Lecture 5: Benefits of Scaling IV

- Announcements:
- HW#1A due Wednesday this week
- HW#1B due Wednesday the week after HW#1A
- Lecture Modules 3 & 4 on Process Modules online
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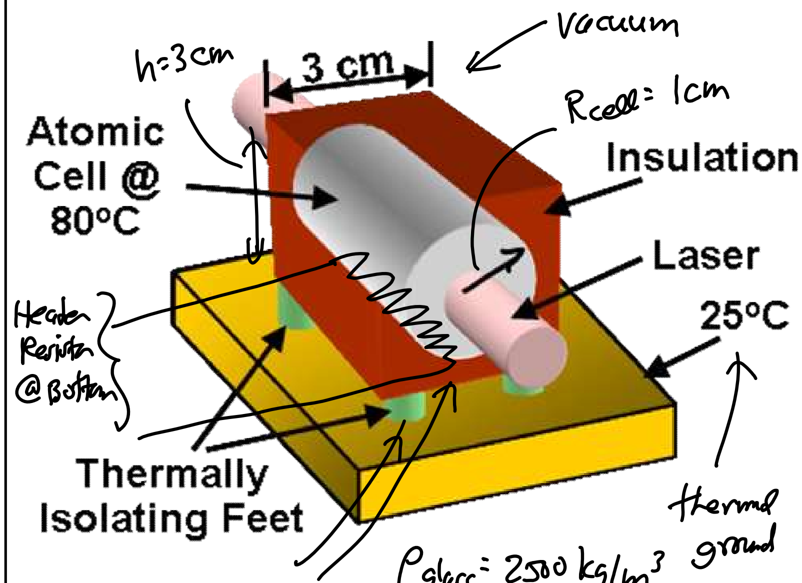
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - —GHz micromechanical resonators
    - —Chip-scale atomic clock
    - —Thermal Circuits
    - —Micro gas chromatograph
- Senturia, Chpt. 3; Jaeger, Chpt. 2, 3, 6
  - Example MEMS fabrication processes
  - Photolithography
  - Etching
  - Oxidation
  - Film Deposition
  - Ion Implantation
  - Diffusion
- ------------------------------------
- Last Time: Thermal circuit modeling

Example: Thermal Clock.

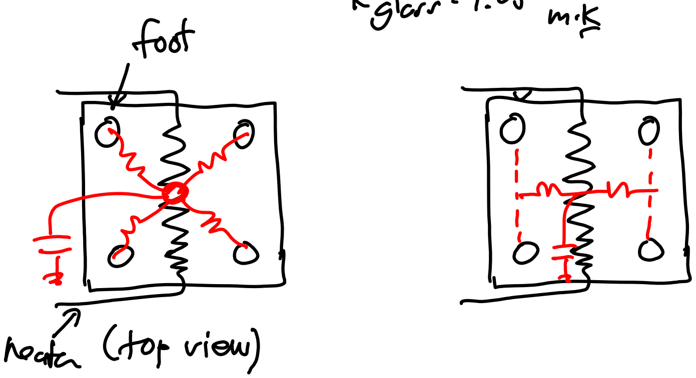
⇒ determine the power needed to get this atomic cell to 80°C (from RT) & how fast

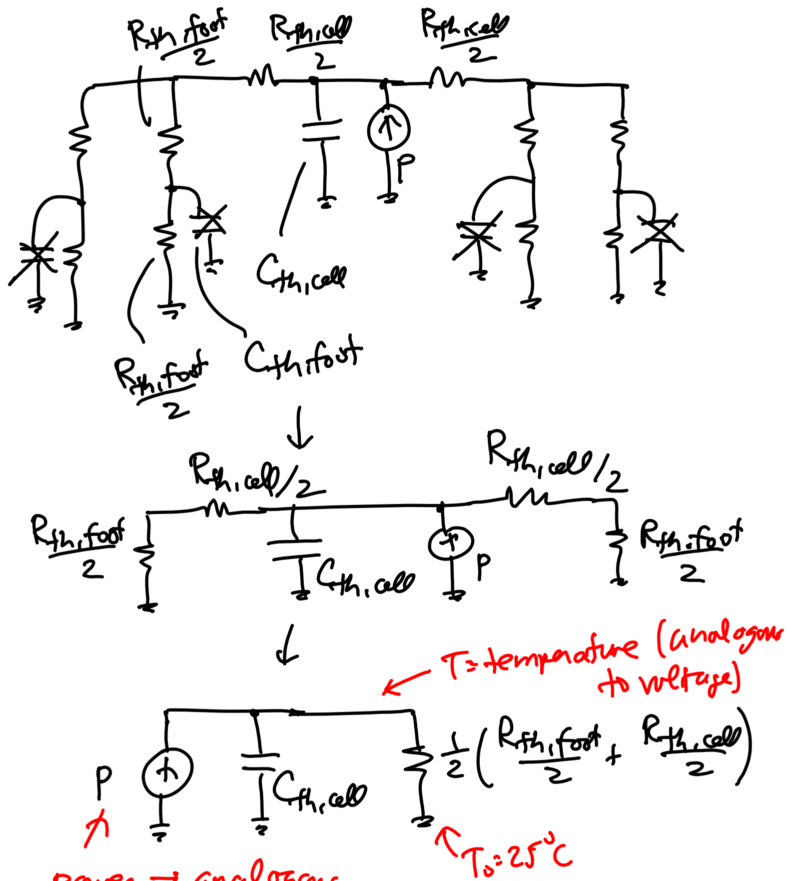


all glass: $\rho_{glass} = 2500\ kg/m^3$

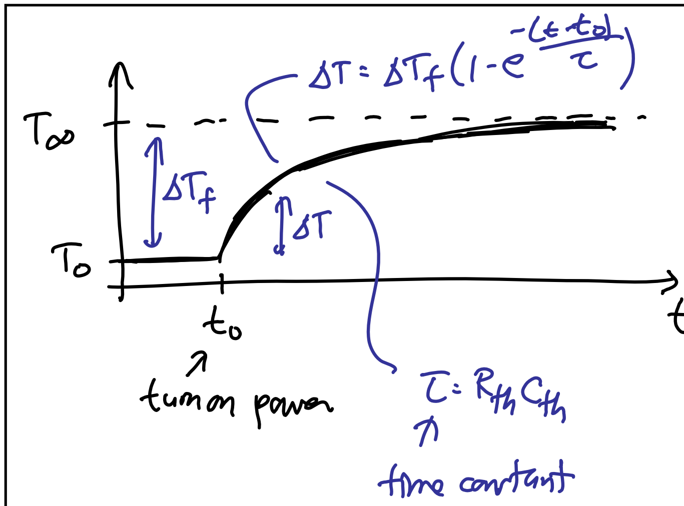
$C_{p,glass} = 0.5\ J/(g \cdot K)$

$K_{glass} = 1.05 \frac{W}{m \cdot K}$



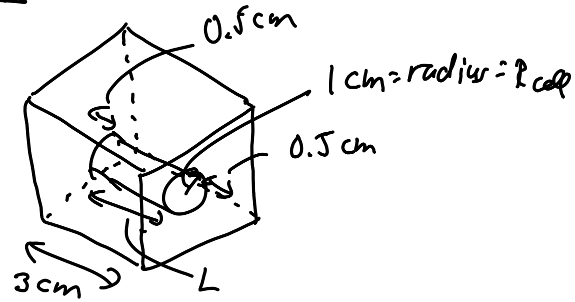


$\Delta T = T - T_0 = P R_{th}$ (in steady-state)



Find $C_{th,cell}$:

First, find the cell volume:

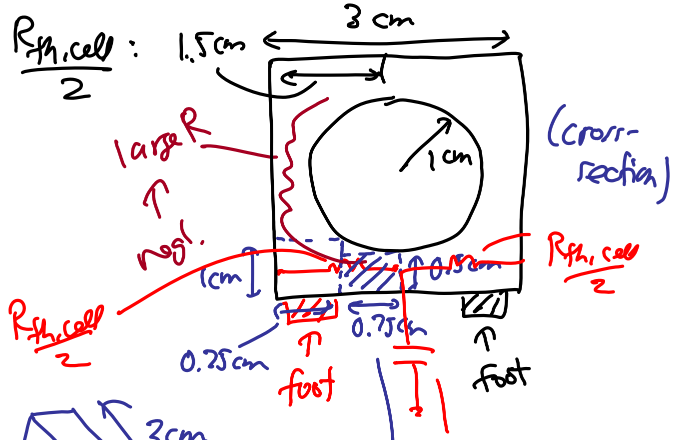


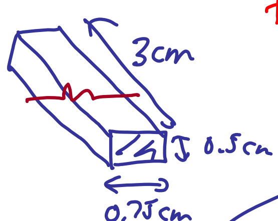
$$V_{cell} = hWL - \pi R_{cell}^2 \cdot L$$
$$= (3cm)(3cm)(3cm) - \pi(1cm)^2(2cm) = 20.7\ cm^3$$

$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$$
$$= \left(2500\ \frac{kg}{\cancel{m^3}}\right)\left(1000\ \frac{g}{\cancel{kg}}\right)\left(\frac{1}{100^3}\ \frac{\cancel{m^3}}{\cancel{cm^3}}\right)$$
$$\times\ (20.7\ \cancel{cm^3})\left(0.5\ \frac{J}{\cancel{g} \cdot K}\right)$$

$$\Rightarrow C_{th,cell} = \underline{\underline{25.9\ J/K}}$$

Find $\frac{R_{th,cell}}{2}$:



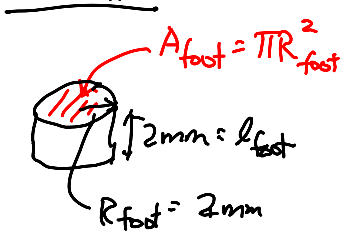


$$\frac{R_{th,cell}}{2} = \frac{\frac{3}{4}}{k(3)(\frac{1}{2})} + \frac{\frac{3}{4}}{k(3)(1)} = \frac{1}{k}\left(\frac{1}{8}+\frac{1}{4}\right) = \frac{3}{8}\frac{1}{k}$$

$\boxed{R_{th} = \frac{\ell}{kA}}$

$$\therefore \frac{R_{th,cell}}{2} = \frac{3}{8}\frac{1}{1.05} \times \left(100\,\frac{cm}{m}\right) = 35.7\ K/W$$

Find $R_{th,foot}$:



$$\therefore R_{th,foot} = \frac{\ell_{foot}}{kA_{foot}} = \frac{(2mm)}{(1.05\frac{W}{m\cdot K})\pi(2mm)^2} = 151.6\,\frac{K}{W}$$

Then:

$$R_{th} = \frac{1}{2}\left(\frac{R_{th,foot}}{2} + \frac{R_{th,cell}}{2}\right)$$

$$= \frac{1}{2}\left(\frac{151.6}{2} + 35.7\right) \rightarrow R_{th} = 55.8\ K/W$$

⇒ Find the power req'd to maintain $T_\infty = 80°C$ in steady-state:

$$P = \frac{T_\infty - T_0}{R_{th}} = \frac{(80-25)}{55.8} = 0.99W \sim 1W$$

⇒ Find the time constant:

$$\tau = R_{th}C_{th,cell} = (55.8\ K/W)(25.9\ J/K) = 24\ min.$$

It takes 3τ to reach steady-state

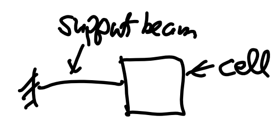
∴ must wait 72 min. before using this atomic cell!

How about using MEMS?

(how about scaling this?)

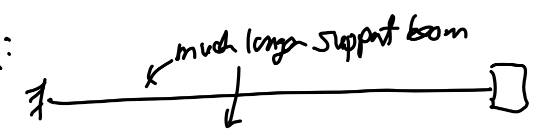
⇒ much smaller cell volume → $V\downarrow \rightarrow C_{th}\downarrow$; → weight $= mg\downarrow$

Macro:



↓ shrink dimensions

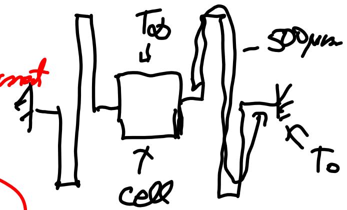
Micro:

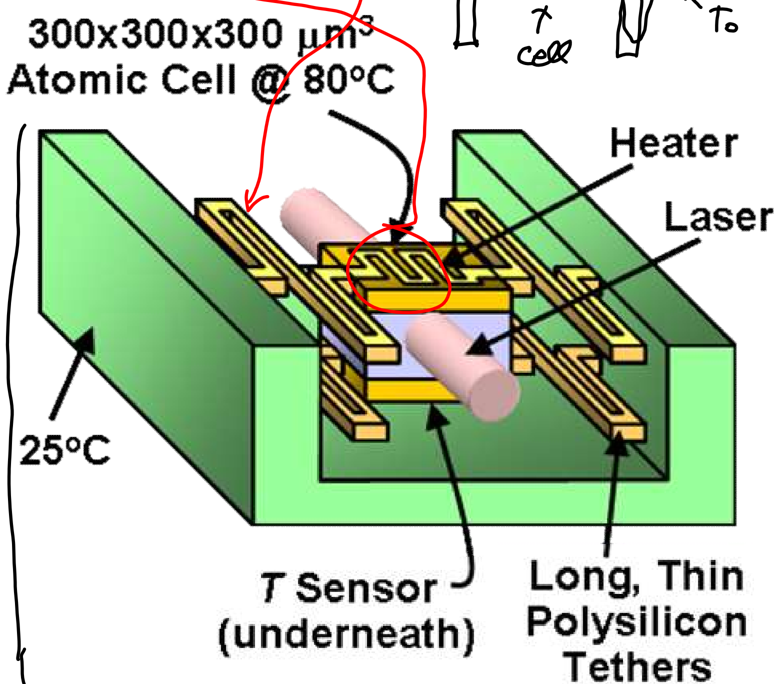


$R_{th}\uparrow \rightarrow P\downarrow$

MEMS Atomic Cell

Assumption: $R_{heater} \gg R_{interconnect}$





Cell is hollow w/ 10μm-thick walls.

Get $C_{th,cell}$:

$$V_{cell} = (300\mu)(300\mu)(300\mu) - (280\mu)(280\mu)(280\mu)$$
$$= 5.048 \times 10^{-12}\, m^3$$

(much smaller than macro!)

$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$$
$$= \left(2500 \frac{kg}{m^3}\right)\left(5.048\times10^{-12}\, m^3\right)\left(500 \frac{J}{kg\cdot K}\right)$$
$$= 6.31\times10^{-6} \frac{J}{K}$$ ← 4 million X smaller than macro

$$R_{th,supp} = \frac{l_{supp}}{k_{polySi} W_{supp} h_{supp}} = \frac{500\mu}{\left(30 \frac{W}{m\cdot K}\right)(20\mu)(10\mu)}$$

$$\Rightarrow R_{th,supp} = 83,333\ K/W$$ ← 548X larger than macro!

... and ...

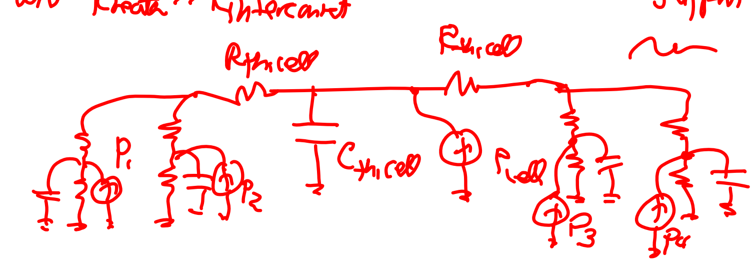
$$P = \frac{(80-25)}{\frac{1}{4}(83,333)} = (2.64mW)4 \approx 10mW$$

four supports — much smaller than 1W!

$$\tau = \frac{0.13}{4}s = 0.03s$$ ← much faster than 72 min.!

All Due to Scaling!

W/O $R_{heater} \gg R_{interconnect}$



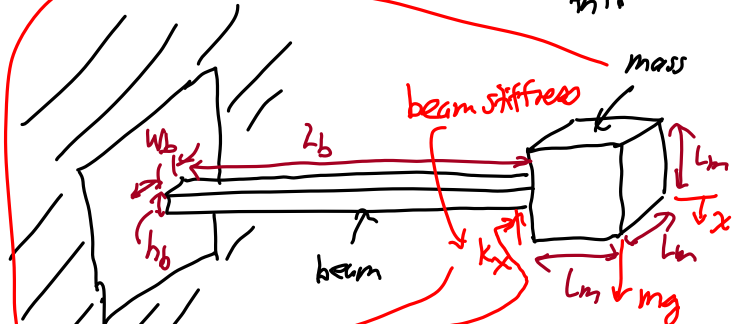
Remarks. (what makes all this possible?)

① scaling reduces $C_{th} \sim \ell^3 \rightarrow s^3$

$\searrow s\downarrow \rightarrow C_{th}\downarrow\downarrow\downarrow$

② scaling allows the use of long, thin, defl. supports

$R_{th}\uparrow\uparrow$



$k_x \triangleq$ stiffness @ this pt. $= \frac{1}{4} E W_b \frac{h_b^3}{L_b^3}$

$s \frac{s^3}{s^3} \sim s$

mass $= \rho L_m^3 \sim s^3$

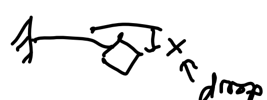
@ static equilibrium:

Force due to Gravity = Spring Force

$mg = kx$ — acceleration due to gravity; displacement

$x = \frac{m}{k} g \sim \frac{s^3}{s} \sim s^2$

as $s\downarrow \rightarrow x\downarrow\downarrow$



$R_{th} = \frac{L_b}{K W_b h_b} \rightarrow$ want to raise this (for lower power consumption atomic cell)

but maintain the same drop x

$$\rho L_m^3 g = \frac{1}{4} E W_b \frac{h_b^3}{L_b^3} x$$

$$\frac{L_b}{W_b h_b} = \frac{1}{4} E \frac{h_b^2}{L_b^2} x \frac{1}{\rho L_m^3 g} \sim \frac{s^2}{s^2} \frac{1}{s^3} \sim \frac{1}{s^3}$$

R_{th}

const.

as $s\downarrow \rightarrow \frac{L_b}{W_b h_b} \sim R_{th}\uparrow\uparrow\uparrow$