Determining Sensor Resolution

Minimum Detectable Signal (MDS)

* Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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Sensed Signal
  +-----------------+  +-----------------+
  | Sensor Scale    |  + Circuit Gain     |
  | Factor          |  + Circuit Output   |
  | Sensor Noise    |  + Noise           |
  |                 |                  |
  +-----------------+  +-----------------+
                  |                  |
                  | Output           |
                  | Includes desired |
                  | output plus      |
                  | noise            |
```

- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Reflected Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
Noise

- Noise: Random fluctuation of a given parameter \( I(t) \)
- In addition, a noise waveform has a zero average value
- We can’t handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

\[
\lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} |I - I_D|^2 \, dt
\]

Noise Spectral Density

- We can plot the spectral density of this mean-square value:

\[
\frac{\mathcal{N}}{\Delta f} \quad [\text{units}^2/\text{Hz}]
\]

- One-sided spectral density
  - used in circuits
  - measured by spectrum analyzers

- Two-sided spectral density (1/2 the one-sided)
  - Often used in systems courses

Noise Sources

- Noise Sources
Thermal Noise

- **Thermal Noise in Electronics:** (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free electrons in a conducting medium
  - Path of electrons randomly oriented due to collisions
- **Thermal Noise in Mechanics:** (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.

- **Properties:**
  - Thermal noise is white (i.e., constant with frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor

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Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R}$ or a shunt current generator $\overline{i_R}$

![Circuit Diagram]

Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

- Actual:
  - $\overline{v_R}$
- Noiseless:
  - $\overline{i_R}$ or $\overline{v_R}$

- 4kT:
  - $\overline{v_R} = 4kTR$
  - $\overline{i_R} = 4kTR$

where $4kT = 1.66 \times 10^{-20} V \cdot C$ and these are spectral densities.

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Noise in Capacitors and Inductors?

- Resistor generates thermal noise
- Capacitors and inductors are noiseless → why?

![Circuit Diagram]

*Now, add a resistor:

![Circuit Diagram]

Need to add a forcing function, like a noise voltage $\overline{v_R}$ to keep the motion going → and this noise source is associated with R

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Why 4kTR?

- Why is $\overline{v_R} = 4kTR$? (a heuristic argument)
- The **Equipartition Theorem** of Statistical Thermodynamics says that there is a mean energy $(1/2)kT$ associated with each degree of freedom in a given system

- An electronic circuit possesses two degrees of freedom:
  - Current, $i$, and voltage, $v$
  - Thus, we can write:
    - $\frac{1}{2}Li^2 = \frac{1}{2}k_BT$
    - $\frac{1}{2}Cv^2 = \frac{1}{2}k_BT$

- Similar expressions can be written for mechanical systems:
  - For example: for displacement, $x$
    - Spring constant $kx^2 = \frac{1}{2}k_BT$
    - $\frac{1}{2}(k_BT)^2$

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**Shot Noise**

- Associated with direct current flow in diodes and bipolar junction transistors.
- Arises from the random nature by which e−’s and h+’s surmount the potential barrier at a pn junction.
- The DC current in a forward-biased diode is composed of h+’s from the p-region and e−’s from the n-region that have sufficient energy to overcome the potential barrier at the junction.

\[ \frac{\Delta I}{\Delta f} = 2qI_D \]

Charge on an e− (≈ 1.6x10⁻¹⁹ C)

**Flicker (1/f) Noise**

- In general, associated w/ random trapping & release of carriers from “slow” states.
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies.
- Often, get a mean-square noise spectral density that looks like this:

\[ \frac{I_n^2}{I_D} \sim \frac{1}{f} \]

\[ I_D = \text{DC current} \]

\[ K = \text{const. for a particular device} \]

\[ a = 0.5 \rightarrow 2 \]

\[ b \sim 1 \]

**Back to Determining Sensor Resolution**

**MEMS-Based Tuning Fork Gyroscope**

[Zaman, Ayazi, et al, MEMS'06]
Drive Axis Equivalent Circuit

* Generates drive displacement velocity \( x_d \) to which the Coriolis force is proportional

To Sense Amplifier (for synchronization)

Drive-to-Sense Transfer Function

Minimum Detectable Signal (MDS)

* Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

Gyro Readout Equivalent Circuit (for a single tine)

* Easiest to analyze if all noise sources are summed at a common node

Noise Sources

\[ \vec{F}_c = m\vec{a}_c = m \cdot (2\vec{x}_d \times \Omega) \]

Gyro Sense Element Output Circuit

Signal Conditioning Circuit (Transresistance Amplifier)

\[ \vec{V}_0 = \vec{A}_r \vec{V}_0 \]

Minimum Detectable Signal (MDS)

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Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element).
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS.

Diagram:

- Sensed Signal
- Sensor Scale Factor
- Sensor Noise
- Circuit Input-Referenced Noise
- Circuit Gain
- Signal Conditioning Circuit
- Output

Includes desired output plus noise.