EE C247B – ME C218
Introduction to MEMS Design
Fall 2016

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Module 17: Noise & MDS

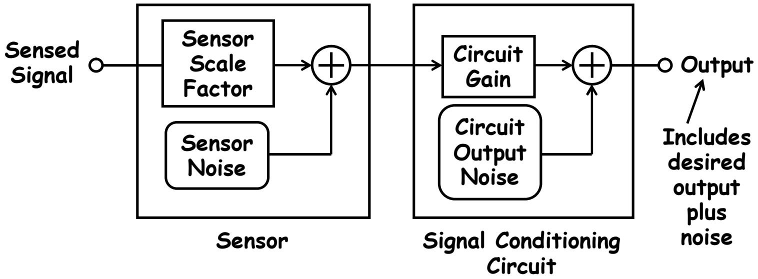
Lecture Outline

- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination

Determining Sensor Resolution

Minimum Detectable Signal (MDS)

- <u>Minimum Detectable Signal</u> (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

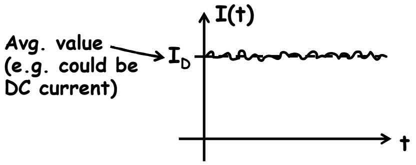


- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Noise

Noise

- **Noise**: Random fluctuation of a given parameter I(t)
- In addition, a noise waveform has a zero average value



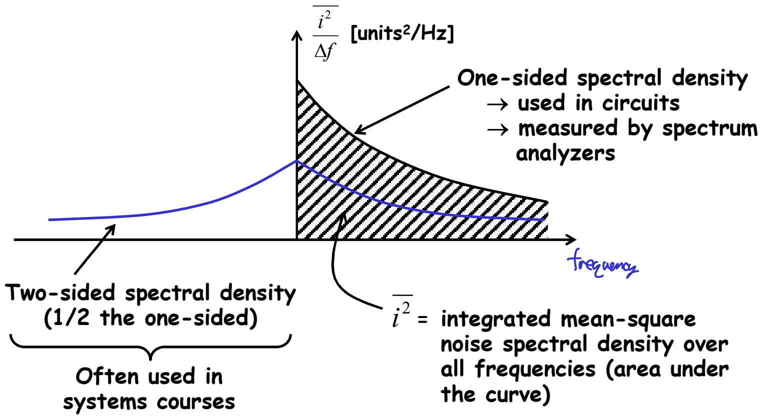
- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T |I - I_D|^2 dt$

Noise Spectral Density

- We can plot the spectral density of this mean-square value:



One-sided spectral density
→ used in circuits
→ measured by spectrum analyzers

Two-sided spectral density
(1/2 the one-sided)

Often used in systems courses

$\overline{i^2}$ = integrated mean-square noise spectral density over all frequencies (area under the curve)

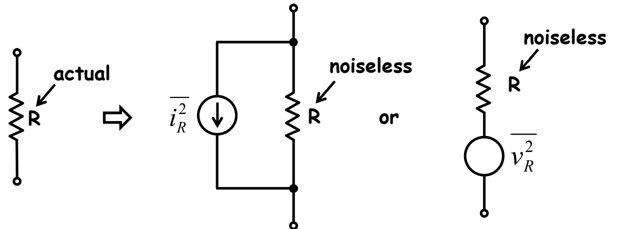
Noise Sources

Thermal Noise

- Thermal Noise in Electronics: (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free e⁻'s in a conducting medium
  - Path of e⁻'s randomly oriented due to collisions
- Thermal Noise in Mechanics: (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.
- Properties:
  - Thermal noise is white (i.e., constant w/ frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor

Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R^2}$ or a shunt current generator $\overline{i_R^2}$



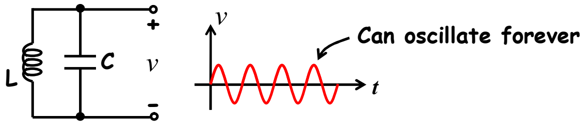
Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

$$\boxed{\frac{\overline{i_R^2}}{\Delta f} = \frac{4kT}{R}} \qquad \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR}$$

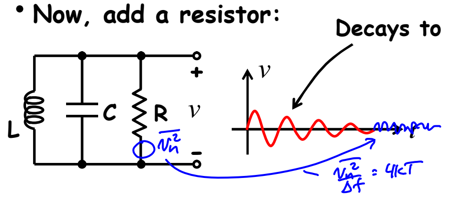
where $4kT = 1.66x10^{-20} V \cdot C$
and where these are spectral densities.

Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?



- Now, add a resistor:



But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature

Need to add a forcing function, like a noise voltage $\overline{v_R^2}$ to keep the motion going → and this noise source is associated with R

Why 4kTR?

- Why is $\overline{v_R^2} = 4kTR\Delta f$ (a heuristic argument)
- The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy (1/2)kT associated w/ each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom:
  - Current, i, and voltage, v
  - Thus, we can write:

$$\frac{1}{2}L\overline{i^2} = \frac{1}{2}k_BT \quad , \quad \overbrace{\frac{1}{2}C\overline{v^2}}^{\text{Energy}} = \frac{1}{2}k_BT$$

- Similar expressions can be written for mechanical systems
  - For example: for displacement, x

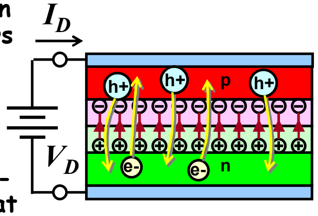
Spring constant

$$\frac{1}{2}k\overline{x^2} = \frac{1}{2}k_BT$$

$\frac{1}{2}\left(\frac{1}{k}\right)F^2$

Shot Noise

- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which e⁻'s and h⁺'s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of h⁺'s from the p-region and e⁻'s from the n-region that have sufficient energy to overcome the potential barrier at the junction → noise process should be proportional to DC current
- Attributes:
  - Related to DC current over a barrier
  - Independent of temperature
  - White (i.e., const. w/ frequency)
  - Noise power ~ I_D & bandwidth



pn-junction

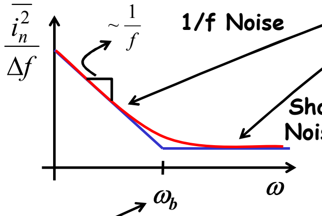
$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D$$

Charge on an e⁻ ($=1.6\times10^{-19}$C)

DC Current

Flicker (1/f) Noise

- In general, associated w/ random trapping & release of carriers from "slow" states
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:



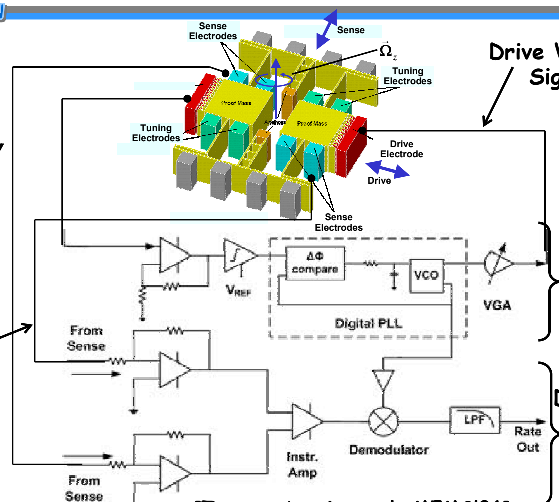
$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D + K\left(\frac{I_D^a}{f^b}\right)$$

I_D = DC current
K = const. for a particular device
a = 0.5 → 2
b ~ 1

1/f Noise Corner Frequency

Back to Determining Sensor Resolution

MEMS-Based Tuning Fork Gyroscope



Drive Voltage Signal

(-) Sense Output Current

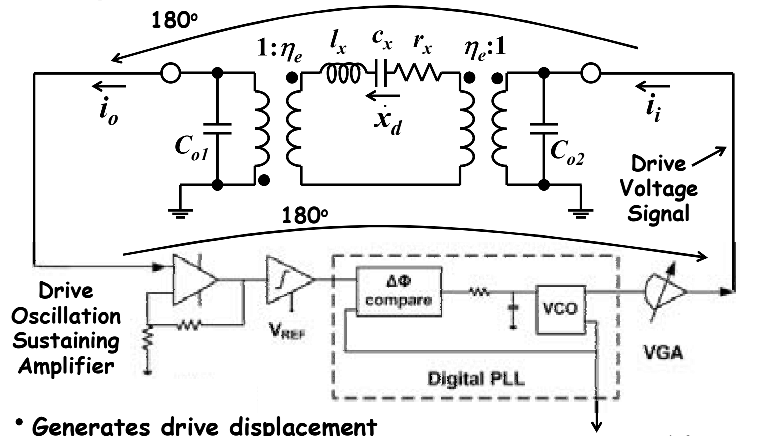
(+) Sense Output Current

Drive Oscillation Sustaining Amplifier

Differential TransR Sense Amplifier

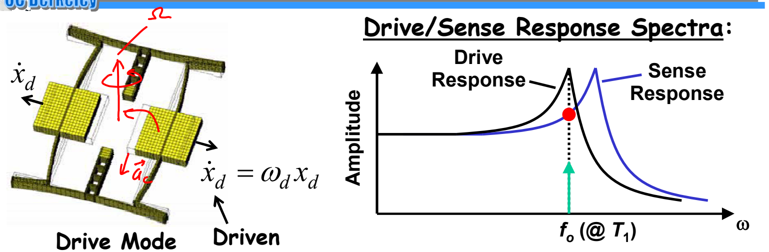
[Zaman, Ayazi, et al, MEMS'06]

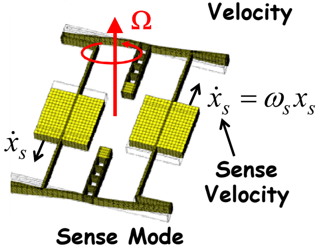
Drive Axis Equivalent Circuit



- Generates drive displacement velocity $\dot{x}_d$ to which the Coriolis force is proportional

Drive-to-Sense Transfer Function





Rotation-Induced Coriolis Force:

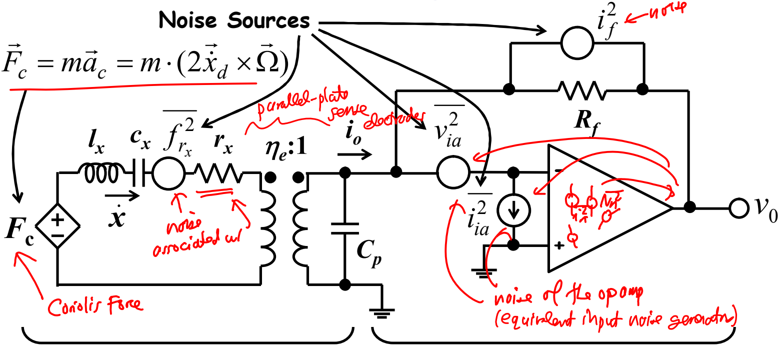
$\vec{a}_c = 2\vec{\dot{x}}_d \times \vec{\Omega} = 2\omega_d \vec{x}_d \times \vec{\Omega}$

⇒ Acts in the sense mode direction

$a_s = 2\omega_d x_d \Omega \sin 90°$

$\boxed{a_s = 2\omega_d x_d \Omega}$

Gyro Readout Equivalent Circuit (for a single tine)



- Easiest to analyze if all noise sources are summed at a common node

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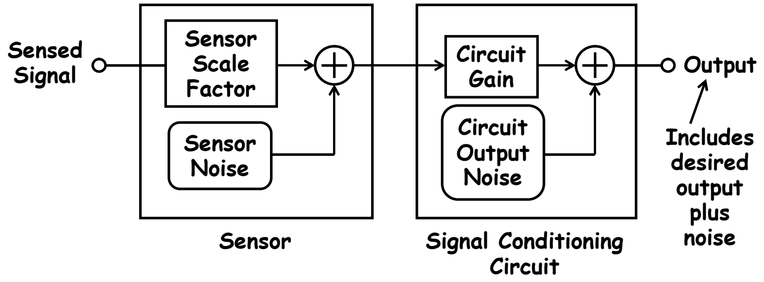

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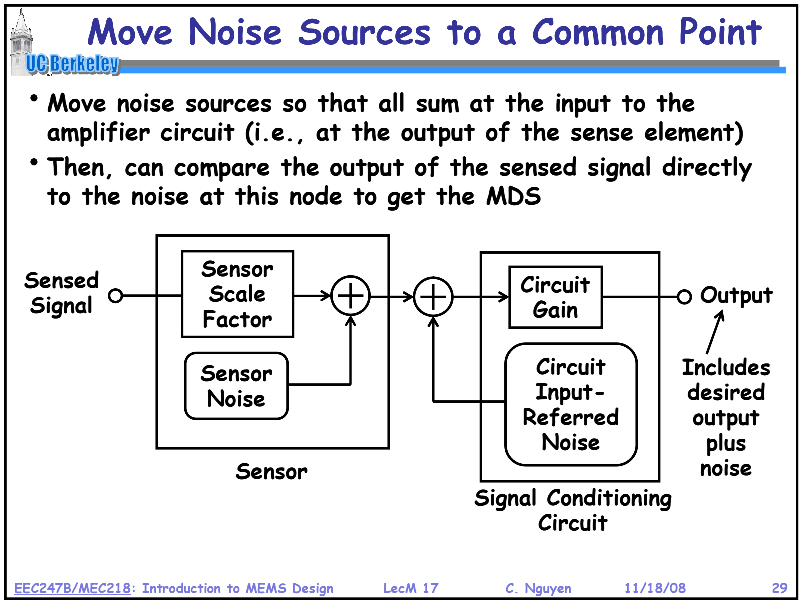

Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

EEC247B/MEC218: Introduction to MEMS Design LecM 17 C. Nguyen 11/18/08 29