,\omega_o',Q')$$

$$\omega_p = \frac{1}{(R_x \| R_D)C_p}$$

Depend on both R_D & C_p.

Impact depends on where ω_p is relative to ω_o

Includes C_o, line C, bond pad C, and next stage C



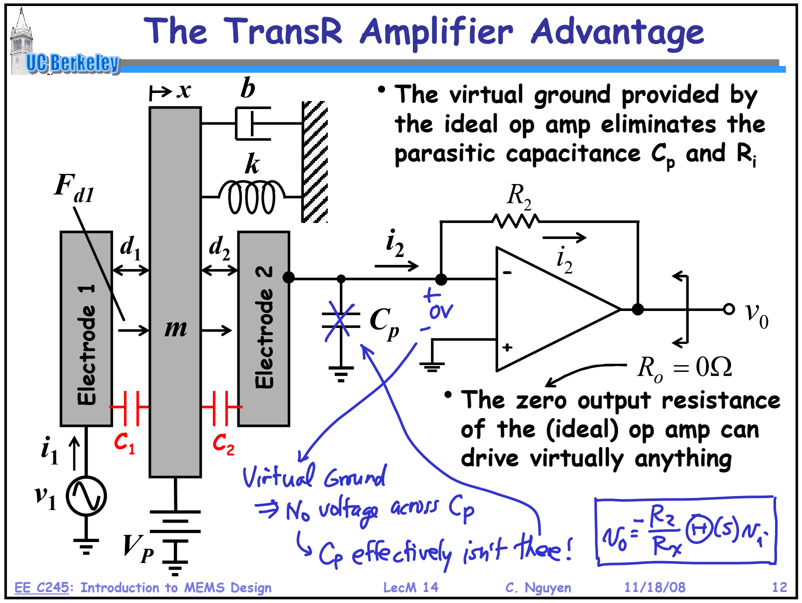
Problems With Purely Resistive Sensing

- In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input R_i of these circuits can load R_D

These change w/ hook-up → not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

Soln: use op amps.



The TransR Amplifier Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p and R_i

$R_o = 0\Omega$

- The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground
⇒ No voltage across C_p
↳ C_p effectively isn't there!

$$\boxed{v_0 = -\frac{R_2}{R_x}\textcircled{H}(s)v_i}$$

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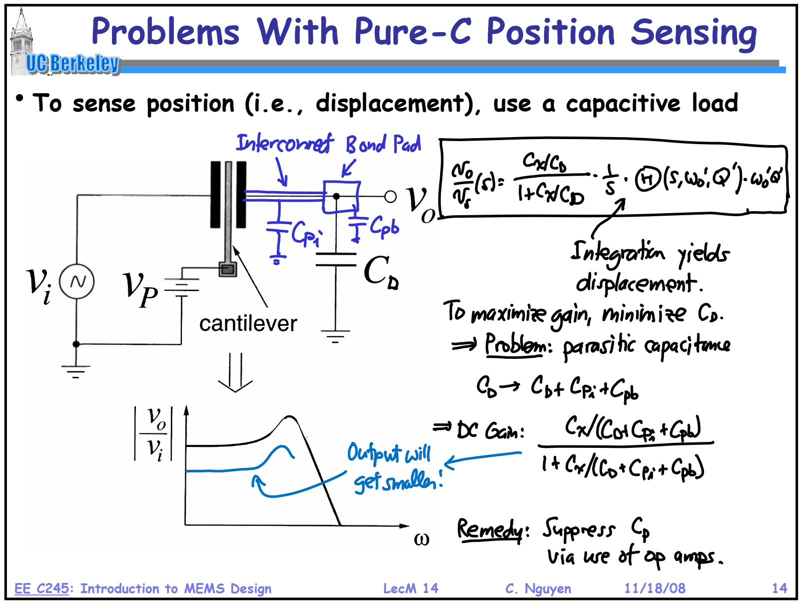
Position Sensing Circuits



Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load

$$\boxed{\frac{v_o}{v_s}(s)=\frac{C_x/C_D}{1+C_x/C_D}\cdot\frac{1}{s}\cdot Ⓗ(s,\omega_o',Q')\cdot\omega_o'Q'}$$

Integration yields displacement.

To maximize gain, minimize C_D.

⇒ Problem: parasitic capacitance

$C_D \rightarrow C_D + C_{pi} + C_{pb}$

⇒ DC Gain: $\dfrac{C_x/(C_D+C_{pi}+C_{pb})}{1+C_x/(C_D+C_{pi}+C_{pb})}$

Remedy: Suppress C_p via use of op amps.

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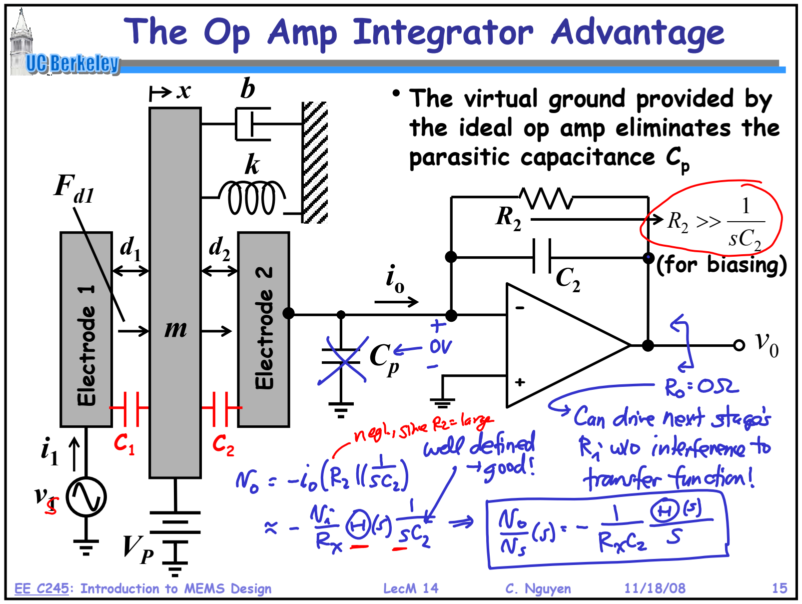

The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p

$R_2 >> \frac{1}{sC_2}$ (for biasing)

$R_o = 0\Omega$

Can drive next stage's R_i w/o interference to transfer function!

$v_o = -i_o\left(R_2 \| \frac{1}{sC_2}\right)$

negl. since R_2 = large

well defined → good!

$\approx -\frac{v_i}{R_x}Ⓗ(s)\frac{1}{sC_2} \Rightarrow \boxed{\frac{v_o}{v_s}(s) = -\frac{1}{R_xC_2}\frac{Ⓗ(s)}{s}}$

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Differential Position Sensing

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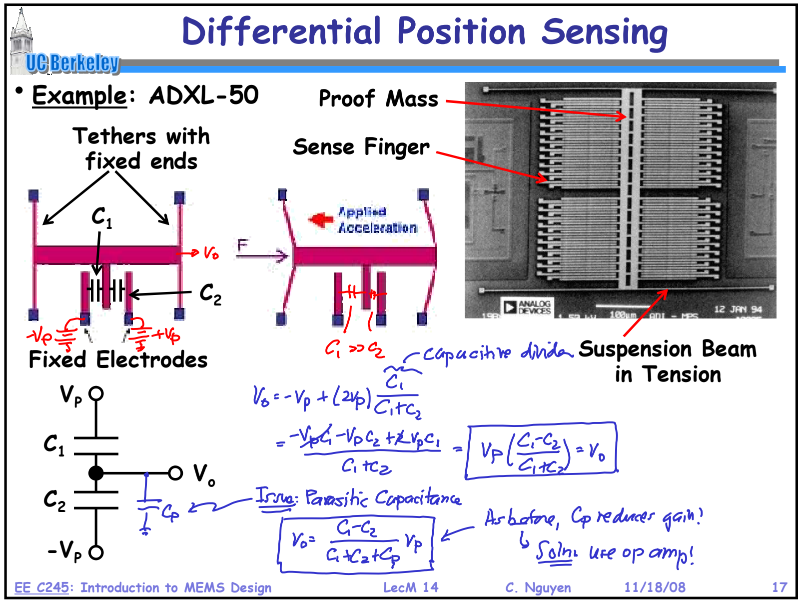

Differential Position Sensing

- **Example**: ADXL-50

capacitive divider

$V_o = -V_p + (2V_p)\frac{C_1}{C_1+C_2}$

$= \frac{\cancel{-V_pC_1} - V_pC_2 + \cancel{2}V_pC_1}{C_1+C_2} = \boxed{V_p\left(\frac{C_1-C_2}{C_1+C_2}\right) = V_o}$

Issue: Parasitic Capacitance

$$\boxed{V_o = \frac{C_1-C_2}{C_1+C_2+C_p}V_p}$$

As before, C_p reduces gain!

↳ Soln: use op amp!

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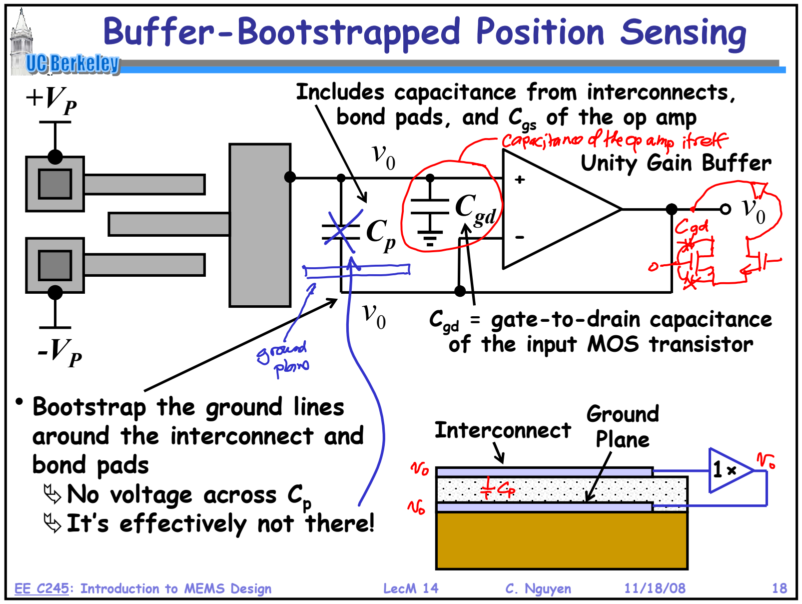
Buffer-Bootstrapped Position Sensing

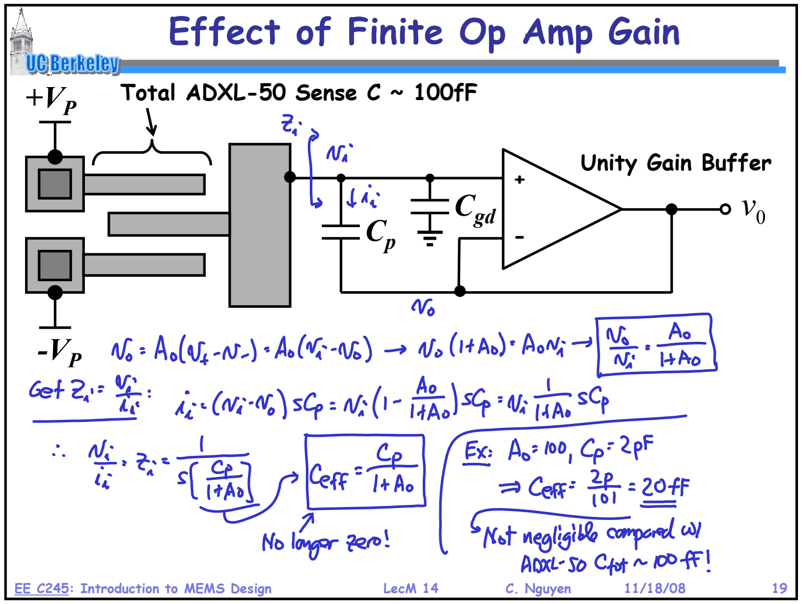


Includes capacitance from interconnects, bond pads, and C_{gs} of the op amp

C_{gd} = gate-to-drain capacitance of the input MOS transistor

- Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across C_p
  - It's effectively not there!

Effect of Finite Op Amp Gain



Total ADXL-50 Sense C ~ 100fF

$$v_o = A_o(v_+ - v_-) = A_o(v_i - v_o) \rightarrow v_o(1+A_o) = A_o v_i \rightarrow \boxed{\frac{v_o}{v_i} = \frac{A_o}{1+A_o}}$$

Get $Z_i = \frac{v_i}{i_i}$:

$$i_i = (v_i - v_o)sC_p = v_i\left(1 - \frac{A_o}{1+A_o}\right)sC_p = v_i \frac{1}{1+A_o} sC_p$$

$$\therefore \frac{v_i}{i_i} = Z_i = \frac{1}{s\left[\frac{C_p}{1+A_o}\right]} \quad \boxed{C_{eff} = \frac{C_p}{1+A_o}}$$

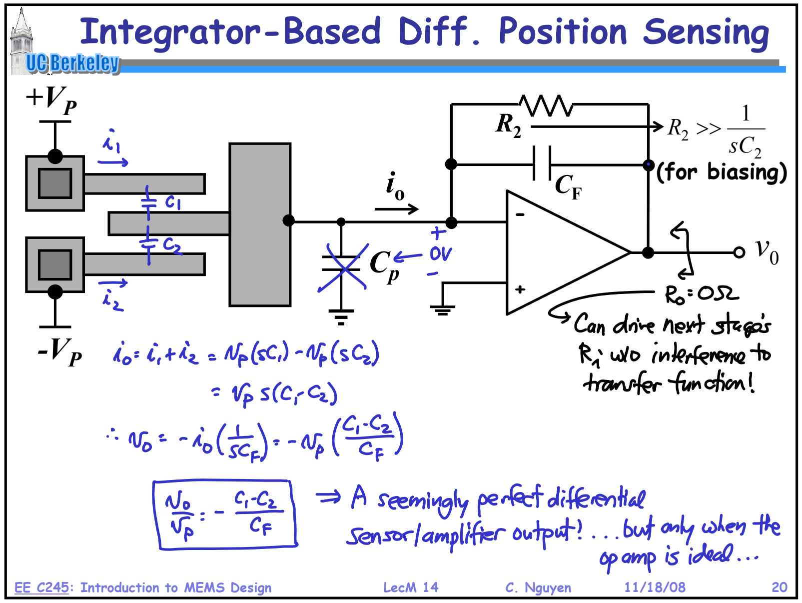
No longer zero!

Ex: $A_o = 100$, $C_p = 2pF$

$$\Rightarrow C_{eff} = \frac{2p}{101} = 20fF$$

Not negligible compared w/ ADXL-50 $C_{tot} \sim 100fF$!

Integrator-Based Diff. Position Sensing



$R_2 >> \frac{1}{sC_2}$ (for biasing)

$R_o = 0\Omega$

Can drive next stage's R_i w/o interference to transfer function!

$$i_o = i_1 + i_2 = v_p(sC_1) - v_p(sC_2)$$
$$= v_p s(C_1 - C_2)$$

$$\therefore v_o = -i_o\left(\frac{1}{sC_F}\right) = -v_p\left(\frac{C_1 - C_2}{C_F}\right)$$

$$\boxed{\frac{v_o}{v_p} = -\frac{C_1 - C_2}{C_F}}$$

⇒ A seemingly perfect differential sensor/amplifier output! ... but only when the op amp is ideal ...