Lecture 2: Benefits of Scaling I

- Announcements:
- The notes from last time are online, as well as the video – both in the Lecture link table
- Modules 1 & 2 are online (also, in the Lecture link table)
- HW#1A (due Wednesday, Feb. 3) online at the Homework link
- HW#1B (due Wednesday, Feb. 10) also online
- As announced last time, I will be traveling next week (at the IEEE MEMS Conference)
  - Next week's lectures will be by recorded video
  - The videos will be online in the Lecture link table in the far right column
  - Please watch the videos before the week after next to avoid falling behind
- Get your computer accounts by following the instructions at the end of the Course Info Sheet (the new one recently uploaded)
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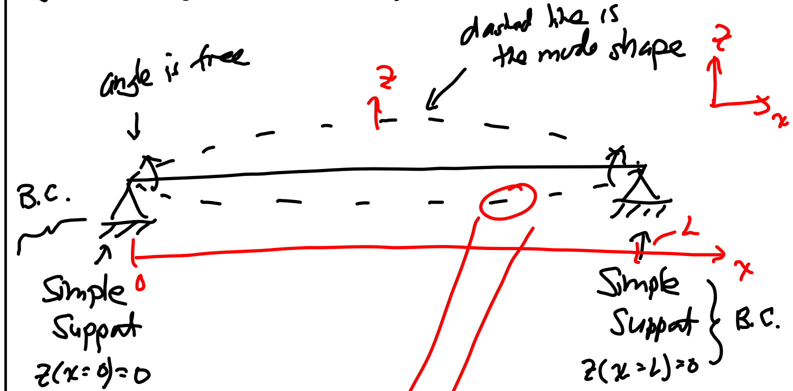
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
- -----------------------------------
- Finish Module 1
- Start going through Module 2

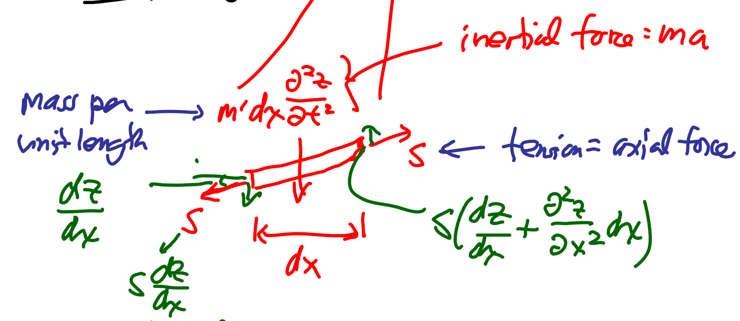
Scaling of Guitar Strings

guitar string ≡ transversely vibrating stretched wire



Get Equation for Resonance Freq.

⇒ free body diagram:



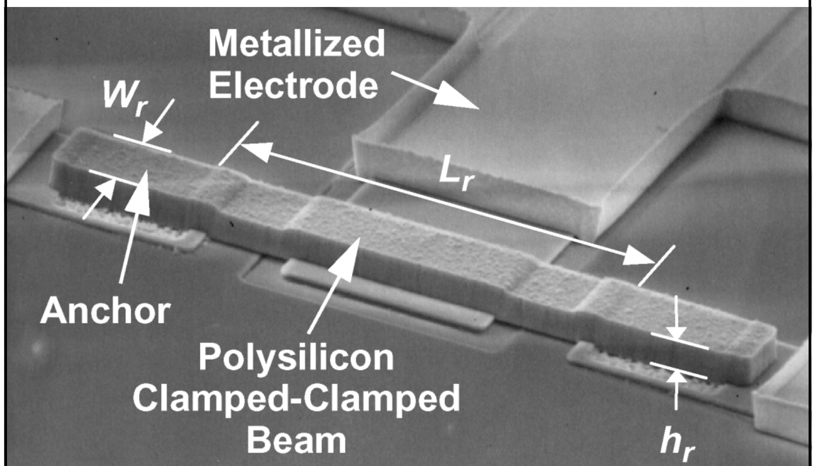
⇒ condition for dynamic equilibrium:

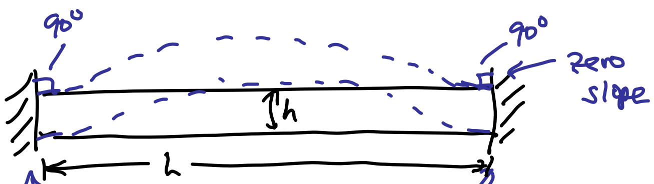
$$S\left(\frac{dz}{dx} + \frac{\partial^2 z}{\partial x^2} dx\right) - S\frac{dz}{dx} - m' dx \frac{\partial^2 z}{\partial t^2} = 0$$

solve ⤷ $\boxed{f_i = \frac{i}{2L}\sqrt{\frac{S}{m'}}}$ → if $L\downarrow$ → $f_i \uparrow$

↑ i^{th} mode

Clamped-Clamped Beam





B.C.'s: Clamped (a Fixed)

$\Rightarrow$ Eq. for Resonance:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2}$$

where $E \triangleq$ Young's modulus [GPa]

$\rho \triangleq$ density [kg/m³]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

Example: $L = 40\mu m$, $h = 2\mu m$

polySi $\rightarrow$ $E = 150GPa$, $\rho = 2300\ kg/m^3$

$$\therefore f_o = (1.03)\underbrace{\sqrt{\frac{\overset{10^9}{150G}}{2300}}}\cdot\frac{2\mu}{(\underset{10^{-6}}{40\mu})^2} \Rightarrow \boxed{f_o = 10.4MHz}$$

$$\sqrt{\frac{E}{\rho}} = \begin{cases}\text{acoustic} \\ \text{velocity}\end{cases}$$

polySi: 8076 m/s

bigger, smaller $\rightarrow$ 2X, ½X $\rightarrow$ S

Scaling:

1. Scale all dimensions equally by a factor S

$$f_o = \underbrace{1.03\sqrt{\frac{E}{\rho}}}_{\text{const.}}\frac{h}{L^2} \sim \frac{S}{S^2} = \frac{1}{S}$$

2. If scale L only: $f_o \sim \frac{1}{S^2}$ $\rightarrow$ much faster rise in f_o
...but...problems...

Example. $L = 4\mu m \rightarrow f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} = 1.04GHz$