Lecture 3: Benefits of Scaling II

- **Announcements:**
- As announced last time, I am on travel right now
- This is a pre-recorded video
- The notes from last time are online, as well as the video – both in the Lecture link table
- Modules 1 & 2 are online (also, in the Lecture link table)
- HW#1A (due Wednesday, Feb. 3) online at the Homework link
- HW#1B (due Wednesday, Feb. 10) also online
- Get your computer accounts by following the instructions at the end of the Course Info Sheet (the new one recently uploaded)
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- **Today:**
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
- -----------------------------------
- **Last Time:**
- Going through Module 2, looking at a clamped-clamped beam resonator example

⇒ Eq. for Resonance Freq:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \quad (1)$$

where $E \triangleq$ Young's modulus of elasticity [GPa]
$\rho \triangleq$ density [kg/m³]
$h \triangleq$ thickness [m]
$L \triangleq$ length [m]

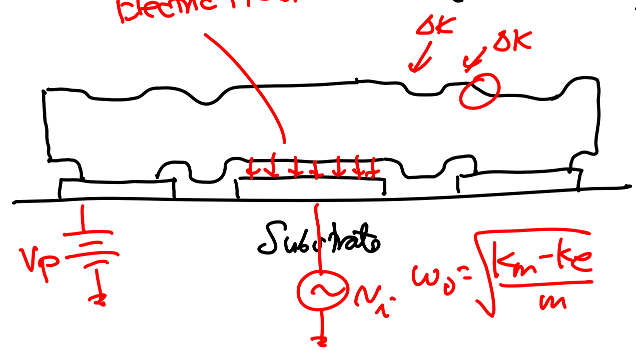
Example: $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150\,GPa$, $\rho = 2300\,kg/m^3$

$$\therefore f_o = (1.03)\sqrt{\frac{150G}{2300}}\frac{2\mu}{(40\mu)^2} \Rightarrow f_o = 10.4MHz$$

Why isn't this 8.5MHz? (as measured)

generates electrical stiffness ← Electric Field



$$\omega_o = \sqrt{\frac{k_m - k_e}{m}}$$

Scalings:

2x, ½x

① Scale all dimensions equally by a factor S

$$f_o \sim \frac{S}{S^2} = \frac{1}{S}$$

② If scale L only: $f_o = \frac{1}{S^2}$ → even faster rise in freq.!

(... but problems ...)

Example:

$L = 4\mu m$ → $f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} =$ **1.04 GHz**

↑ ignore width effect → really need ~3μm

questionable thing to do

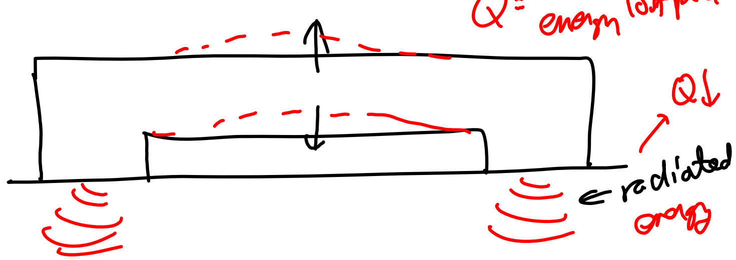
Remarks:

① Eq.(1) not accurate when $L \simeq W \simeq h$

② When $L \simeq h$ (or when it isn't more than 10×h)

↳ get anchor loss problems

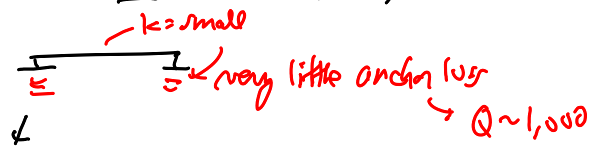
$$Q = \frac{\text{energy per cycle}}{\text{energy lost per cycle}}$$



↓ Solns:

③ Solution: use nanodimensions! ✓

↳ ex.: h=300nm, L~1μm



Problem: power handling ↓ when size ↓

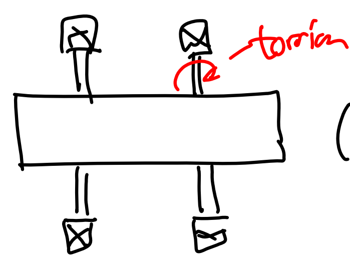
↳ Soln: use massive numbers in arrays!

④ Better Soln: use other geometries

Free-Free Beam: nodal points

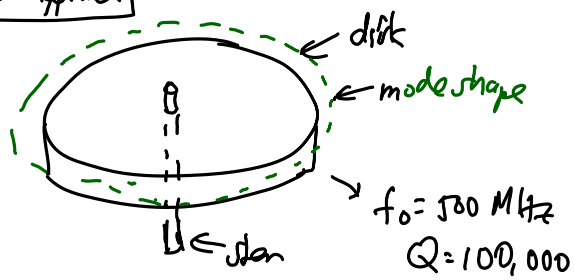
(side view)

no vertical motion → less loss from pumping into the substrate



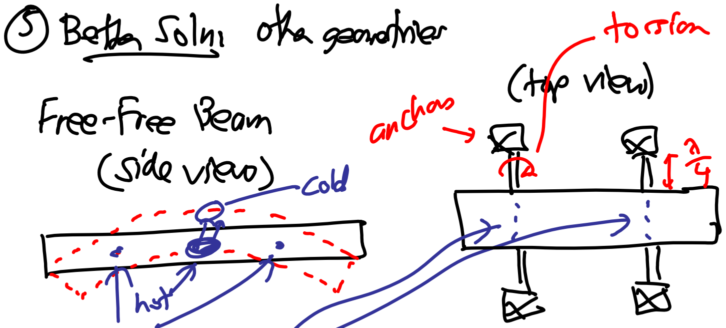
(top view)

Even Better Approach



(5) Better Soln: other geometries

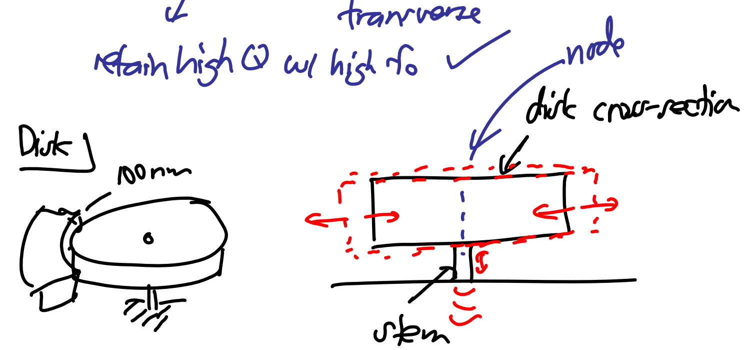
Free-Free Beam (side view)



nodal locations → no motion (transverse)

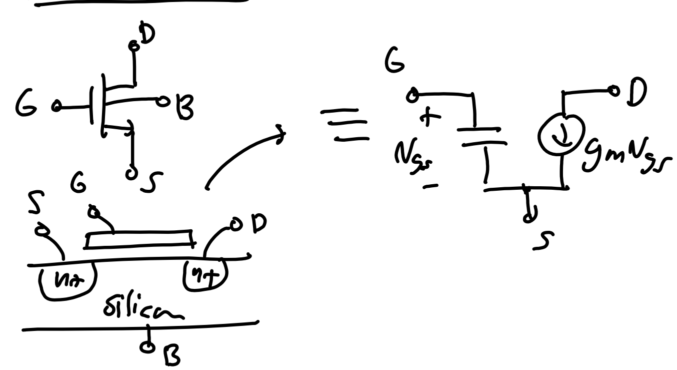
retain high Q w/ high f_o ✓

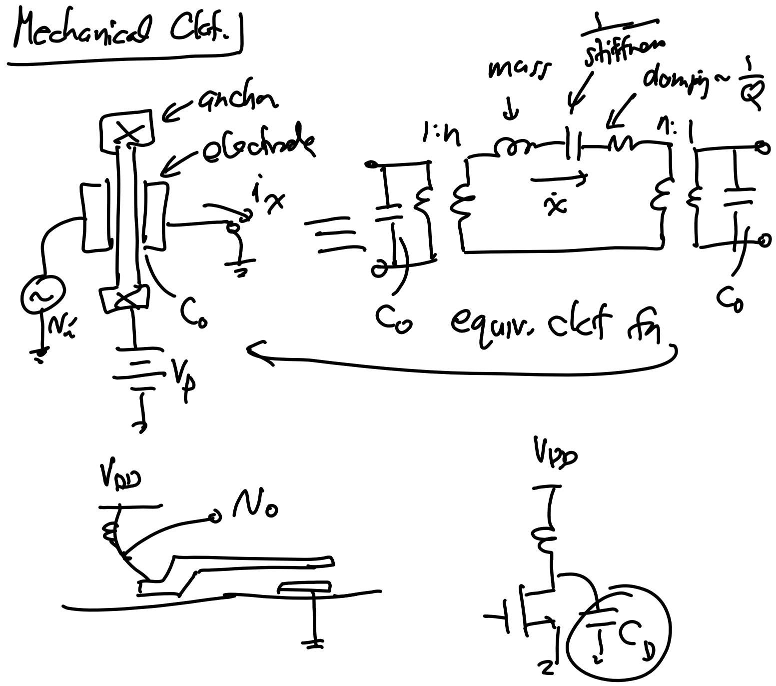
Disk



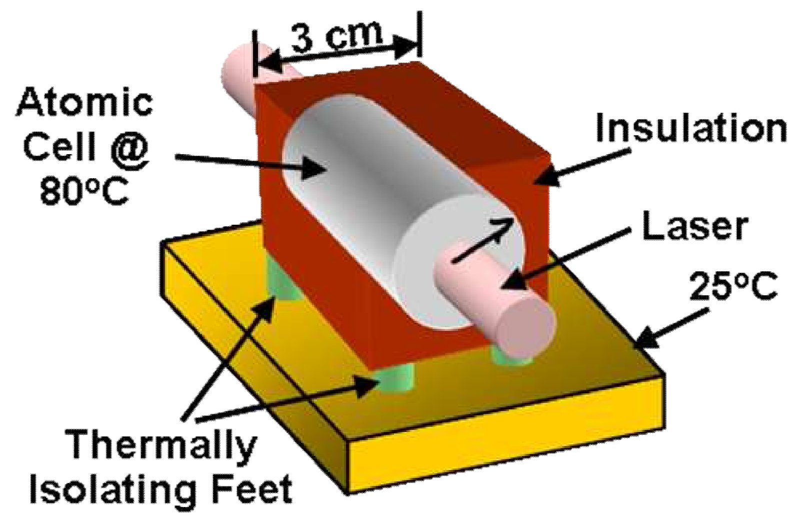
Circuit Design

Transistor Ckt.



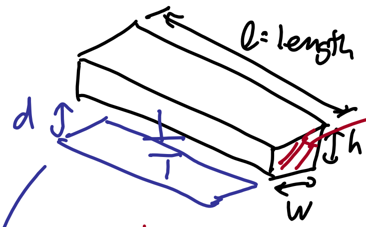


- Now go through Module 2, slides 9-30



Review Electrical Resistance First

(then attack the thermal R analogy)



cross-sectional area = $A = hW$

$R_e \triangleq$ electrical resistance $= \dfrac{l}{\sigma A}$

σ ↑ electrical conductivity

C_e = capacitance = $\dfrac{\epsilon_o W L}{d}$