Lecture Outline

* Reading: Senturia, Chpt. 14
* Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
Velocity-to-Voltage Conversion

To convert velocity to a voltage, use a resistive load.
**Velocity-to-Voltage Conversion**

*To convert velocity to a voltage, use a resistive load*

![Diagram of velocity-to-voltage conversion](image1)

- **Brute force approach:**
  \[
  \frac{N_0}{N_1}(s) = \frac{R_0}{R_0 + \frac{1}{sC_x} + sL_x + R_0} = \frac{R_0}{R_0 + \frac{s(L_0 + R_0)}{L_x}} \times sC_x \]

**Sine this structure has completely symmetrical I/O port:**

- \( R_x \rightarrow Q_x \)
- \( R_x R_0 \rightarrow \frac{R_x R_0}{L_x} \rightarrow \frac{Q_x}{Q_x} \)

- \( Q_x \rightarrow \frac{R_x}{R_x + R_0} \)

**Position-to-Voltage Conversion**

*To sense position (i.e., displacement), use a capacitive load*

![Diagram of position-to-voltage conversion](image2)

- **Brute force approach:**
  \[
  \frac{N_0}{N_1}(s) = \frac{C_D}{sC_D + 1 + sL_x + \frac{1}{sC_D}} = \frac{C_D}{1 + \frac{sL_x}{C_D} + s^2 L_x C_D} \]

**Again, I/O port symmetry:**

- \( Q_x \rightarrow Q_x \)
- \( Q_x \rightarrow \frac{Q_x}{Q_x} \)
- \( Q_x \rightarrow \frac{Q_x}{Q_x + R_x} \)

- \( \frac{Q_x}{Q_x} \rightarrow \frac{Q_x}{Q_x + \frac{R_x}{C_D}} \)

- \( \frac{Q_x}{Q_x} \rightarrow \frac{Q_x}{Q_x + \frac{R_x}{C_D}} \)

**Notes:**

- Velocity-to-voltage conversion uses resistive loads.
- Position-to-voltage conversion uses capacitive loads.
- Both conversions involve symmetry in their I/O ports.

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**Position-to-Voltage Conversion**

To sense position (i.e., displacement), use a capacitive load.

\[
V_o(s) = \frac{C_D}{1 + \frac{C_D}{C_L}} \frac{(s \omega)^2}{s^2 + (s \omega)^2 + (s \omega)^2}
\]

**To maximize gain \( \omega \to 1 \), need \( C_D \gg C_L \).

**Note:** Can use similar short-cut to the 
R case.

1. Get DC response \( C \)'s dominate.
2. Then:

\[
\frac{V_o(s)}{V_i(s)} = \frac{1}{s} \mathcal{Z}(s, \omega_0 \omega) \cdot \omega_0^2
\]

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**Velocity Sensing Circuits**
**Velocity-to-Voltage Conversion**

To convert velocity to a voltage, use a resistive load.

\[ V_o = \frac{V_i}{R} \]

Since this structure has completely symmetrical I/O ports:

\[ \frac{V_o}{V_i} = \frac{1}{R} \]

Work @ resonance: (to simplify the analysis)

\[ \frac{V_o}{V_i} = \frac{R_o}{R_o + R_c} \]

Then, generator is off resonance:

\[ \frac{V_o}{V_i} = \frac{R_o}{R_o + R_c + \sigma(\omega, \theta)} \]

where \( \sigma(\omega, \theta) = \frac{R_o}{R_o + R_c} \)

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**Problems With Purely Resistive Sensing**

Now we get: (approximately)

\[ \frac{V_o}{V_i} \approx \frac{R_o}{R_o + \frac{1}{\frac{d}{d\theta}}} \]

\[ \omega_p^2 = \frac{1}{(R_o + R_c)C_p} \]

Depend on both \( R_o + R_c \).

Impact depends on \( \omega_p \) relative to \( \omega_0 \).

Includes \( C_o \), line \( C \), bond pad \( C \), and next stage \( C \).
Problems With Purely Resistive Sensing

In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input $R_i$ of these circuits can load $R_D$

These change w/ hook-up → not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

**Soln:** use op amps.

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The TransR Amplifier Advantage

* The virtual ground provided by the ideal op amp eliminates the parasitic capacitance $C_p$ and $R_i$

The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground ⇒ $V_o$ voltage across $C_p$

$C_p$ effectively isn't there!

$N_0 = \frac{R_2}{R_x} \Theta(s) t^s$
Position Sensing Circuits

Problems With Pure-C Position Sensing

* To sense position (i.e., displacement), use a capacitive load

\[
\begin{align*}
\Delta V_{o}(s) &= \frac{C_{0}C_{D}}{1+\frac{C_{0}C_{D}}{C_{P}}} \Delta \phi(s) \\
\text{Integration yields} & \quad \text{displacement.} \\
\text{To maximize gain, minimize} & \quad C_{0}. \\
\Rightarrow \text{Problem: parasitic capacitance} & \quad C_{0} \rightarrow C_{b} + C_{P} + C_{Pb} \\
\Rightarrow \text{DC Gain:} & \quad \frac{C_{x}}{C_{0}(C_{b} + C_{P} + C_{Pb})} \\
\text{Remedy:} & \quad \text{Suppress} C_{P} \text{ via use of op amps.}
\end{align*}
\]
The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance $C_p$

\[ \tau = \frac{1}{sC_2} \]

(for biasing)

Differential Position Sensing
Differential Position Sensing

Example: ADXL-50

Tethers with fixed ends

Fixed Electrodes

\[ V_p \]

\[ C_1 \]

\[ C_2 \]

\[ -V_p \]

\[ V_0 \]

\[ \frac{C_1}{C_1+C_2} \]

\[ \frac{C_2}{C_1+C_2} \]

\[ \frac{C_p}{C_1+C_2+C_p} \]

\[ V_0 = -V_p + \left( \frac{C_1}{C_1+C_2} \right) \frac{2(V_p)}{C_1+C_2} \]

\[ V_0 = \frac{C_1-C_2}{C_1+C_2+C_p} V_p \]

\[ V_0 = \frac{C_1-C_2}{C_1+C_2+C_p} V_p \]

\[ \text{No voltage across } C_p \]

\[ \text{It's effectively not there!} \]

Buffer-Bootstrapped Position Sensing

Includes capacitance from interconnects, bond pads, and \( C_{gs} \) of the op amp

Unity Gain Buffer

\[ V_0 \]

\[ C_{gd} \]

\[ C_p \]

\[ C_{gd} = \text{gate-to-drain capacitance of the input MOS transistor} \]

Ground Plane

\[ \times \]

\[ +V_p \]

\[ -V_p \]

\[ \text{Bootstrap the ground lines around the interconnect and bond pads} \]

\[ \text{No voltage across } C_p \]

\[ \text{It's effectively not there!} \]
**Effect of Finite Op Amp Gain**

- Total ADXL-50 Sense C ~ 100fF

\[ V_P \]

Unity Gain Buffer

\[ C_P \]

\[ C_{gd} \]

\[ V_0 \]

\[ N_0 = A_0(N_c - N_i) \times A_0(N_i - N_c) \rightarrow N_0 (1 + A_0) \times A_0 N_i \rightarrow A_0 N_i = \frac{N_0 N_i}{1 + A_0} \]

Get \[ z_{in} = \frac{N_i}{z_i} \]

\[ A_0 (N_i - N_c) sC_P \]

\[ = N_c \left( 1 - \frac{A_0}{1 + A_0} \right) sC_P \]

\[ N_c \frac{sC_P}{1 + A_0} \]

\[ C_{eff} = \frac{sC_P}{1 + A_0} \]

Ex: \( A_0 = 10 \), \( C_P = 2pF \)

\[ \Rightarrow C_{eff} = \frac{2pF}{10} = 0.2pF \]

Not negligibly compared with ADXL-50 Ceff ~ 100 fF!

**Integrator-Based Diff. Position Sensing**

\[ V_P \]

\[ i_0 \]

\[ R_2 \]

\[ C_F \]

\[ R_0 \]

\[ V_0 \]

\[ + \]

\[ - \]

\[ 0V \]

Can drive next stage \( R_2 \) without interfering to transfer function!

\[ \frac{V_0}{V_P} = -\frac{C_2}{C_F} \Rightarrow A \text{ seemingly perfect differential sensor/amp output} \ldots \text{but only when the op amp is ideal} \ldots \]