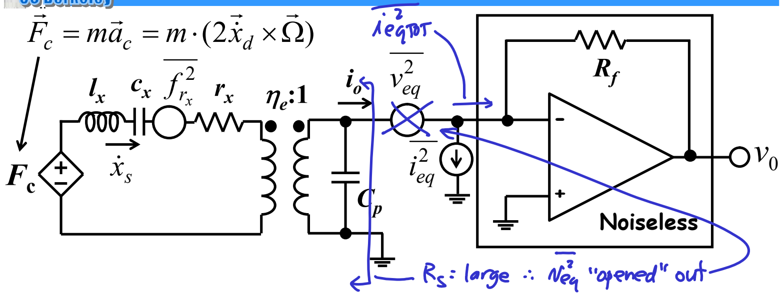
Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m\vec{a}_c = m\cdot(2\dot{\vec{x}}_d \times \vec{\Omega})$

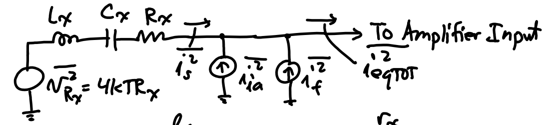


- Now, find the i_{eqTOT} entering the amplifier input:

$i_{eqTOT} = i_s + i_{eq} \rightarrow \overline{i_{eqTOT}^2} = \overline{i_s^2} + \overline{i_f^2} + \overline{i_{ia}^2} + \frac{\overline{v_{ia}^2}}{R_f^2}$ $\frac{\overline{f_{r_x}^2}}{\Delta f} = 4kTr_x$

Brownian motion noise of the sense element → determined entirely by the noise in $r_x \rightarrow \overline{f_{r_x}^2}$
↳ easiest to convert to an all electrical equiv. ckt.

Example: Gyro MDS Calculation (cont)



where $L_x = \frac{l_x}{\eta_e^2}$, $C_x = \eta_e^2 c_x$, $R_x = \frac{r_x}{\eta_e^2}$

$\therefore\ i_s = v_{R_x}\left(\frac{1}{R_x}\right)\Theta(j\omega_d) \rightarrow \frac{\overline{i_s^2}}{\Delta f} = 4kTR_x\left(\frac{1}{R_x^2}\right)|\Theta(j\omega_d)|^2$

$\Rightarrow \boxed{\frac{\overline{i_s^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta(j\omega_d)|^2}$

Thus:

$$\boxed{\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)}$$

learn to get these from EE240.
↳ or just get them from a data sheet...

LF356 Op Amp Data Sheet

LF155/LF156/LF256/LF257/LF355/LF356/LF357
JFET Input Operational Amplifiers

General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

Features

Advantages

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers
- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits

Common Features

- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: 0.01 pA/√Hz
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

$\sqrt{\frac{\overline{i_{ia}^2}}{\Delta f}} = 0.01\ pA/\sqrt{Hz}$

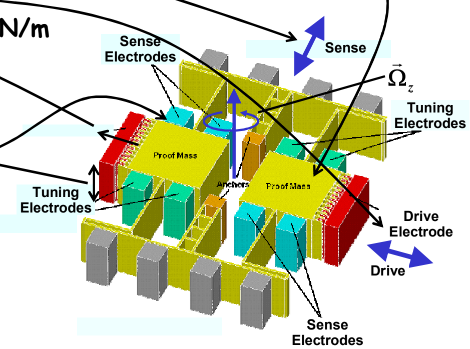
Uncommon Features

	LF155/ LF355	LF156/ LF256/ LF356	LF257/ LF357 (A_V=5)	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	μs
Fast slew rate	5	12	50	V/μs
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	nV/√Hz

$\sqrt{\frac{\overline{v_{ia}^2}}{\Delta f}} = 12\ nV/\sqrt{Hz}$

Example ARW Calculation

- Example Design:
  - Sensor Element:
    - m = (100μm)(100μm)(20μm)(2300kg/m³) = 4.6x10⁻¹⁰kg
    - ω_s = 2π(15kHz)
    - ω_d = 2π(10kHz)
    - $k_s = \omega_s^2 m$ = 4.09 N/m
    - x_d = 20 μm
    - Q_s = 50,000
    - V_P = 5V
    - h = 20 μm
    - d = 1 μm
  - Sensing Circuitry:
    - R_f = 1MΩ
    - i_{ia} = 0.01 pA/√Hz
    - v_{ia} = 12 nV/√Hz



Example ARW Calculation (cont)

Get rotation rate to output current scale factor:

$$A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e |\Theta(j\omega_d)| = 2\left(\frac{10k}{15k}\right)(50k)(20\mu)(5)(2000\epsilon_o)(0.000024) = \underline{2.83\times10^{-12}C}$$

$$\left[\Theta(j\omega_d) = \frac{(j\omega_d)(\omega_s/Q_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10k)(15k)/(50k)}{(15k)^2-(10k)^2 + \frac{j(10k)(15k)}{50k}} = \frac{j(3k)}{1.25\times10^8 + j(3k)}\right.$$

$$\left.\rightarrow |\Theta(j\omega_d)| = \frac{3k}{\sqrt{(1.25\times10^8)^2 + (3k)^2}} = 0.000024\right]$$

$$\left[\frac{\partial C}{\partial x} = \frac{C_o}{d} = \frac{\epsilon_o h W_p}{d} = \frac{\epsilon_o(20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_o \rightarrow \eta_e = V_p\frac{\partial C}{\partial x} = \overbrace{5(2000\epsilon_o)}^{8.854\times10^{-8}F/m}\right]$$

Assume electrode covers the whole sidewall.

ϵ_o = 8.854x10^{-12} F/m

Then, get noise:

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)$$

Example ARW Calculation (cont)

$$\left[R_x = \frac{\omega_s m}{Q_s\eta_e^2} = \frac{2\pi(15k)(4.6\times10^{-10})}{(50k)(8.854\times10^{-8})^2} = 110.6k\Omega\right]$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6k)}(0.000024)^2}_{8.64\times10^{-35}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

Sensor element noise insignificant

Noise from R_f dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.68\times10^{-26}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 1.30\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{1.30\times10^{-13}}{2.83\times10^{-12}}(3600)\left(\frac{180}{\pi}\right) = \underline{9448\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(9448) = 157\ ^\circ/\sqrt{hr} = ARW$$

⇒ Almost turned around in 1 hour!

What if $\omega_d = \omega_s$?

If $\omega_d = \omega_s = 15kHz$, then $|\Theta(j\omega_d)| = 1$ and

$$A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e |\Theta(j\omega_d)| = 2Q_s x_d \eta_e = 2(50k)(20\mu)(5)(2000\epsilon_o) = \underline{1.77\times10^{-7}C}$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6k)}(1)^2}_{1.51\times10^{-25}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

Now, the sensor element dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.67\times10^{-25}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 4.08\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{4.08\times10^{-13}}{1.77\times10^{-7}}(3600)\left(\frac{180}{\pi}\right) = \underline{0.476\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(0.476) = 0.0079\ ^\circ/\sqrt{hr} = ARW$$

⇒ Navigation grade!