Minimum Detectable Signal (MDS)

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Determined Sensor Resolution

Lecture Outline

- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination

Determining Sensor Resolution
Noise

- Noise: Random fluctuation of a given parameter $I(t)$
- In addition, a noise waveform has a zero average value (e.g., could be DC current)

We can handle noise at instantaneous times.
But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation.
Thus, represent noise by its mean-square value:

$$
\int_{-\infty}^{\infty} \left| I(t) - I_D \right|^2 dt
$$

Noise Spectral Density

- We can plot the spectral density of this mean-square value:

$$
\frac{1}{T} \int_{0}^{T} \left| I(t) - I_D \right|^2 dt
$$

Circuit Noise Calculations

- Deterministic: $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- Random: $S_o(\omega) = |H(j\omega)|^2 S_i(\omega)$

**How is it we can do this?**
Handling Noise Deterministically

* Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

\[
\frac{S(f)}{\Delta f} = S_0(f) \rightarrow v_{n1} = \sqrt{S_0(f) \cdot B}
\]

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

\[v_n(t) = A \cos \omega t\]

Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period \(1/B\).

[This is actually the principle by which oscillators work, i.e., oscillators are just noise going through a tiny bandwidth filter]

Systematic Noise Calculation Procedure

1. For \(i_{n1}\), replace w/ a deterministic source of value \(i_{n1} = \frac{I_{n1}^2}{\Delta f} \cdot (1 \text{ Hz})\)

2. Calculate \(v_{n0} = i_{n0} H(j\omega)\) (treating it like a deterministic signal)

3. Determine \(v_{n0}^2 = \left| H(j\omega) \right|^2\)

4. Repeat for each noise source: \(i_{n1}, v_{n2}, v_{n3}\)

5. Add noise power (mean square values)

\[
\begin{align*}
\bar{v}_{nTOT}^2 &= v_{n1TOT}^2 + v_{n2TOT}^2 + v_{n3TOT}^2 + \cdots \\
\bar{v}_{nTOT} &= \sqrt{\bar{v}_{n1TOT}^2 + \bar{v}_{n2TOT}^2 + \bar{v}_{n3TOT}^2 + \cdots}
\end{align*}
\]

Total rms value

Noise Sources
Thermal Noise

- Thermal Noise in Electronics: (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free e-’s in a conducting medium
  - Path of e-’s randomly oriented due to collisions
- Thermal Noise in Mechanics: (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.

* Properties:
  - Thermal noise is white (i.e., constant w/ frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor

Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?

Why 4kTR?

- Why is $\sqrt{V} = 4kTR\Delta f$ (a heuristic argument)
- The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy $(1/2)kT$ associated w/ each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom: current, i, and voltage, v
- Thus, we can write:
  $\frac{1}{2}Li^2 = \frac{1}{2}k_BT$, $\frac{1}{2}Cv^2 = \frac{1}{2}k_BT$

Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\frac{\Delta V}{R}$ or a shunt current generator $\frac{\Delta I}{R}$

Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

where $4kT = 1.66 \times 10^{-20} V \cdot C$ and where these are spectral densities.
**Why 4kTR? (cont)**

* Why is $v_T^2 = 4kTR\Delta f$? (a heuristic argument)
* Consider an RC circuit:

\[ \frac{R}{C} \mathcal{R} \quad \frac{1}{\tau = RC} \quad \frac{v}{C} \quad \frac{v_T}{C} \]

\[ E = \frac{1}{2} kT + \frac{1}{2} C v_T^2 \]

\[ \therefore v_T^2 = \frac{E}{C} \]

- integrated noise over all frequencies

**Question:** What value of $\frac{v_T^2}{\Delta f}$ (assuming white noise) gives us this? 

**Shot Noise**

* Associated with direct current flow in diodes and bipolar junction transistors
* Arises from the random nature by which e-’s and h+’s surmount the potential barrier at a pn junction
* The DC current in a forward-biased diode is composed of h+’s from the p-region and e-’s from the n-region that have sufficient energy to overcome the potential barrier at the junction → noise process should be proportional to DC current
* Attributes:
  - Related to DC current over a barrier
  - Independent of temperature
  - White (i.e., const. w/ frequency)
  - Noise power ~ $I_D$ & bandwidth

**Flicker (1/f) Noise**

* In general, associated w/ random trapping & release of carriers from “slow” states
* Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
* Often, get a mean-square noise spectral density that looks like this:

\[ \frac{I_n^2}{\Delta f} = 2qI_D + K \left( \frac{f}{f_c} \right)^a \]

$I_D$ = DC current
$K = $ const. for a particular device
$a = 0.5 \rightarrow 2$
$b = 1$
**Example: Typical Noise Numbers**

* Hookup the circuit below and make some measurements

![Circuit Diagram]

- Measure w/ AC voltmeter
- Measure w/ spectrum analyzer

![Noise Equation]

\[
V_{n} = \sqrt{(100 \times 10^{-12}) (kT)}
\]

\[
V_{n} = \frac{4 kT R}{C}
\]

\[
V_{n} = 64 \mu V \text{ rms}
\]

**Example: Typical Noise Numbers**

* Hookup the circuit below and make some measurements

![Circuit Diagram]

- Measure w/ AC voltmeter
- Measure w/ spectrum analyzer

![Noise Equation]

\[
V_{n} = \sqrt{(100 \times 10^{-12}) (kT)}
\]

\[
V_{n} = \frac{4 kT R}{C}
\]

\[
V_{n} = 64 \mu V \text{ rms}
\]

**Back to Determining Sensor Resolution**

![MEMS-Based Tuning Fork Gyroscope Diagram]

- (-) Sense Output Current
- (+) Sense Output Current
- Drive Voltage Signal
- Drive Oscillation Sustaining Amplifier
- Differential TransR Sense Amplifier

[Zaman, Ayazi, et al., MEMS'06]
Drive Axis Equivalent Circuit

![Drive Axis Equivalent Circuit Diagram](image1)

Drive Oscillation Sustaining Amplifier

- Generates drive displacement velocity \( x_d \) to which the Coriolis force is proportional
- To Sense Amplifier (for synchronization)

Drive Mode

- Drive Oscillation
- Sustaining Amplifier
- Drive Voltage Signal

Sense Mode

- Digital PLL

Drive-to-Sense Transfer Function

![Drive-to-Sense Transfer Function Diagram](image2)

Minimum Detectable Signal (MDS)

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity
- Sensor Scale Factor
- Sensor Noise
- Circuit Gain
- Circuit Output Noise
- Output

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system
Move Noise Sources to a Common Point

* Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
* Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

Equivalent Input-Referred Voltage and Current Noise Sources

Calculation of \( v_{eq}^2 \) and \( i_{eq}^2 \)

a) To get \( v_{eq}^2 \) for a two-port:

1) Short input, find \( v_{ID}^2 \) (or \( i_{ID}^2 \))
2) For eq. network, short input, find \( v_{ID}^2 \) (or \( i_{ID}^2 \))
   \[ f\left(v_{eq}^2\right) \]
3) Set \( v_{ID} = v_{eq} \) → solve for \( v_{eq}^2 \) (or \( i_{eq}^2 \))
**Calculation of $v_{in}^2$ and $i_{in}^2$ (cont)**

b) To get $i_{in}$ for a 2-port:

1) Open input, find $v_{ij}^2$ (or $i_{ij}^2$).
2) Open input for eq. circuit, find $v_{ij}^2$ (or $i_{ij}^2$).
3) Set $v_{ij}^2 = v_{ij}^2 (i_{ij}) \rightarrow$ solve for $i_{ij}^2$ (or $i_{ij}^2$).

*Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated.*

---

**Cases Where Correlation Is Not Important**

2) $R_S = \text{large}$ (Ideally = $\infty$ for an ideal current source)

\[ v_{in}^{(2)} = \frac{R_S}{R_S + R_m} v_{in}, \quad v_{in}^{(2)} \text{ effectively "opened out"}. \]

\[ v_{in}^2 = \frac{R_S}{R_S + R_m} v_{in}^2, \quad v_{in}^2 = 0! \]

\[ v_{in} = \frac{R_S}{R_S + R_m} v_{in} = 0! \]

\[ \therefore \text{For } R_S = \text{large}, \quad v_{in}^2 \text{ can be neglected} \rightarrow \text{only } i_{in}^2 \text{ is important}. \]

(And again, we need not deal with correlation)

---

**Cases Where Correlation Is Not Important**

2) $R_S = \text{small}$ (Ideally = 0 for an ideal voltage source):

\[ R_S = \text{small, } i_{in}^{(2)} \text{ can be neglected} \rightarrow \text{only } v_{in}^2 \text{ is important}. \]

(Thus, we need not deal with correlation)

---

**Example: TransR Amplifier Noise**

**Input-referred current noise:**

Open input, equal output voltage noise.

**Case I:**

\[ N_i = \frac{v_{in}^2}{2} R_f \]

**Case II:**

\[ N_i = \frac{v_{in}^2}{2} R_f \]

**This is unity gain!**

---

Copyright © 2016 Regents of the University of California
* To summarize, for a transresistance amplifier, the equivalent input-referred current and voltage noise generators are given by:

\[ \overline{v_{eq}^2} = \frac{v_{eq}^2}{R_f} \]

\[ \overline{i_{eq}^2} = i_{eq}^2 + i_{eq}^2 + \frac{v_{eq}^2}{R_f^2} \]

Input-referred voltage noise:

\[ \overline{v_{eq}} = \overline{v_{eq}^2} \]

\[ \overline{i_{eq}} = \overline{i_{eq}^2} \]

\[ \overline{v_{eq}^2} = v_{eq}^2 \]

\[ \overline{i_{eq}^2} = i_{eq}^2 + i_{eq}^2 + \frac{v_{eq}^2}{R_f^2} \]

\[ \overline{v_{eq}} = \overline{v_{eq}^2} \]

\[ \overline{i_{eq}} = \overline{i_{eq}^2} \]

\[ v_{eq}^2 = v_{eq}^2 \]