Lecture 3: Benefits of Scaling II

Announcements:

- As announced last time, I am on travel right now.
- This is a pre-recorded video.
- The notes from last time are online, as well as the video - both in the Lecture link table.
- Modules 1 & 2 are online (also, in the Lecture link table).
- HW#1A (due Wednesday, Feb. 3) online at the Homework link.
- HW#1B (due Wednesday, Feb. 10) also online.
- Get your computer accounts by following the instructions at the end of the Course Info Sheet (the new one recently uploaded).

Today:

- Reading: Senturia, Chapter 1.
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples:
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph

Last Time:

- Going through Module 2, looking at a clamped-clamped beam resonator example.

Equation for Resonance Freq:

\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 1.03 \sqrt{\frac{E}{\rho L^2}} \]

where:

- \( E \) = Young's modulus of elasticity [GPa]
- \( \rho \) = density [kg/m^3]
- \( h \) = thickness [m]
- \( L \) = length [m]

Example: \( L = 49\mu m, h = 2\mu m \)

\[ f_0 = (1.03) \sqrt{\frac{150}{2300}} \frac{2\mu}{(4\mu)^2} = f_0 = 10.4MHz \]

Why isn't this 0.5MHz?

(as measured)
Scalings

1. Scale all dimensions equally by a factor \( s \)
   \[ f_0 \sim \frac{s}{s^2} = \frac{1}{s} \]

2. If scale \( L \) only: \( f_0 = \frac{1}{s^2} \) — even small increase in freq.
   (...) but problem...

Example:

\( L = 4 \, \mu m \) — \( f_0 = (1.03)(8076) \frac{2\pi}{(4\mu m)^2} = 1.04 \, GHz \)

Remove:

1. Eq. (i) not accurate when \( L \approx W \approx h \)
2. When \( L \approx h \), can't cut more than \( 10 \times h \)
   
   May anchor less problems

Remedy:

\( Q = \frac{\text{energy per cycle}}{\text{energy lost per cycle}} \)

Solution:

- We homodimensional!
  
  \( \xi \) small

- May little anchor loss

Problem: power handle to sustain flow

Solution: use more number in array!

Both sides: use other geometry

Free-Free Beam: nodal points

Ride view

No vertical motion — low loss from pumping into substrate

Top view
Even Better Approach

Disk

\[ f_0 = 580 \text{ MHz} \]
\[ Q = 100,000 \]

Circuit Design
Transistor Circuit

\[ G \]

\[ \frac{V_{DS}}{V_{GS}} \]

\[ N_{S} \]

\[ G \]

\[ V_{DS} \]

\[ V_{GS} \]

\[ V_{IN} \]
• Now go through Module 2, slides 9-30
Review Electrical Resistance First

(Then address the thermal R analogy)

\[ R_e = \text{electrical resistance} = \frac{l}{\sigma A} \]

\[ C_e = \text{capacitance} = \frac{\varepsilon W L}{d} \]

\[ \text{cross-sectional area} = A = hw \]