EE C247B – ME C218
Introduction to MEMS Design
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Module 17: Noise & MDS

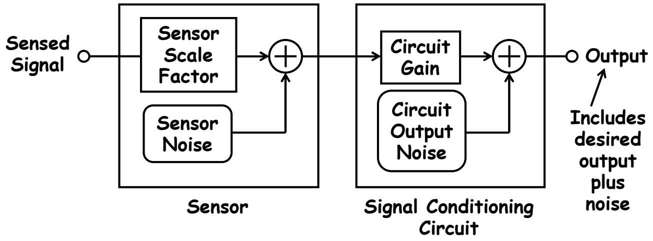
Lecture Outline

- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination

Determining Sensor Resolution

Minimum Detectable Signal (MDS)

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

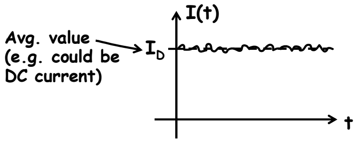


- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Noise

Noise

- **Noise**: Random fluctuation of a given parameter I(t)
- In addition, a noise waveform has a zero average value



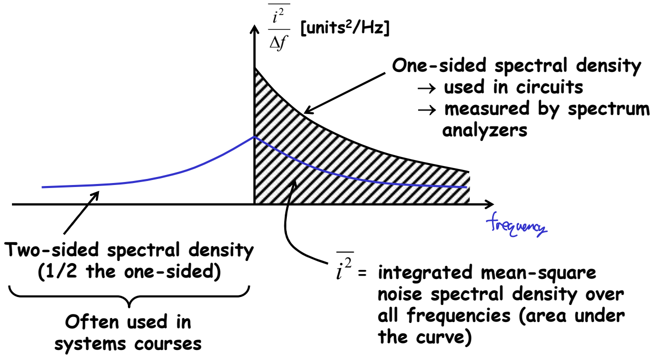
- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T |I - I_D|^2\, dt$

Noise Spectral Density

- We can plot the spectral density of this mean-square value:



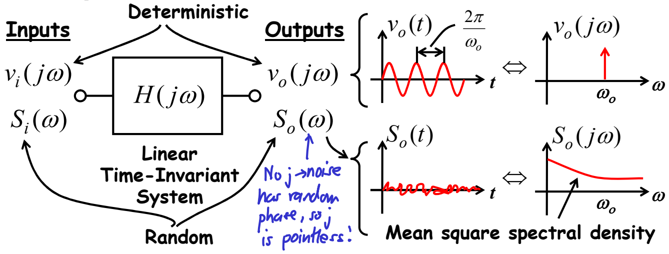
One-sided spectral density
→ used in circuits
→ measured by spectrum analyzers

Two-sided spectral density (1/2 the one-sided)

Often used in systems courses

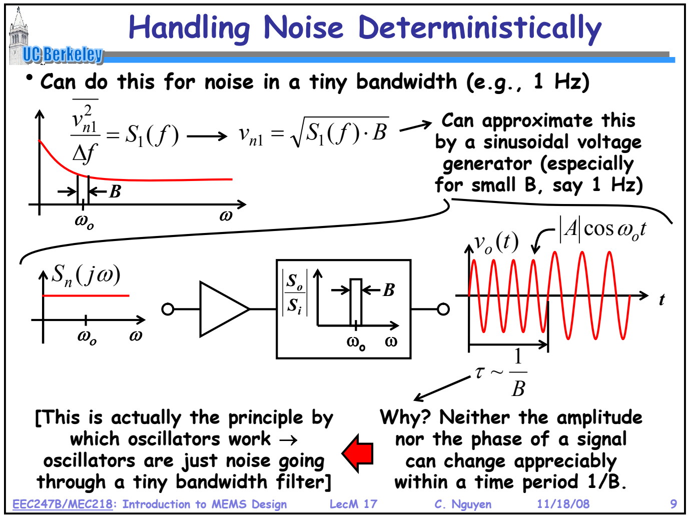
$\overline{i^2}$ = integrated mean-square noise spectral density over all frequencies (area under the curve)

Circuit Noise Calculations



- **Deterministic:** $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- **Random:** $S_o(\omega) = \left[H(j\omega)H^*(j\omega)\right]S_i(\omega) = \left|H(j\omega)\right|^2 S_i(\omega)$

$$\underbrace{\sqrt{S_o(\omega)} = \left|H(j\omega)\right|\sqrt{S_i(\omega)}}_{\text{Root mean square amplitudes}} \longrightarrow \text{How is it we can do this?}$$



Handling Noise Deterministically

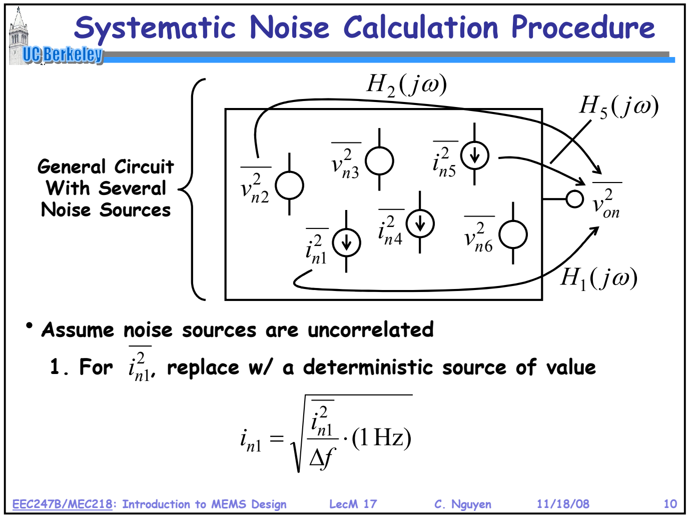
- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

$$\frac{\overline{v_{n1}^2}}{\Delta f} = S_1(f) \longrightarrow v_{n1} = \sqrt{S_1(f)\cdot B}$$

→ Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter]

Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.



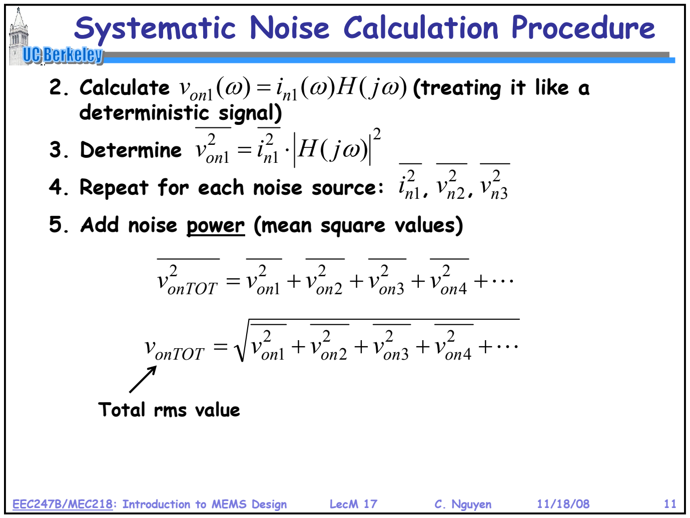
Systematic Noise Calculation Procedure

General Circuit With Several Noise Sources

- Assume noise sources are uncorrelated

1. For $\overline{i_{n1}^2}$, replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f}\cdot(1\,\text{Hz})}$$



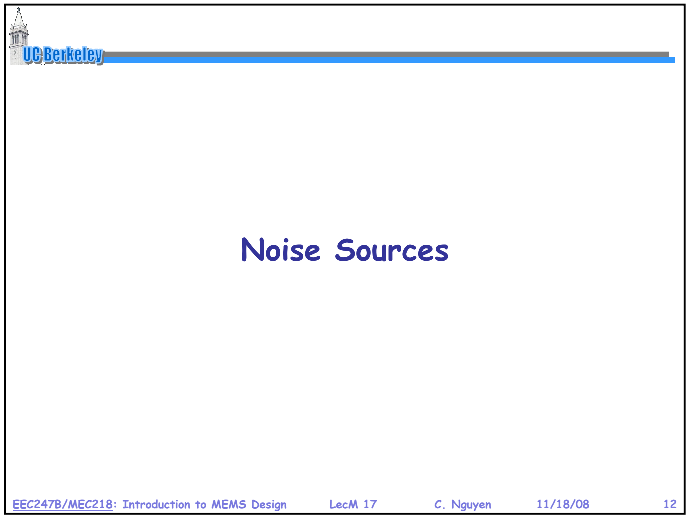
Systematic Noise Calculation Procedure

2. Calculate $v_{on1}(\omega) = i_{n1}(\omega)H(j\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2}\cdot|H(j\omega)|^2$
4. Repeat for each noise source: $\overline{i_{n1}^2}$, $\overline{v_{n2}^2}$, $\overline{v_{n3}^2}$
5. Add noise <u>power</u> (mean square values)

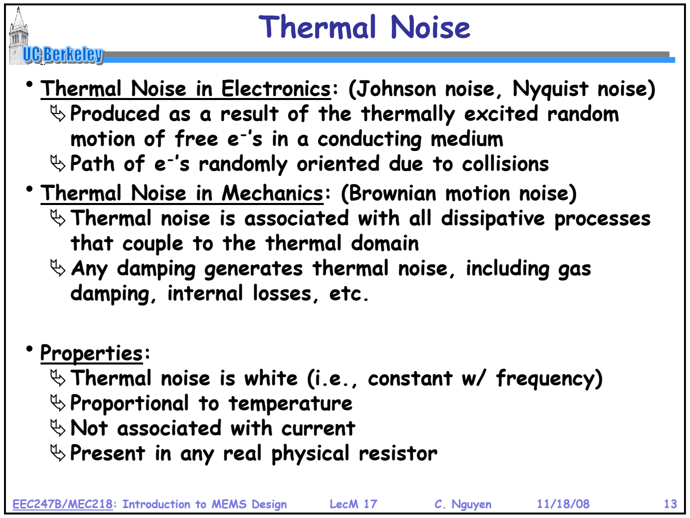
$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots}$$

↗ Total rms value



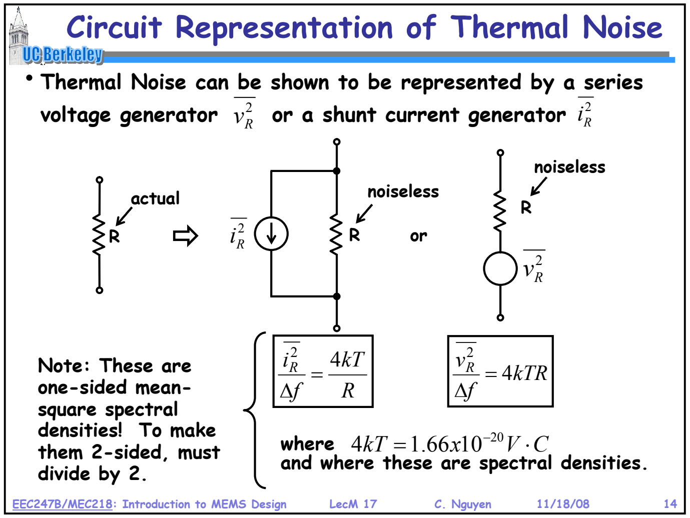
Noise Sources



Thermal Noise

- Thermal Noise in Electronics: (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free e⁻'s in a conducting medium
  - Path of e⁻'s randomly oriented due to collisions
- Thermal Noise in Mechanics: (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.

- Properties:
  - Thermal noise is white (i.e., constant w/ frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor



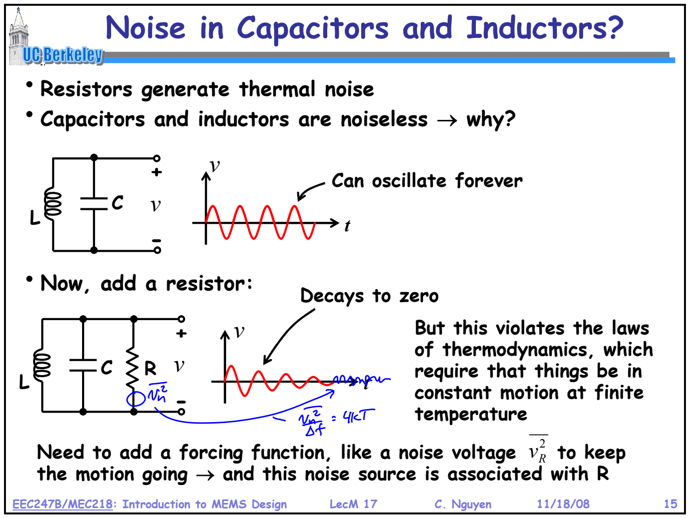
Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R^2}$ or a shunt current generator $\overline{i_R^2}$

Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

$$\boxed{\frac{\overline{i_R^2}}{\Delta f} = \frac{4kT}{R}} \qquad \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR}$$

where $4kT = 1.66x10^{-20} V \cdot C$
and where these are spectral densities.



Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?

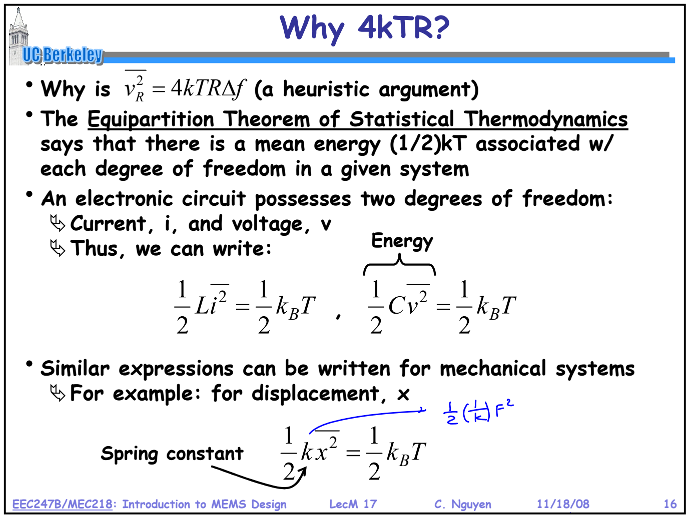
Can oscillate forever

- Now, add a resistor:

Decays to zero

But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature

Need to add a forcing function, like a noise voltage $\overline{v_R^2}$ to keep the motion going → and this noise source is associated with R



Why 4kTR?

- Why is $\overline{v_R^2} = 4kTR\Delta f$ (a heuristic argument)
- The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy (1/2)kT associated w/ each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom:
  - Current, i, and voltage, v
  - Thus, we can write:

$$\frac{1}{2}L\overline{i^2} = \frac{1}{2}k_BT \quad , \quad \overbrace{\frac{1}{2}C\overline{v^2}}^{\text{Energy}} = \frac{1}{2}k_BT$$

- Similar expressions can be written for mechanical systems
  - For example: for displacement, x

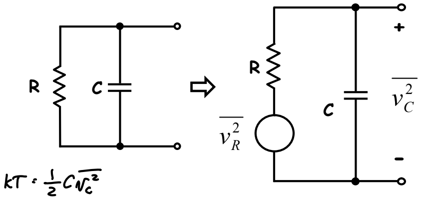
$$\frac{1}{2}k\overline{x^2} = \frac{1}{2}k_BT$$

Spring constant

$\frac{1}{2}\left(\frac{1}{k}\right)F^2$

Why 4kTR? (cont)

- Why is $\overline{v_R^2} = 4kTR\Delta f$? (a heuristic argument)
- Consider an RC circuit:



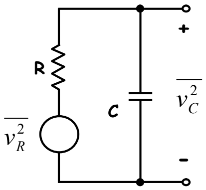
$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_c^2}$

$\therefore \overline{v_c^2} = \frac{kT}{C}$ ← integrated noise over all freqs.
(total mean square voltage integrated over all freqs.)

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ (assuming white noise) gives us this ?

Why 4kTR? (cont)

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ (assuming white noise) gives us $\overline{v_c^2} = \frac{kT}{C}$?



$$\overline{v_c^2} = \int_0^\infty \left|\frac{1}{1+j\omega RC}\right|^2 \frac{\overline{v_R^2}}{\Delta f}\, df$$

[noise is white]
$[\omega_b = \frac{1}{RC}]$

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega^2}\, d\omega$$

$$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

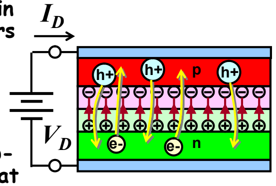
$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\frac{\omega_b^2}{\omega_b}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Big|_0^\infty = \frac{1}{2\cancel{\pi}}\frac{\overline{v_R^2}}{\Delta f}\left(\frac{\cancel{\pi}}{2}\omega_b - 0\right)$$

$$= \frac{1}{4}\omega_b\frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C} \longrightarrow \frac{\overline{v_R^2}}{\Delta f} = 4kT\left(\frac{\omega_b}{C}\right) \Rightarrow \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR} \checkmark$$

↑
R

Shot Noise

- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which e^-'s and h^+'s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of h^+'s from the p-region and e^-'s from the n-region that have sufficient energy to overcome the potential barrier at the junction → noise process should be proportional to DC current
- **Attributes**:
  - Related to DC current over a barrier
  - Independent of temperature
  - White (i.e., const. w/ frequency)
  - Noise power ~ I_D & bandwidth



pn-junction

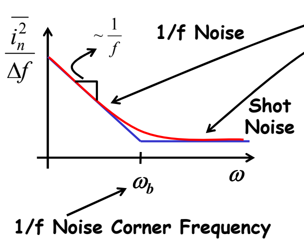
$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D$$

Charge on an e- (=1.6x10-19C)

DC Current

Flicker (1/f) Noise

- In general, associated w/ random trapping & release of carriers from "slow" states
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:

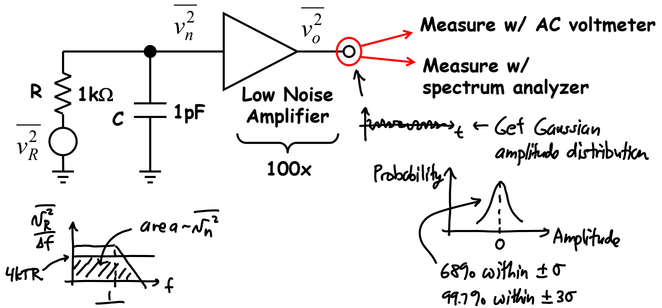


$$\frac{\overline{i_n^2}}{\Delta f} = 2qI_D + K\left(\frac{I_D^a}{f^b}\right)$$

I_D = DC current
K = const. for a particular device
a = 0.5 → 2
b ~ 1

Example: Typical Noise Numbers

- Hookup the circuit below and make some measurements

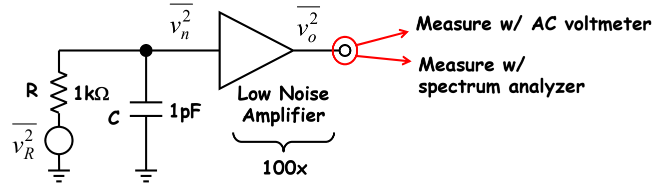


$\sqrt{(1.66\times10^{-20})(1k)}$ → 1kΩ: $4nV/\sqrt{Hz}$ (for every 1K of R)

1pF: $\sqrt{\frac{kT}{C}} = 64\mu V$ rms

Example: Typical Noise Numbers

- Hookup the circuit below and make some measurements

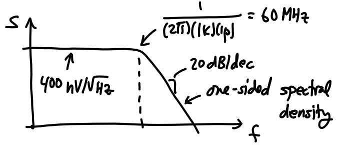


AC Voltmeter

$\sqrt{\overline{v_o^2}} = (100)(64\mu V\ rms)$

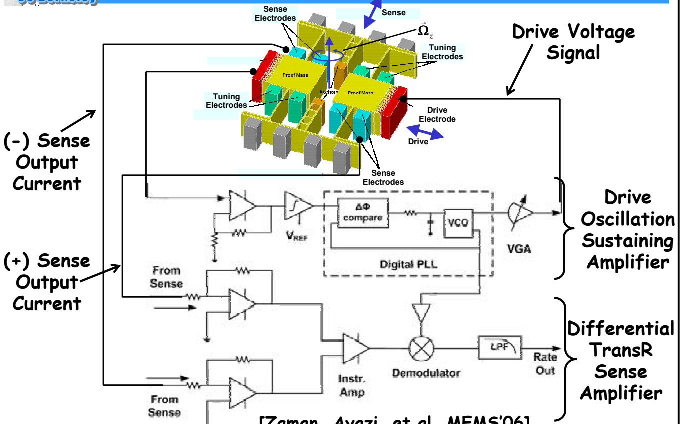
$= 6.4mV$ rms

Spectrum Analyzer

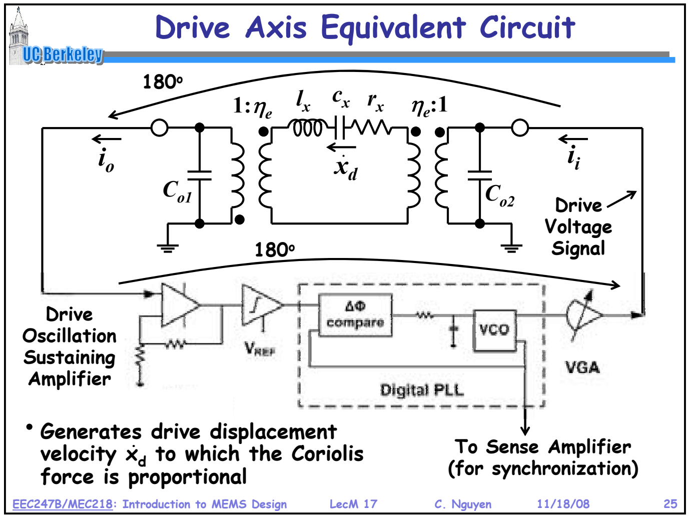


Back to Determining Sensor Resolution

MEMS-Based Tuning Fork Gyroscope



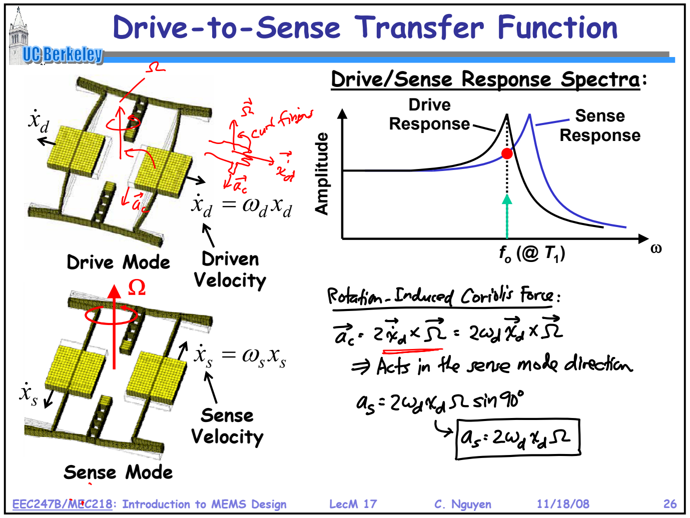
[Zaman, Ayazi, et al, MEMS'06]



Drive Axis Equivalent Circuit

- Generates drive displacement velocity $\dot{x}_d$ to which the Coriolis force is proportional

To Sense Amplifier (for synchronization)



Drive-to-Sense Transfer Function

Drive/Sense Response Spectra:

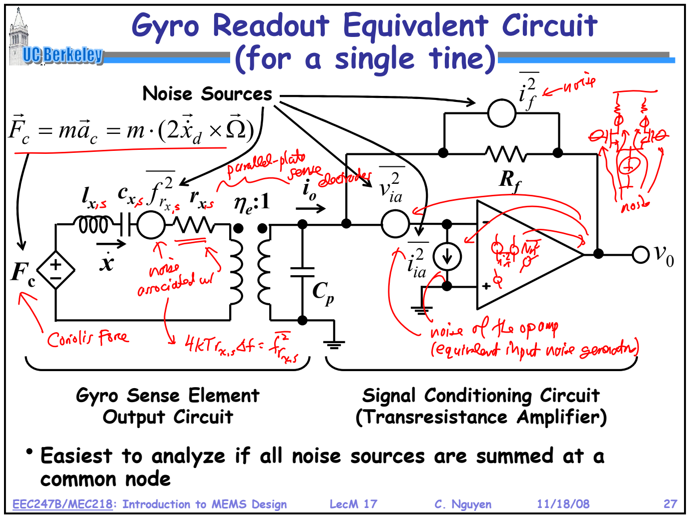
Rotation-Induced Coriolis Force:

$\vec{a}_c = 2\vec{\dot{x}}_d \times \vec{\Omega} = 2\omega_d \vec{x}_d \times \vec{\Omega}$

$\Rightarrow$ Acts in the sense mode direction

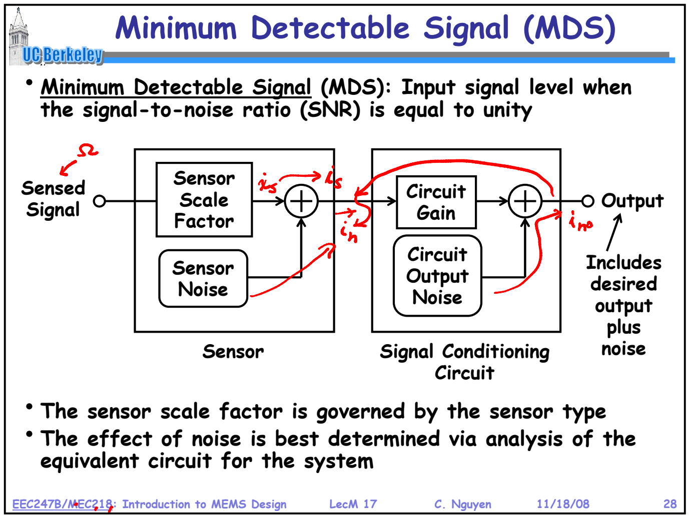
$a_s = 2\omega_d x_d \Omega \sin 90°$

$\hookrightarrow \boxed{a_s = 2\omega_d x_d \Omega}$



Gyro Readout Equivalent Circuit (for a single tine)

- Easiest to analyze if all noise sources are summed at a common node



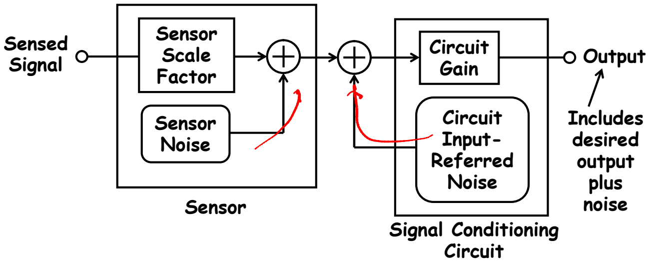
Minimum Detectable Signal (MDS)

- **Minimum Detectable Signal** (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Move Noise Sources to a Common Point

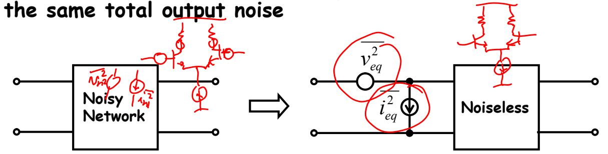
- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS



Equivalent Input-Referred Voltage and Current Noise Sources

Equivalent Input v, i Noise Generators

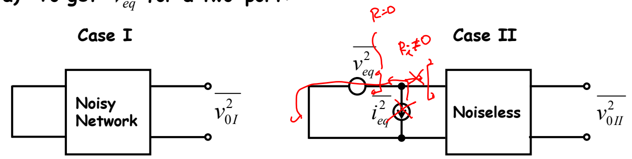
- Take a noisy 2-port network and represent it by a noiseless network with input *v* and *i* noise generators that generate the same total output noise



- **Remarks**:
  1. Works for linear time-invariant networks
  2. v_{eq} and i_{eq} are generally correlated (since they are derived from the same sources)
  3. In many practical circuits, one of v_{eq} and i_{eq} dominates, which removes the need to address correlation
  4. If correlation is important → easier to return to original network with internal noise sources

Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$

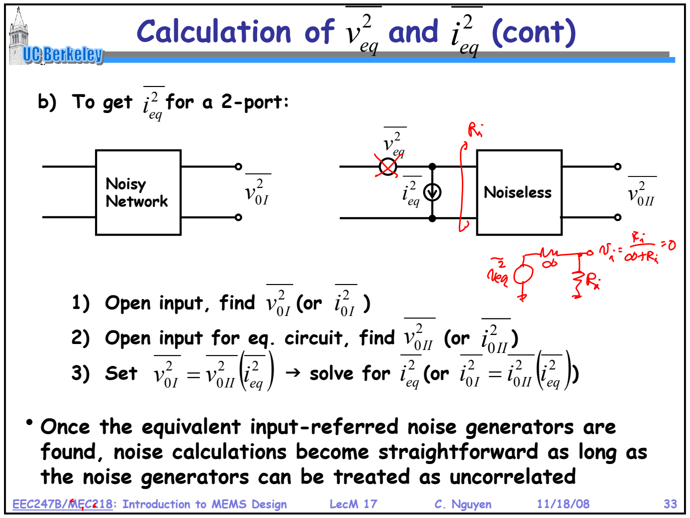
a) To get $\overline{v_{eq}^2}$ for a two-port:



1) Short input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
2) For eq. network, short input, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)

$$\overline{v_{0II}^2} = f\left(\overline{v_{eq}^2}\right) \qquad \overline{i_{0II}^2} = f\left(\overline{v_{eq}^2}\right)$$

3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}$ → solve for $\overline{v_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}$)

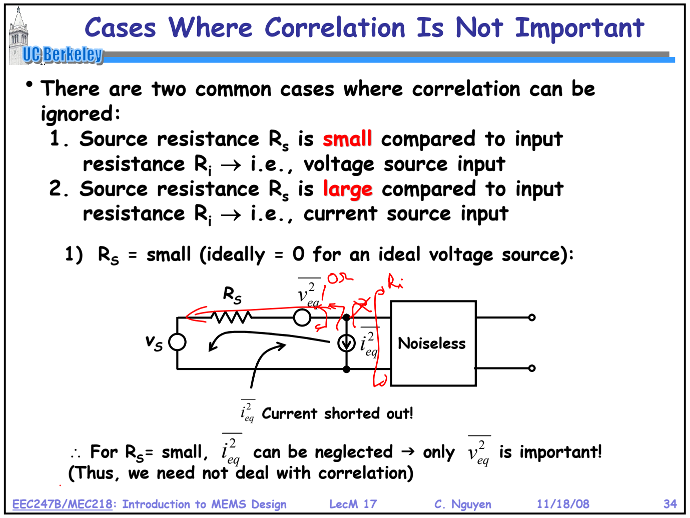


Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

b) To get $\overline{i_{eq}^2}$ for a 2-port:

1) Open input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
2) Open input for eq. circuit, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}\left(\overline{i_{eq}^2}\right)$ → solve for $\overline{i_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}\left(\overline{i_{eq}^2}\right)$)

- Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated



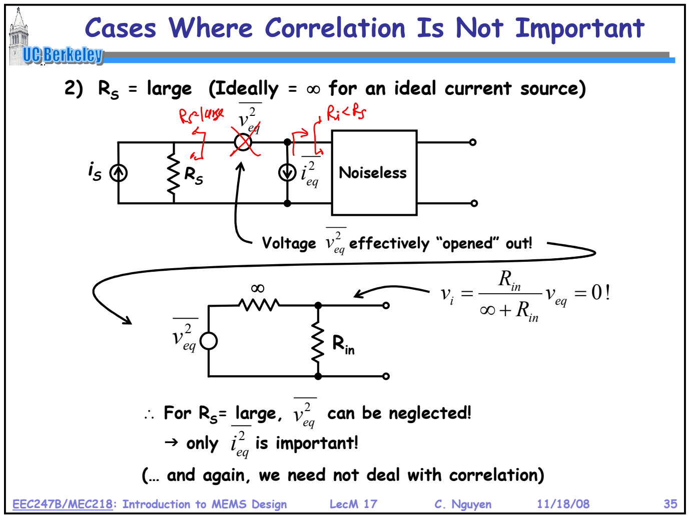
Cases Where Correlation Is Not Important

- There are two common cases where correlation can be ignored:
  1. Source resistance R_s is **small** compared to input resistance R_i → i.e., voltage source input
  2. Source resistance R_s is **large** compared to input resistance R_i → i.e., current source input

1) R_S = small (ideally = 0 for an ideal voltage source):

$\overline{i_{eq}^2}$ Current shorted out!

∴ For R_S= small, $\overline{i_{eq}^2}$ can be neglected → only $\overline{v_{eq}^2}$ is important!
(Thus, we need not deal with correlation)



Cases Where Correlation Is Not Important

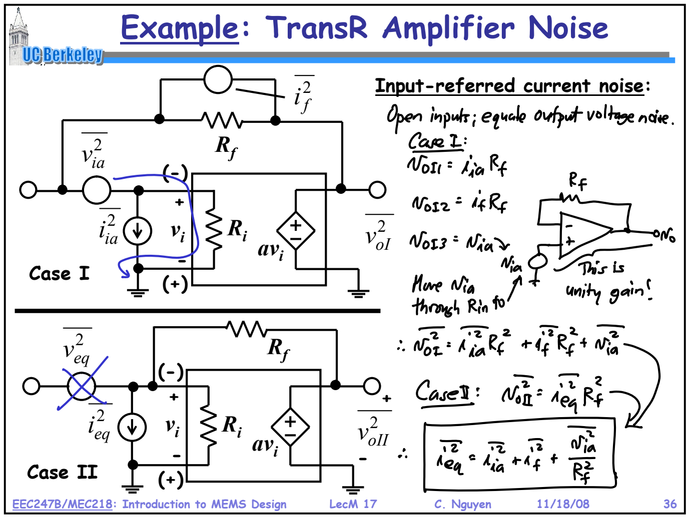
2) R_S = large (Ideally = ∞ for an ideal current source)

Voltage $\overline{v_{eq}^2}$ effectively "opened" out!

$$v_i = \frac{R_{in}}{\infty + R_{in}} v_{eq} = 0!$$

∴ For R_S= large, $\overline{v_{eq}^2}$ can be neglected!
→ only $\overline{i_{eq}^2}$ is important!

(… and again, we need not deal with correlation)



Example: TransR Amplifier Noise

Input-referred current noise:

Open inputs; equate output voltage noise.

Case I:

$v_{oI1} = i_{ia} R_f$

$v_{oI2} = i_f R_f$

$v_{oI3} = v_{ia}$

Move v_{ia} through Rin to

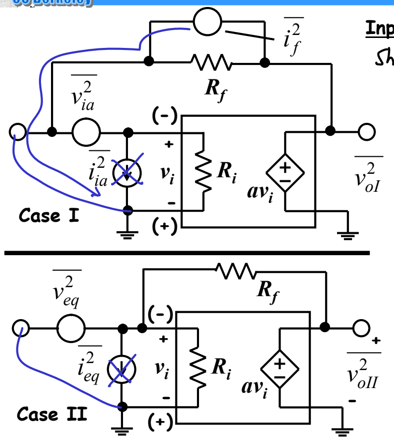
This is unity gain!

$$\therefore \overline{v_{oI}^2} = \overline{i_{ia}^2} R_f^2 + \overline{i_f^2} R_f^2 + \overline{v_{ia}^2}$$

Case II: $\overline{v_{oII}^2} = \overline{i_{eq}^2} R_f^2$

$$\therefore \boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}}$$

Example: TransR Amplifier Noise (cont)



Input-referred voltage noise:

Short inputs; equate output voltage noise

Case I:

$\overline{v_{oI}^2} = \overline{v_{ia}^2} a^2$

(Both $\overline{i_{ia}^2}$ & $\overline{i_f^2}$ are shorted out.)

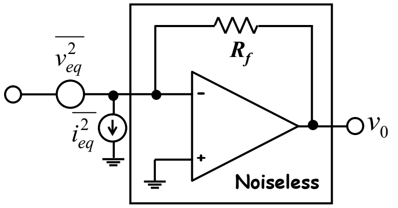
Case II:

$\overline{v_{oII}^2} = \overline{v_{eq}^2} a^2$

$\therefore \boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$

Example: TransR Amplifier Noise (cont)

- To summarize, for a transresistance amplifier, the equivalent input-referred current and voltage noise generators are given by:



$$\boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}}$$

$$\boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$$