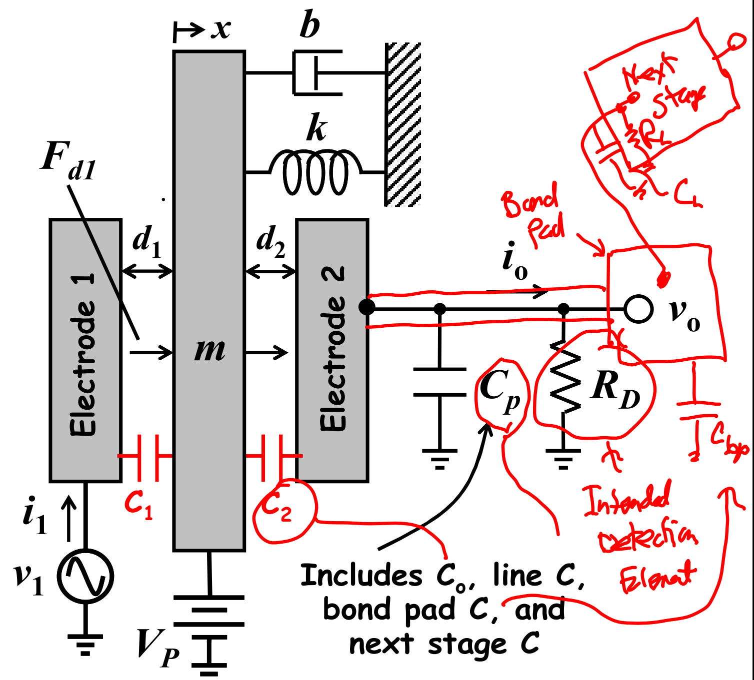
Lecture 25: Sensing Circuits II

- Announcements:
- HW#6 online and due Friday, April 22
- Module 16 on "Sensing Circuit Non-Idealities and Integration" is online
- Project slide #3 due Friday, April 29
- I am at the EECS Retreat today; this is a pre-recorded lecture
- ---
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
  - MEMS-Transistor Integration
    - Mixed
    - MEMS-First
    - MEMS-Last
- ---
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
- ---

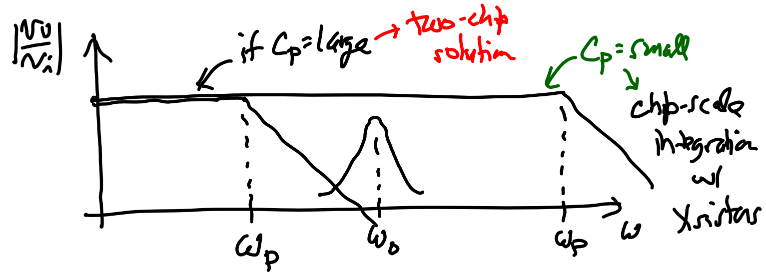
- Last Time:
- Discussing velocity sensing; simplest way is via a resistive output load R_D
- Problem: parasitics (resistance and capacitance)



Now, we get:

$$\frac{v_o}{v_1}(s) \sim \frac{R_D}{R_x+R_D} \cdot \frac{1}{1+\frac{s}{\omega_p}} \cdot H(s,\omega_o',Q')$$

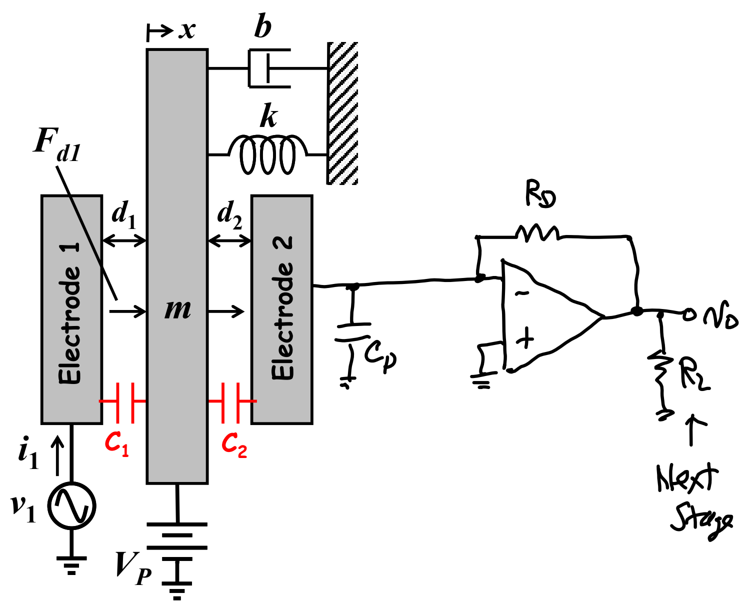
$$\omega_p = \frac{1}{(R_x \| R_D)C_p}$$



Problems w/ Purely Resistive Detection

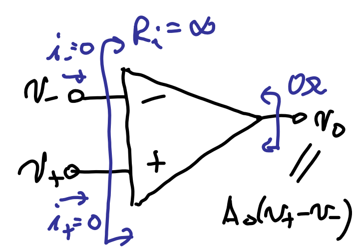
1. Need large R_D for large gain ... but ...
2. $R_D \uparrow \rightarrow Q \downarrow$
3. $R_D \uparrow \rightarrow \omega_p \doteq \frac{1}{R_D C_p} \downarrow \rightarrow$ get undesirable LPF cut-off
4. Load $R_L \rightarrow$ affects gain!

Solution: use Xsistors → op amp



Ideal Op Amp Laws

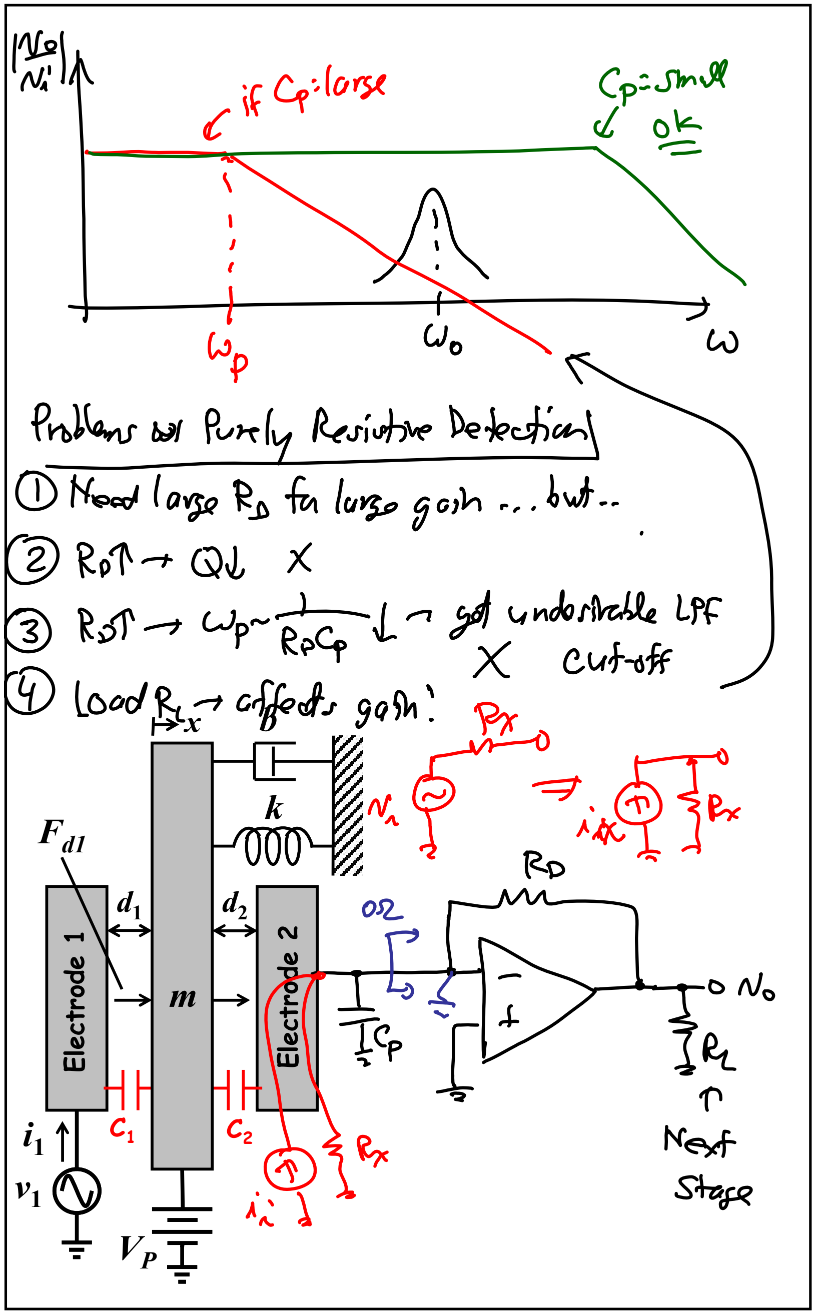
1. $R_i = \infty \rightarrow i_- = i_+ = 0$
2. $R_o = 0$
3. Gain $= A_o = \infty$

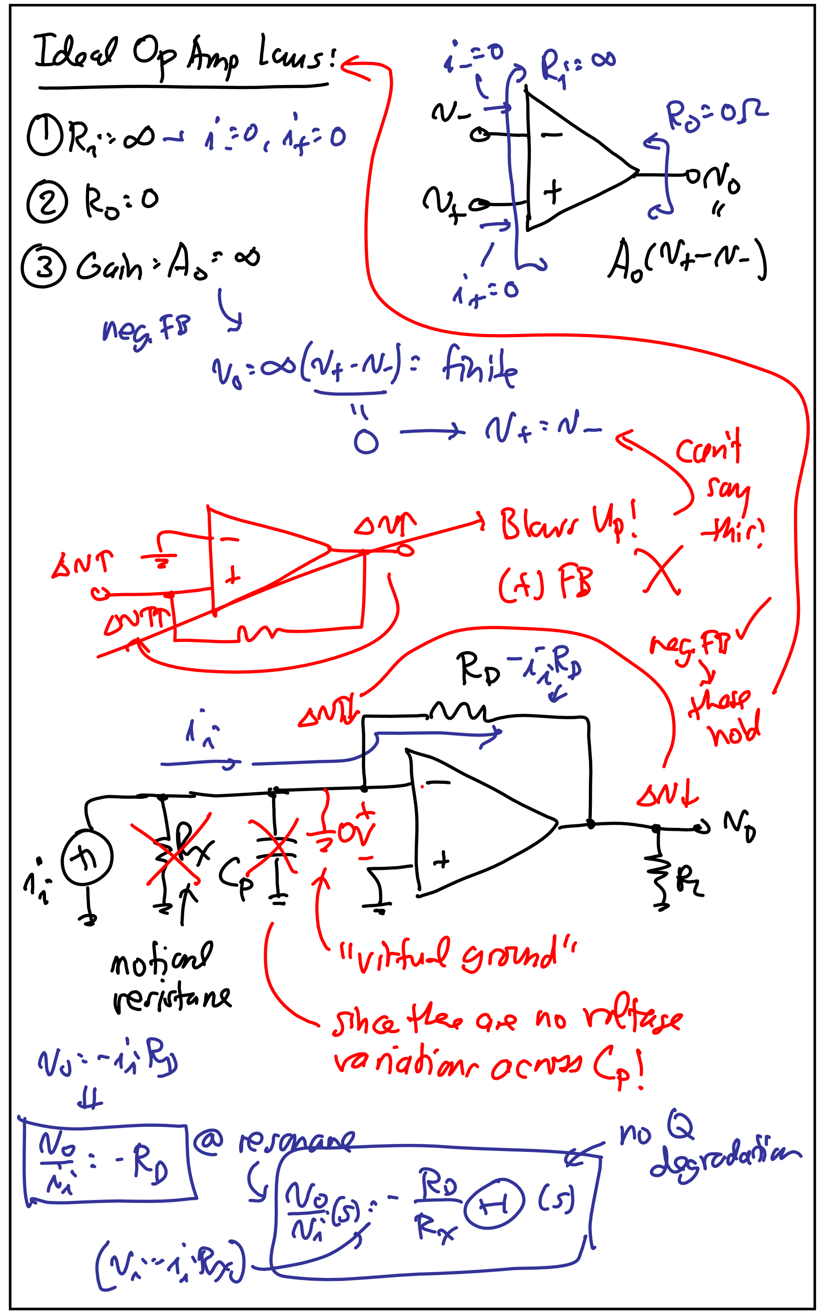


neg. FB $\rightarrow v_+ = v_-$

v_o = finite $\quad \infty \underbrace{(v_+ - v_-)}_{= 0} = $ finite

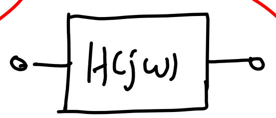
- Transitioned to pre-recorded version at this point, which starts on the next page


|vo/vi|
if Cp: large
Cp: small
ok
ωp
ωo
ω
Problems w/ Purely Resistive Detection
① Need large RD for large gain ... but ...
② RD↑ → Q↓ X
③ RD↑ → ωp ~ 1/(RD Cp) ↓ → get undesirable LPF cut-off X
④ Load RL → affects gain!
Rx
vi
ix
Rx
x
b
k
Fd1
d1
d2
m
Electrode 1
Electrode 2
C1
C2
i1
v1
VP
Cp
RD
0Ω
vo
RL
Next Stage
ii
Rx


Ideal Op Amp Laws!
① Ri = ∞ → i− = 0, i+ = 0
② Ro = 0
③ Gain: Ao = ∞
i− = 0
Ri = ∞
Ro = 0Ω
v−
v+
vo
Ao(v+ − v−)
i+ = 0
neg. FB
vo = ∞(v+ − v−) = finite
0 → v+ = v−
can't say this!
Blows Up!
(+) FB
neg. FB ✓
these hold
RD
ii
Rx
Cp
0V
RL
vo
motional resistance
"virtual ground"
since there are no voltage variations across Cp!
vo = −ii RD
vo/vi = −RD @ resonance
no Q degradation
vo/vi(s) = −RD/Rx H(s)
(vi = ii Rx)

Circuit Noise Calculations

Inputs — Deterministic — Outputs

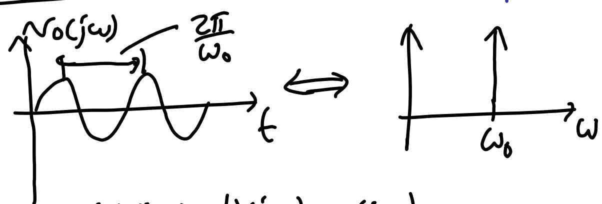


$S_i(\omega)$ Linear Time-Invariant System $S_o(\omega)$

Random

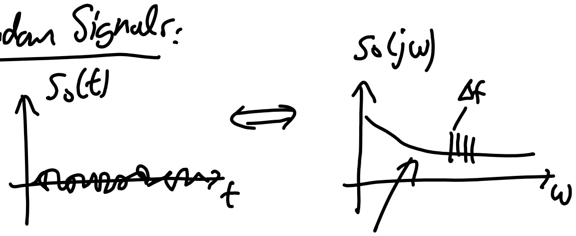
No j → noise has random phase, so j is pointless!

Deterministic Signals:



$$v_o(j\omega) = H(j\omega)\, v_i(j\omega)$$

Random Signals:



Mean-Square Spectral Density

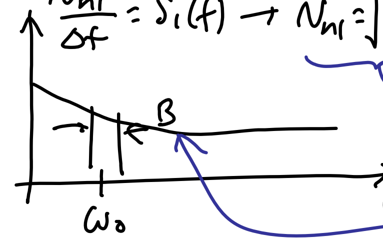
$$S_o(\omega) = [H(j\omega)H^*(j\omega)]\,S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$$

$$\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$$ → How is it we can do this?

Root-mean-square amplitudes

Handling Noise Deterministically

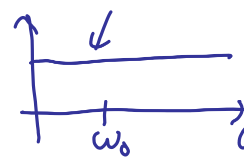
$$\frac{\overline{v_{ni}^2}}{\Delta f} = S_i(f) \rightarrow v_{ni} = \sqrt{S_i(f)\,B}$$

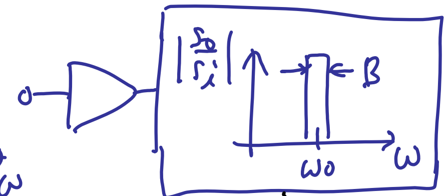


Can approximate this by a sinusoidal voltage generator (esp. when B is small, say 1Hz)

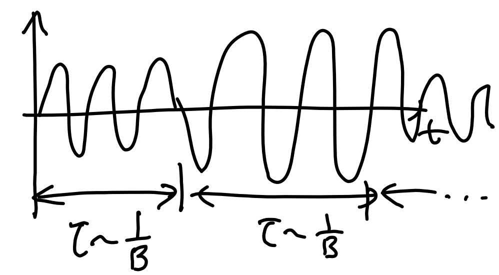
Why is this the case?

white noise



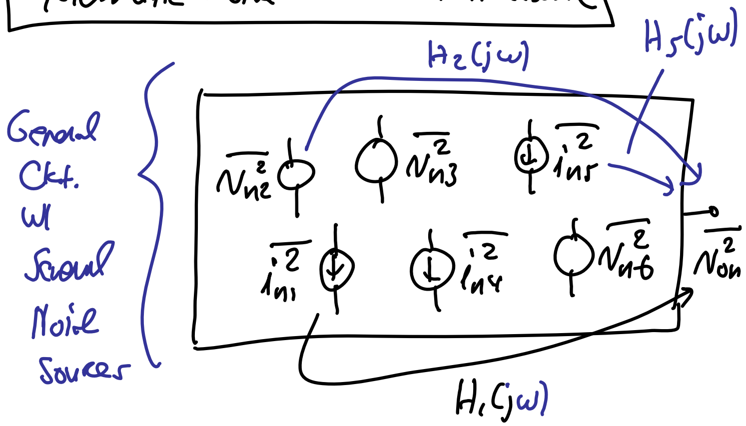


$v_o(t)$



Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B!

Systematic Noise Calculation Procedure



Assume noise sources are uncorrelated.

① For $\overline{i_{n1}^2}$, replace w/ a deterministic source of value $i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f}\cdot(1\,\text{Hz})}$

② Calculate $v_{on1}(\omega) = i_{n1}(\omega)\, H_1(j\omega)$
(treating it like a deterministic signal)

③ Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2}\cdot|H_1(j\omega)|^2$

④ Repeat for each noise source:
$\overline{v_{n2}^2},\ \overline{v_{n3}^2},\ \overline{i_{n4}^2}, \ldots \rightarrow$ output

⑤ Add noise powers (mean-square values)

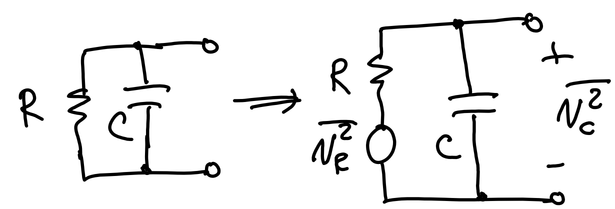
$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots}$$
↑
total rms value

- Go through Module 17, slides 12-16

$\boxed{\text{Why } \frac{\overline{v_{nR}^2}}{\Delta f} = 4kTR?}$ (a heuristic argument)

Consider an RC ckt:



$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_c^2}$

$\therefore\ \boxed{\overline{v_c^2} = \frac{kT}{C}}$ ← integrated noise over all freqs. (total mean-square voltage integrated over all freqs.)

*

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ gives us this (assuming white noise) (*

$$\overline{v_c^2} = \int_0^\infty \left|\frac{1}{1+j\omega RC}\right|^2 \frac{\overline{v_R^2}}{\Delta f}\, df$$

[noise is white] [$\omega_b = \frac{1}{RC}$] $\rightarrow$

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega}\, d\omega$$

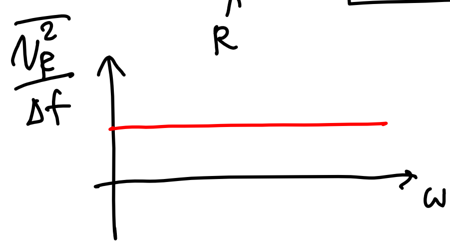
$$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\frac{\omega_b^2}{\omega_b}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Big|_0^\infty$$

$$= \frac{1}{2\cancel{\pi}}\frac{\overline{v_R^2}}{\Delta f}\left(\frac{\cancel{\pi}}{2}\omega_b - 0\right) = \frac{1}{4}\omega_b\frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C}$$

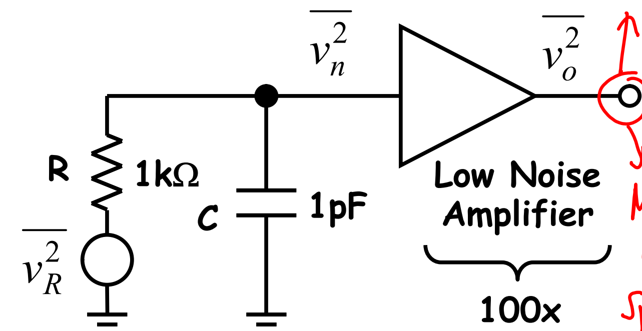
$$\frac{\overline{v_R^2}}{\Delta f} = 4kT\left(\frac{\omega_b}{C}\right) \Rightarrow \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR}$$

R



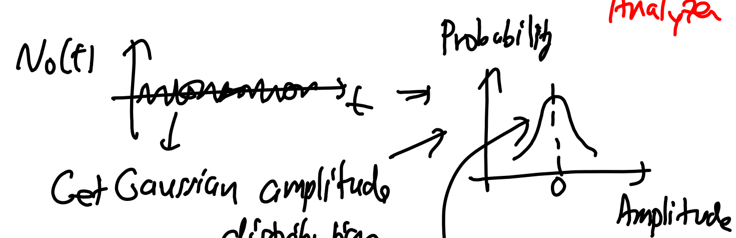
- Go through Module 17, slides 19-20

Example. Typical Noise Numbers



Measure w/ AC voltmeter

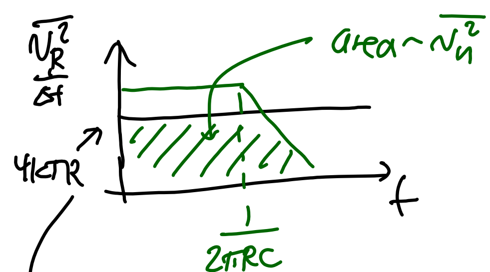
Measure on a Spectrum Analyzer



Get Gaussian amplitude distribution

68% within ±σ

99.7% within ±3σ



R=1kΩ → $\sqrt{(1.66\times10^{-20})(1k)}$

*

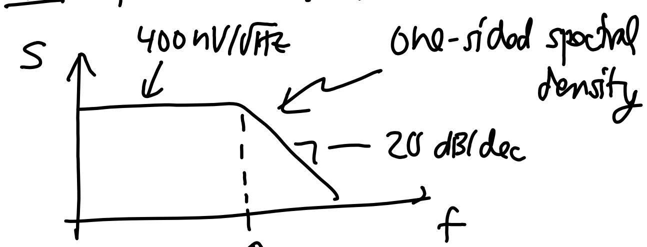
* — 1kΩ: 4nV/√Hz (for every 1K of R)

1pF: $\sqrt{\frac{kT}{C}} = 64\mu V_{rms}$

Case: AC Voltmeter

$\sqrt{\overline{N_o^2}} = (100)(64\mu V_{rms}) = 6.4mV\ rms$

Case: Spectrum Analyzer



$\frac{1}{2\pi(1k)(1p)} = 60MHz$

- Go through Module 17, slides 23-29