


EE C247B - ME C218
Introduction to MEMS Design
Spring 2016

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Lecture Module 11: Equivalent Circuits I

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Lecture Outline

- Reading: Senturia, Chpt. 5
- Lecture Topics:
 - ↗ Lumped Mass
 - ↗ Lumped Stiffness
 - ↗ Lumped Damping
 - ↗ Lumped Mechanical Equivalent Circuits
 - ↗ Electromechanical Analogies

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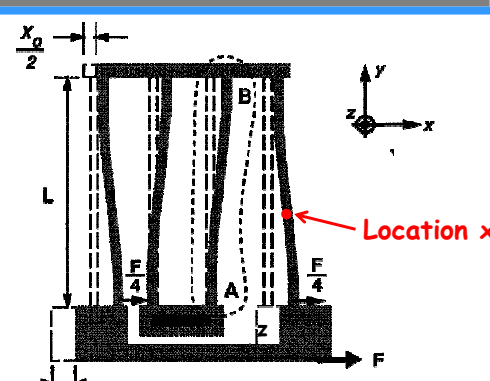
Lumped Parameter Mechanical Equivalent Circuit

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Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy \rightarrow $K.E.$

Equivalent Mass = $M_{eq\ x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^L V^2(x) dx}{\frac{1}{2}V_x^2}$

Maximum Velocity @ location x \rightarrow $\frac{1}{2}V_x^2$

Density \rightarrow $\frac{1}{2}\rho A \int_0^L V^2(x) dx$

Maximum Velocity Function \rightarrow $\frac{1}{2}V_x^2$

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Equivalent Dynamic Mass

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- For the folded-beam structure, we've already determined the maximum kinetic energy
- And in our resonance frequency analysis, we've already determined expressions for velocity

Location on the Truss:

$$M_{eq(truss)} = \frac{KE_{max}}{\frac{1}{2}V_{truss}^2} = \frac{\omega_o^2 x_o^2 (\frac{L}{2}) [M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]}{\frac{1}{2}(\frac{L}{2})\omega_o^2 x_o^2}$$

$$\therefore M_{eq(truss)} = 4 [M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]$$

Location on the Shuttle:

$$M_{eq(shuttle)} = \frac{KE_{max}}{\frac{1}{2}V_{shuttle}^2} = \frac{\omega_o^2 x_o^2 (\frac{L}{2}) [M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]}{\frac{1}{2}\omega_o^2 x_o^2}$$

$$\therefore M_{eq(shuttle)} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$$

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Equivalent Dynamic Stiffness & Damping

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- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_o = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_o^2 M_{eq}(x) \quad \begin{matrix} \Rightarrow \text{large equiv. mass} \\ \text{large stiffness go} \\ \text{hand-in-hand} \end{matrix}$$

- And damping also follows readily from knowledge of Q or other loss measurands

$$Q = \frac{\omega_o M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$

↑
damping

- With mass, stiffness, and damping \Rightarrow lumped parameter equivalent circuit

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Get Potential Energy & Frequency

Truss Parameters:
 $K_{eq(truss)} = 19.2 \text{ N/m}$
 $M_{eq(truss)} = 8.64 \times 10^{-11} \text{ kg}$
 $C_{eq(truss)} = 4.08 \times 10^{-10} \text{ kg/s}$

Shuttle Parameters:
 $K_{eq(shuttle)} = 4.8 \text{ N/m}$
 $M_{eq(shuttle)} = 2.16 \times 10^{-11} \text{ kg}$
 $C_{eq(shuttle)} = 1.02 \times 10^{-10} \text{ kg/s}$

Other Parameters:
 $Q \approx 100K$
 $h = \text{thickness} = 2 \mu\text{m}$
 $\text{Area} = 4,000 \mu\text{m}^2$
 $K_{eq} = M_{eq} = C_{eq} = \infty$

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Electromechanical Analogies

Equation of Motion:
 $F(t) = F \cos(\omega t) \rightarrow x(t) = X \cos \omega t$
 $m_{eq} \ddot{x} + C_{eq} \dot{x} + k_{eq} x = F(t)$
 ⇒ using phasor concepts:
 $F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} x + C_{eq} \dot{x}$
 ⇒ by analogy:

$F \rightarrow N$	$m_{eq} \rightarrow l_x$	$C_{eq} \rightarrow r_x$
$\dot{x} \rightarrow i$	$k_{eq} \rightarrow \frac{1}{C_x}$	

Impedance looking in:
 $N(t) = V \cos \omega t \rightarrow i(t) = I \cos \omega t$
 $\frac{N}{i} = j\omega l_x + \frac{1}{j\omega C_x} + r_x$
 $N = j\omega l_x i + \frac{(1/C_x)}{j\omega} i + r_x i$

Parameter Relationships in the Current Analogy

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Electromechanical Analogies (cont)

• Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness ⁻¹ , k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

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Bandpass Biquad Transfer Function

$$F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} x + c_{eq} \dot{x}$$

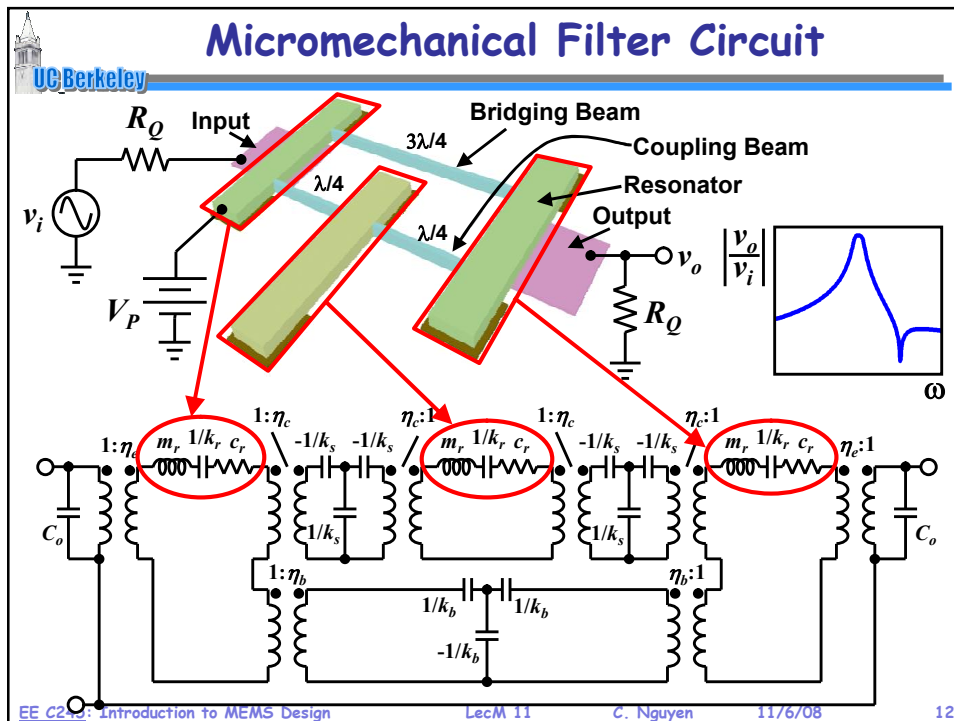
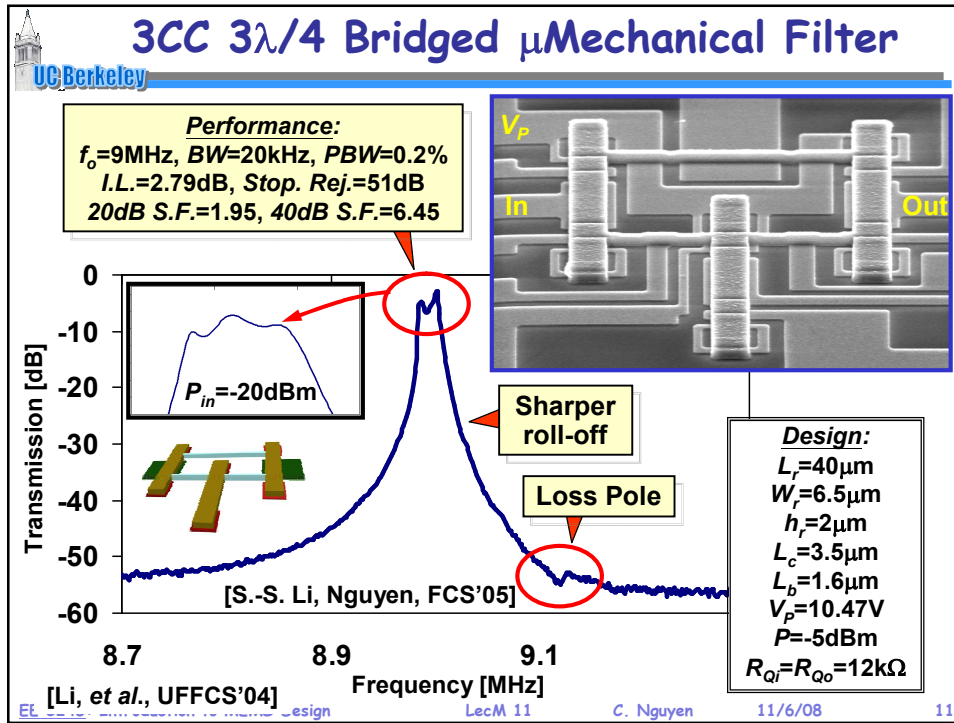
$$\Rightarrow \text{converting to full phasor form:}$$

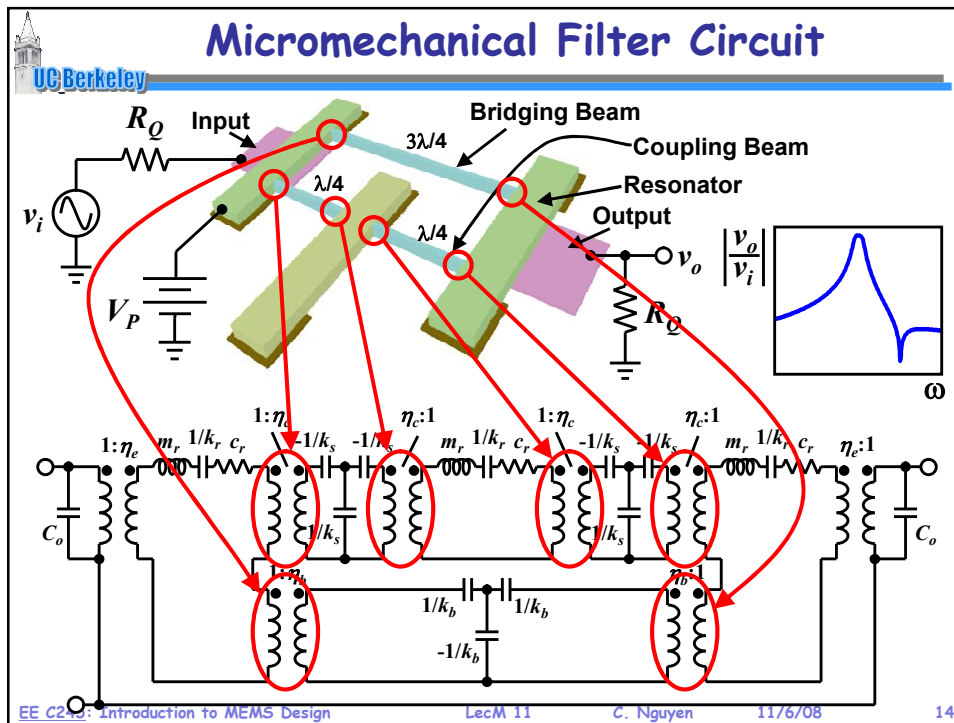
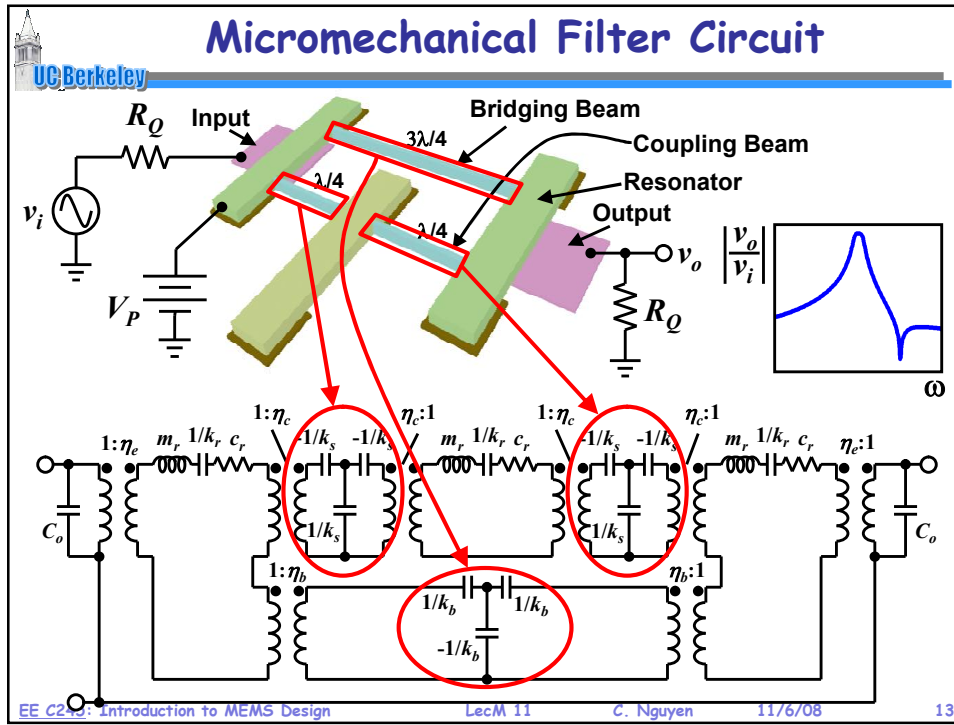
$$F = (j\omega)(j\omega x) m_{eq} + \frac{k_{eq}}{j\omega} (j\omega x) + c_{eq} (j\omega x)$$

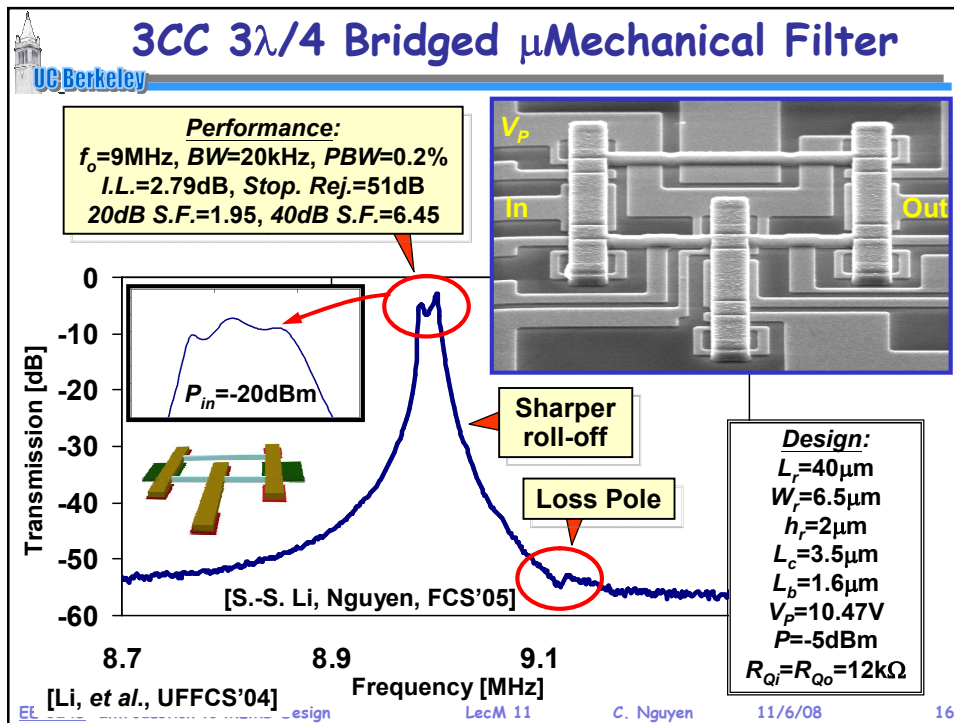
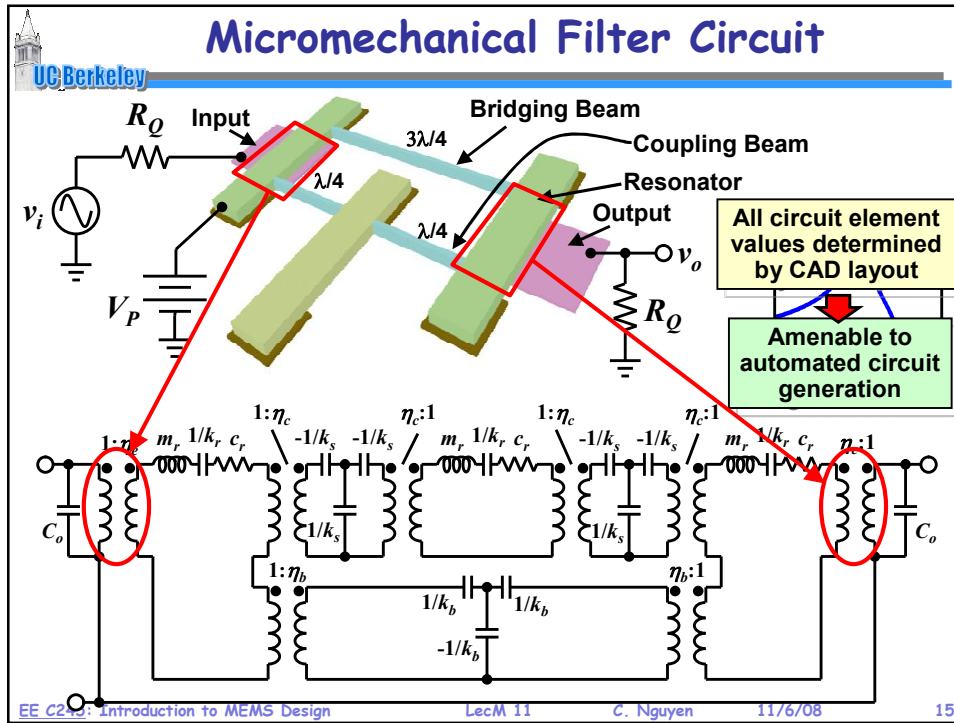
$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}} \left[-\omega^2 \frac{m_{eq}}{k_{eq}} + 1 + j \frac{c_{eq}\omega}{k_{eq}} \right]^{-1} = \frac{1}{k_{eq}} \left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j \frac{\omega}{Q\omega_0} \right]^{-1}$$


$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2, Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0 \right]$$

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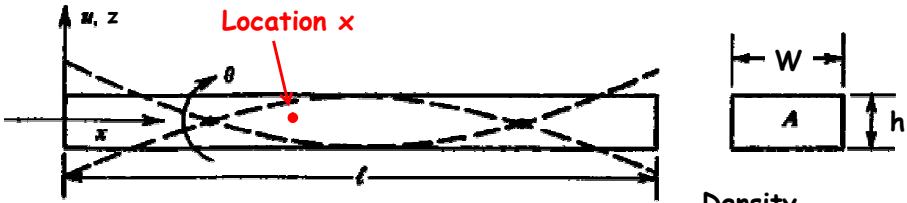
Beam Resonator Equivalent Circuits (Pretty Much the Same Stuff)

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Equivalent Dynamic Mass

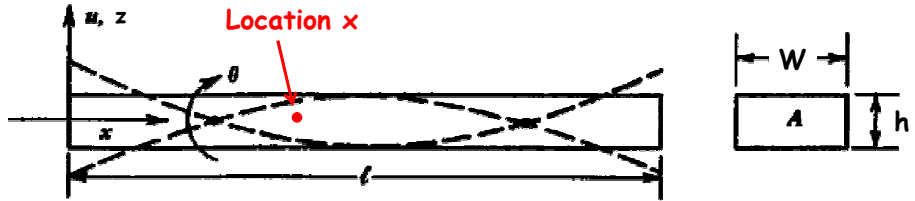
- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy \rightarrow $\frac{1}{2} \rho A \int_0^l v^2(x) dx$
 Density \rightarrow ρ
 Equivalent Mass = $M_{eq\ x} = \frac{K.E.}{\frac{1}{2} V_x^2} = \frac{\frac{1}{2} \rho A \int_0^l v^2(x) dx}{\frac{1}{2} V_x^2}$
 Maximum Velocity @ location x \rightarrow V_x Maximum Velocity Function \rightarrow $v(x)$

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Equivalent Dynamic Mass



- We know the mode shape, so we can write expressions for displacement and velocity at resonance

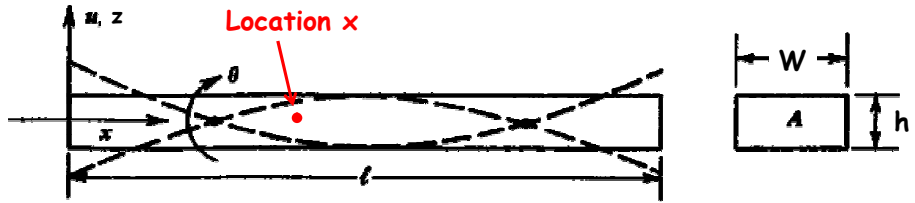
Displacement: $u(x) = B \left[S(\cosh kx + \cos kx) + (\sinh kx + \sin kx) \right], S = \frac{A}{B}$

$[V(x) = \omega u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{\frac{1}{2} \rho A \int_0^l \omega^2 [u(x')]^2 dx'}{\frac{1}{2} \omega^2 [u(x)]^2}$

$$M_{eq}(x) = \frac{\rho A \int_0^l B^2 [S(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')]^2 dx'}{B^2 [S(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}$$

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Equivalent Dynamic Stiffness & Damping



- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_0^2 M_{eq}(x)$$

- And damping also follows readily

$$Q = \frac{\omega_0 M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$

\uparrow damping

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Equivalent Lumped Mechanical Circuit

$K_{eq}(x) = \omega_o^2 M_{eq}(x)$

$M_{eq}(x) = \frac{\rho A \int_0^l [u(x')]^2 dx'}{[u(x)]^2}$

$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$

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Equivalent Lumped Mechanical Circuit

**Example: Polysilicon w/ $l=14.9\mu\text{m}$,
 $W=6\mu\text{m}$, $h=2\mu\text{m} \rightarrow 70\text{ MHz}$**

$K_{eq}(\text{node}) = \infty$
 $M_{eq}(\text{node}) = \infty$
 $C_{eq}(\text{node}) = \infty$

$K_{eq}(0) = 19,927\text{ N/m}$
 $M_{eq}(0) = 1.03 \times 10^{-13}\text{ kg}$
 $C_{eq}(0) = 5.66 \times 10^{-9}\text{ kg/s}$

$K_{eq}(l/2) = 53,938\text{ N/m}$
 $M_{eq}(l/2) = 2.78 \times 10^{-13}\text{ kg}$
 $C_{eq}(l/2) = 1.53 \times 10^{-8}\text{ kg/s}$

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