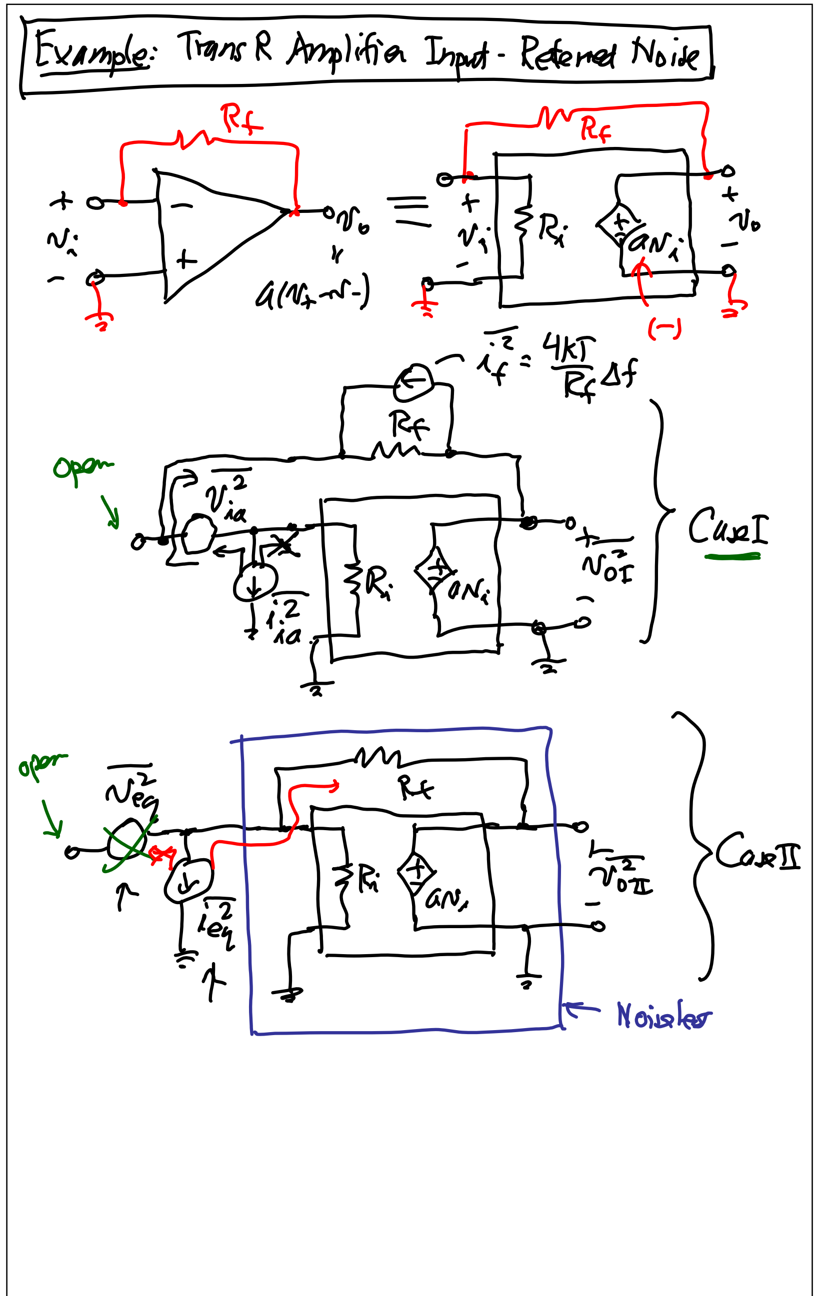
Lecture 27: Gyro Minimum Detectable Signal (MDS)

- Announcements:
- HW#7 online and due Friday, May 6
  - No need to turn in HW#7; we'll just give you time to do it, then release solutions on May 6
  - You will need to understand this material for the Final Exam, so it's in your best interest to do the homework
- Project slide #3 due Friday, April 29, at 5 p.m.
- Project out-brief sign up sheet on Prof. Nguyen's office door
  - People have already taken slots
- Old Final Exams passed out
- Final Exam Info Sheet online
- ------------------------------------
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
  - Course Wrap Up (Final Exam Info)
  - Course Evaluations
- ------------------------------------
- Last Time:
- Going through input referred noise
- Now, continue with this



Input-Referred Current Noise:

Open inputs; equate output voltage noise for Case I & Case II → solve for $\overline{i_{eq}^2}$

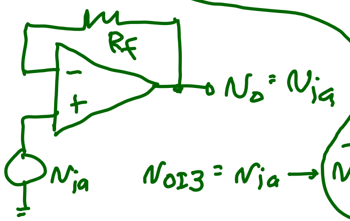
Case I: (use superposition)

$\overline{i_{ia}^2}$: $v_{oI1} = i_{ia} R_f \rightarrow \overline{v_{oI1}^2} = \overline{i_{ia}^2} R_f^2$

$\overline{i_f^2}$: $v_{oI2} = i_f R_f \rightarrow \overline{v_{oI2}^2} = \overline{i_f^2} R_f^2$

power @ output generated by noise sources

$\overline{v_{ia}^2}$:



$v_{oI3} = v_{ia} \rightarrow \overline{v_{oI3}^2} = \overline{v_{ia}^2}$

$\therefore \overline{v_{oI}^2} = \overline{i_{ia}^2} R_f^2 + \overline{i_f^2} R_f^2 + \overline{v_{ia}^2}$

Case II: $v_{oII} = i_{eq} R_f \rightarrow \overline{v_{oII}^2} = \overline{i_{eq}^2} R_f^2$

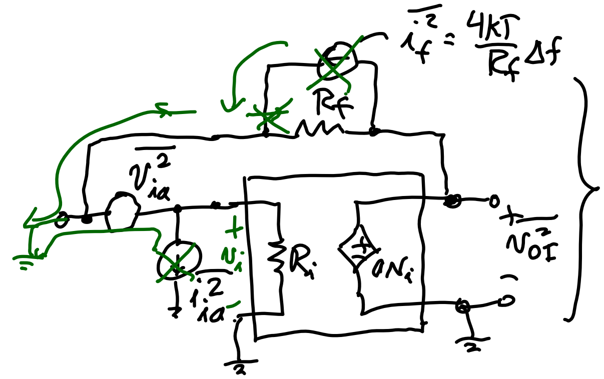
Now, set $\overline{v_{oI}^2} = \overline{v_{oII}^2}$:

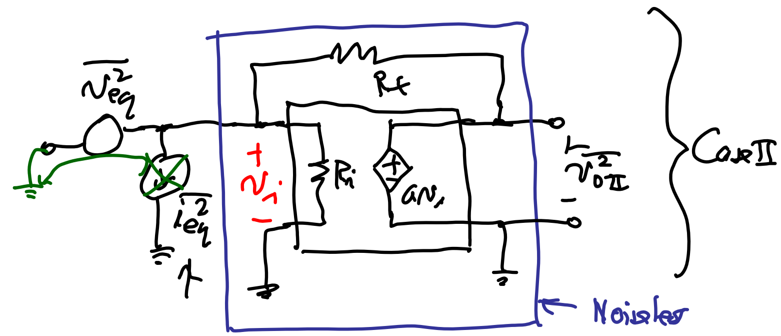
$$\boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}}$$

Now, get the input-referred voltage noise $\overline{v_{eq}^2}$:

Short inputs; equate output voltage noise for Cases I & II

$\overline{i_f^2} = \frac{4kT}{R_f} \Delta f$





Case I: $v_{oI} = a v_{ia} \rightarrow \overline{v_{oI}^2} = a^2 \overline{v_{ia}^2}$

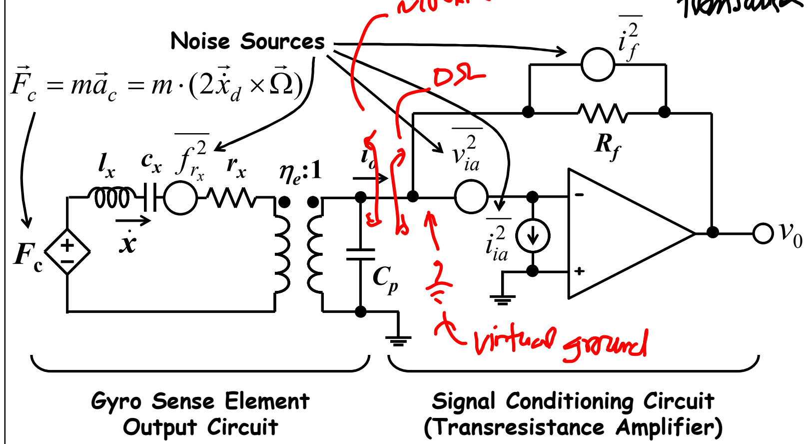
Case II: $v_{oII} = a v_{eq} \rightarrow \overline{v_{oII}^2} = a^2 \overline{v_{eq}^2}$

$\overline{v_{oI}^2} = \overline{v_{oII}^2} \rightarrow \boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$

Gyro Sensing Ckt → Want MDS

⇒ First, need $\frac{i_o}{\Omega}(s)$. ← rotation input

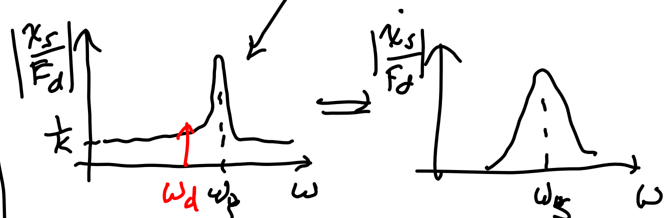
ensures current generated @ output capacitive transducer



⇒ Find the rotation-to-i_o (Ω-to-i_o) transfer fcn:

$$\dot{x}_s = \frac{F_c\omega_s}{k_{eff}} = F_c\left(\frac{\omega_s Q_s}{k_s} H_s(j\omega_d)\right) \qquad x_s = \frac{F_c}{k}$$

$[F_c = m a_c = 2\omega_d x_d \Omega m]$



$$\dot{x}_s = \frac{\omega_s Q_s}{k_s}\cdot 2\omega_d x_d \Omega m \; H_s(j\omega_d), \qquad \frac{m}{k_s} = \frac{1}{\omega_s^2}$$

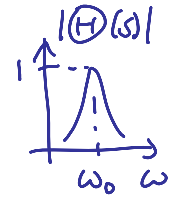
*

$$H(s) = \frac{s(\omega_o/Q)}{s^2 + s(\omega_o/Q) + \omega_o^2}$$

$s=0$: $H(0) = 0$

$s=j\omega_o$: $H(j\omega_o) = 1$

$s=\infty$: $H(\infty) = 0$



* $\dot{x}_s = 2\frac{\omega_d}{\omega_s}Q_s x_d H_s(j\omega_d)\cdot\Omega$

$$i_o = \eta_e\dot{x}_s = \underbrace{2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e H_s(j\omega_d)}_{A \,\triangleq\, \text{scale factor}}\cdot\Omega$$

$$\therefore\ \boxed{i_o = A\Omega,\ \text{where } A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e H_s(j\omega_d)}$$

input rotation — sense freq. — drive freq.

When $\Omega = \Omega_{min} \triangleq$ MDS → $i_o = i_{eqTOT}$

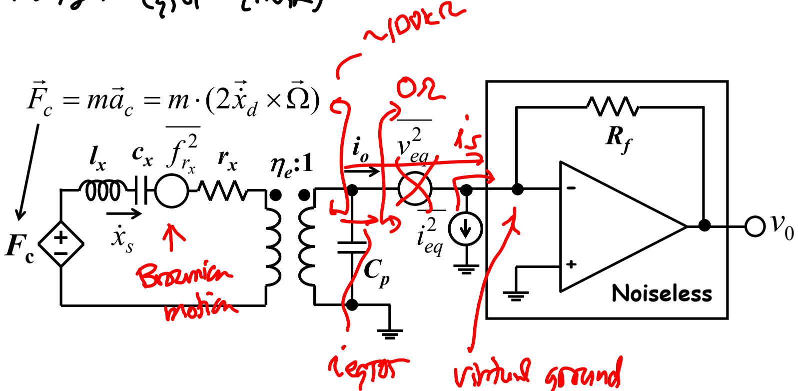
input-referred noise current entering the sense amplifier (in pA/√Hz)

want small as possible

$$\boxed{\Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600\,s}{hr}\right)\left(\frac{180°}{\pi}\right)\ \left[(°/hr)/\sqrt{Hz}\right]}$$

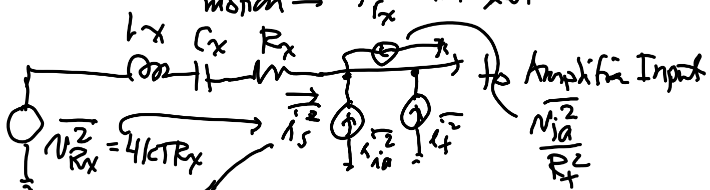
$$\boxed{\text{Angle Random Walk (ARW)} = \frac{1}{60}\Omega_{min}\ \left[°/\sqrt{hr}\right]}$$

Now get i_{eqTOT}: (noise)



$$\overline{i_{eqTOT}^2} = \overline{i_s^2} + \overline{i_{eq}^2} = \overline{i_s^2} + \overline{i_f^2} + \overline{i_{ia}^2} + \frac{\overline{v_{ia}^2}}{R_f^2}$$

Brownian motion → $\overline{f_{r_x}^2} = 4kTr_x\Delta f$



Where $L_x = \frac{l_x}{\eta_e^2}$, $C_x = \eta_e^2 c_x$, $R_x = \frac{r_x}{\eta_e^2}$

$$\therefore\ i_s = v_{R_x}\left(\frac{1}{R_x}\right)\Theta_s(j\omega_d) \rightarrow \frac{\overline{i_s^2}}{\Delta f} = 4kTR_x\left(\frac{1}{R_x^2}\right)\left|\Theta_s(j\omega_d)\right|^2$$

$$\Rightarrow \boxed{\frac{\overline{i_s^2}}{\Delta f} = \frac{4kT}{R_x}\left|\Theta_s(j\omega_d)\right|^2}$$

- Go through slides 43-49 in Module 17

- Related courses at UC Berkeley:
  - EE 143: Microfabrication Technology
  - EE 147/247A: Introduction to MEMS
  - ME 119: Introduction to MEMS (mainly fabrication)
  - BioEng 121: Introduction to Micro and Nano Biotechnology and BioMEMS
  - ME C219 – EE C246: MEMS Design
  - EE 290M?