EE C247B – ME C218
Introduction to MEMS Design
Spring 2016

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Lecture Module 14: Sensing Circuits

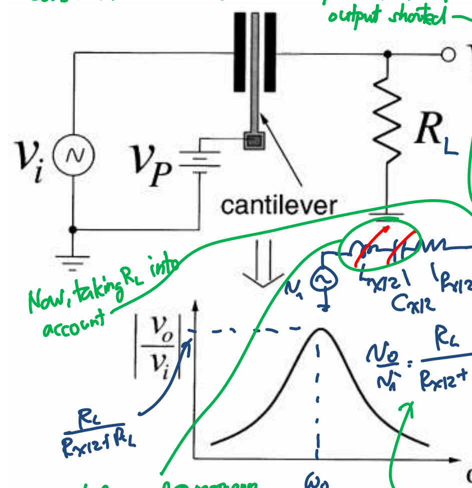
Lecture Outline

- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, w/ output shorted



$$\frac{\dot{x}}{F_{d1}}(s) = \frac{\omega_0 Q}{k}\,\Theta(s)$$

$$\left[F_{d1} = \eta_{e1} v_i\right] \Rightarrow \frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_0 Q}{k}\,\Theta(s)$$

$$\left[i_o = \eta_{e2}\dot{x}\right] \Rightarrow \frac{i_o}{v_i}(s) = \eta_{e1}\eta_{e2}\frac{\omega_0 Q}{k}\,\Theta(s) = \frac{\eta_{e1}\eta_{e2}Q}{m\omega_0}\,\Theta(s)$$

$\frac{1}{R_{x12}}$ → $\frac{\eta_{e1}\eta_{e2}Q}{m\omega_0}$

Now, taking R_L into account

$$\frac{i_o}{v_i}(s) = \frac{1}{R_{x12}}\,\Theta(s)$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_{x12}+R_L}$$

$$\frac{v_o}{v_i}(s) = \frac{R_L}{R_{x12}+R_L}\,\Theta(s)$$

$$Q' = Q\left(\frac{R_{x12}}{R_{x12}+R_L}\right)$$

w/ altered Q

$\frac{R_L}{R_{x12}+R_L}$

L_x & C_x cancel @ resonance

Solve the problem @ resonance first, then multiply by $\Theta(s)$

Velocity-to-Voltage Conversion

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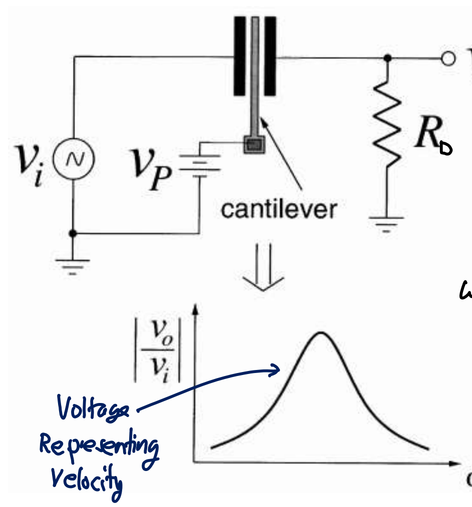
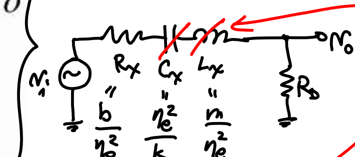


Voltage Representing Velocity

Since this structure has completely symmetrical I/O ports:



$R_x = \frac{b}{\eta_e^2}$, $C_x = \frac{\eta_e^2}{k}$, $L_x = \frac{m}{\eta_e^2}$

C_x & L_x cancel

Work @ resonance: (to simplify the analysis)

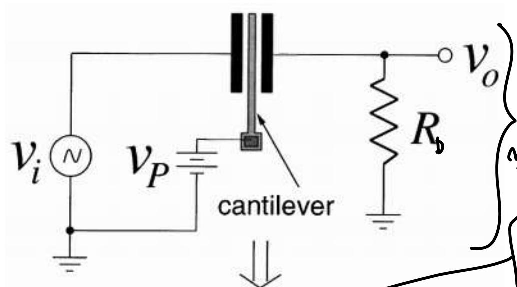
$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D} \quad (\text{@ resonance})$$

Then, generalize to off resonance:

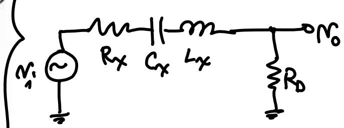
$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D}\,\Theta(s,Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



Since this structure has completely symmetrical I/O ports:



$$\left[Q = \frac{\omega_o L_x}{R_x} \rightarrow Q' = \frac{\omega_o L_x}{R_x + R_D} \rightarrow \frac{R_x+R_D}{L_x} = \frac{\omega_o}{Q'}\right]$$

$$\left[\frac{1}{L_x C_x} = \omega_o^2\right]$$

Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_D C_x}{sR_x C_x + 1 + s^2 L_x C_x + sR_D C_x} = \frac{s\frac{R_D}{L_x}}{s^2 + s\frac{R_x+R_D}{L_x} + \frac{1}{L_x C_x}}$$

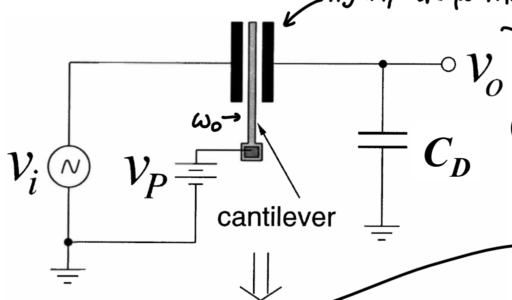
$$= \frac{R_D}{R_x+R_D}\,\frac{s\left(\frac{R_x+R_D}{L_x}\right)}{s^2 + s\left(\frac{R_x+R_D}{L_x}\right) + \frac{1}{L_x C_x}} \Rightarrow \frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D}\,\frac{s(\omega_o/Q')}{s^2 + s(\omega_o/Q') + \omega_o^2} = \frac{R_D}{R_x+R_D}\,\Theta(s, Q')$$

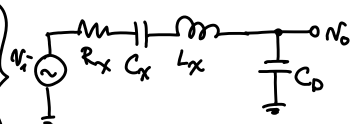
$$Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:





Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

$$\frac{v_o}{v_i}(s) = \frac{\frac{sC_x}{sC_D}}{sR_x C_x + 1 + s^2 L_x C_x + \frac{sC_x}{sC_D}} = \frac{C_x/C_D}{1 + C_x/C_D}\,\frac{1}{1 + s\frac{R_x C_x}{1+C_x/C_D} + s^2\frac{L_x C_x}{1+C_x/C_D}}$$

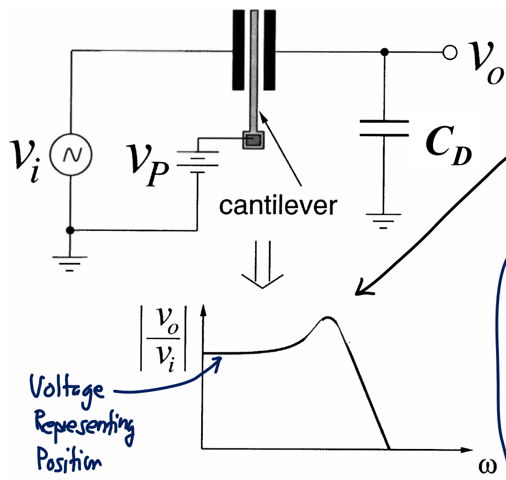
$$= \frac{C_x/C_D}{1+C_x/C_D}\,\frac{\frac{1+C_x/C_D}{L_x C_x}}{s^2 + s\left(\frac{R_x}{L_x}\right) + \frac{(1+C_x/C_D)}{L_x C_x}}$$

$$\left[\omega_o^2 = \frac{1}{L_x C_x} \rightarrow (\omega_o')^2 = \omega_o^2(1 + C_x/C_D)\right]$$

$$\left[Q' = \frac{\omega_o' L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o'}{Q'},\ Q' = Q\sqrt{1 + C_x/C_D}\right]$$ over

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load



$$\frac{V_o}{V_i}(s) = \underbrace{\frac{C_x/C_D}{1+C_x/C_D}}_{\text{DC Gain Term}} \cdot \underbrace{\frac{(\omega_o')^2}{s^2 + \left(\frac{\omega_o'}{Q'}\right)s + (\omega_o')^2}}_{\text{Low-Pass Biquad}}$$

To maximize gain → 1, need $C_D \ll C_x$. (must minimize C_D)

Note: Can we use similar short-cut to the R case.

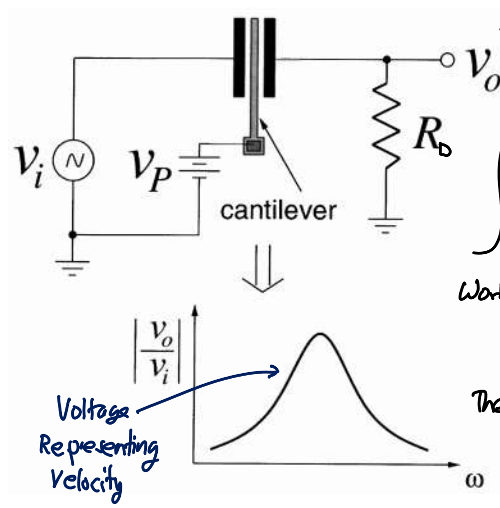
1. Get DC response → C's dominate.
2. Then:

$$\frac{V_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, \omega_o', Q') \cdot \omega_o' Q'$$

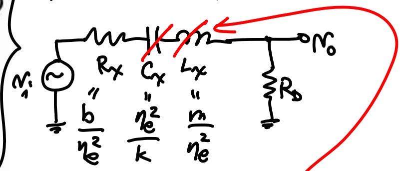
Velocity Sensing Circuits

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



Since this structure has completely symmetrical I/O ports:



C_x & L_x cancel

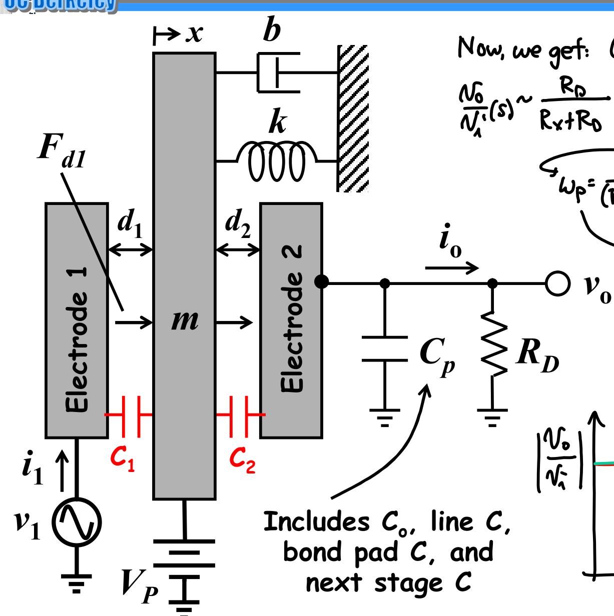
Work @ resonance: (to simplify the analysis)

$$\frac{v_o}{v_i} = \frac{R_D}{R_x + R_D} \quad (\text{@ resonance})$$

Then, generalize to off resonance:

$$\frac{v_o}{v_i} = \frac{R_D}{R_x + R_D} \, H(s, Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x + R_D}\right)$$

Problems With Purely Resistive Sensing



Includes C_o, line C, bond pad C, and next stage C

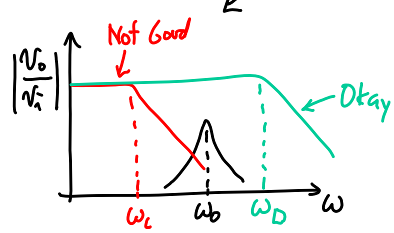
Now, we get: (approximately)

$$\frac{v_o}{v_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot H(s, \omega_o', Q')$$

$$\omega_p = \frac{1}{(R_x \| R_D) C_p}$$

Depend on both R_D & C_p.

Impact depends on where ω_p is relative to ω_o



Problems With Purely Resistive Sensing

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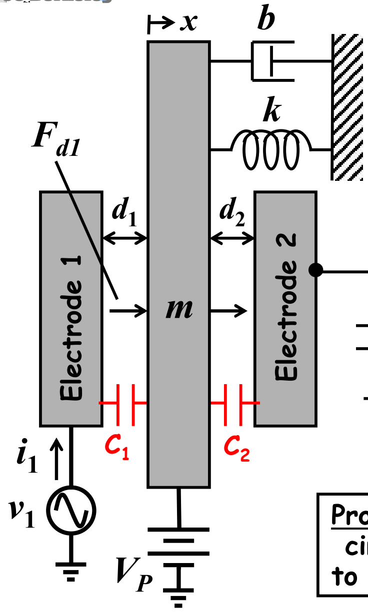
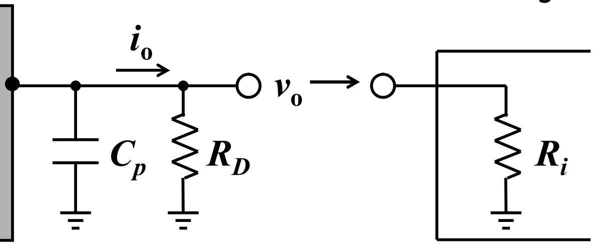


- In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input R_i of these circuits can load R_D



These change w/ hook-up → not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

Soln: use op amps.

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The TransR Amplifier Advantage

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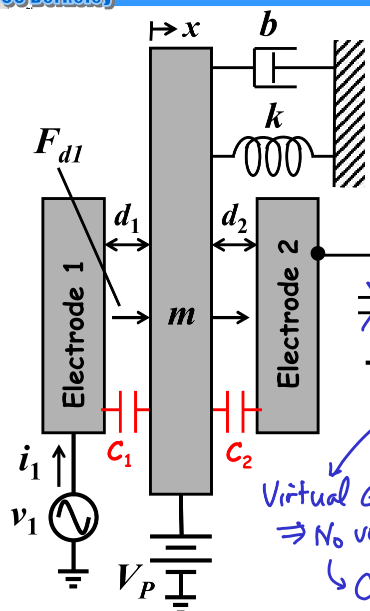
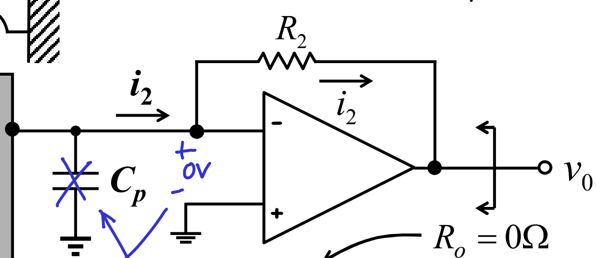


- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p and R_i



- The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground
⇒ No voltage across C_p
↳ C_p effectively isn't there!

$$v_0 = -\frac{R_2}{R_x} \Theta(s) v_i$$

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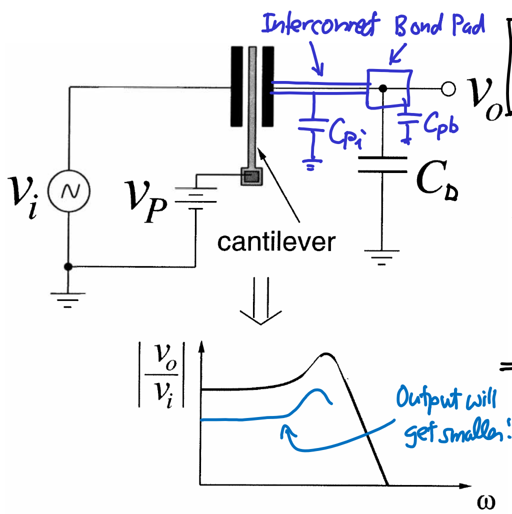
Position Sensing Circuits

Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load



$$\frac{v_o}{v_i}(s) = \frac{C_x/C_D}{1+C_x/C_D} \cdot \frac{1}{s} \cdot \text{Ⓗ}(s, \omega_o', Q') \cdot \omega_o' Q'$$

Integration yields displacement.

To maximize gain, minimize C_D.

⇒ Problem: parasitic capacitance

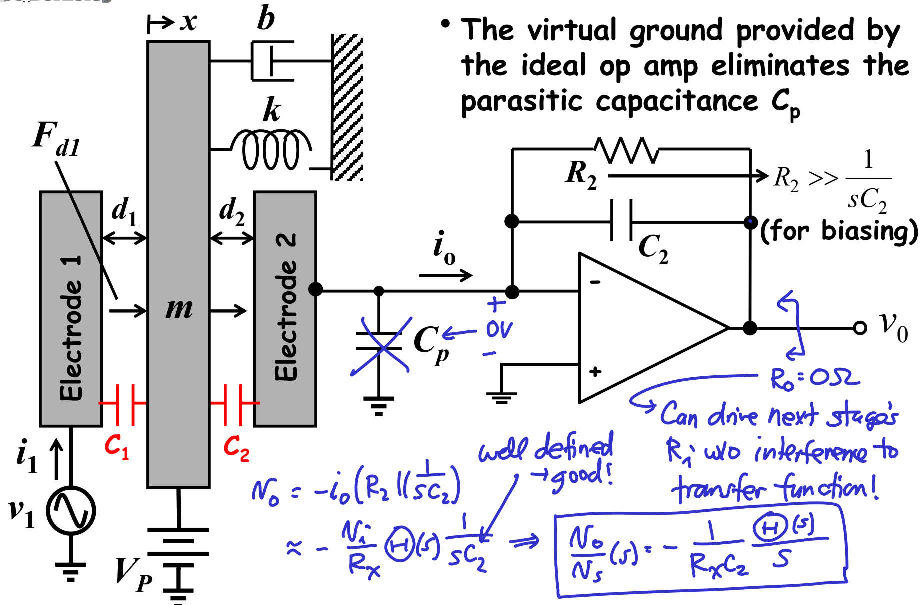
$C_D \rightarrow C_D + C_{Pi} + C_{pb}$

⇒ DC Gain: $\dfrac{C_x/(C_D + C_{Pi} + C_{pb})}{1 + C_x/(C_D + C_{Pi} + C_{pb})}$

Remedy: Suppress C_p via use of op amps.

The Op Amp Integrator Advantage

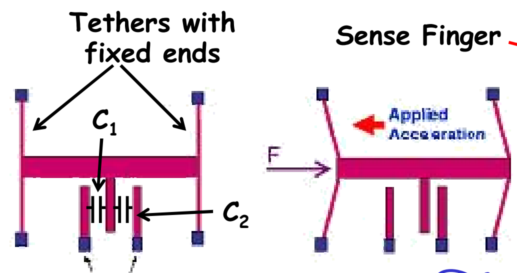
- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p

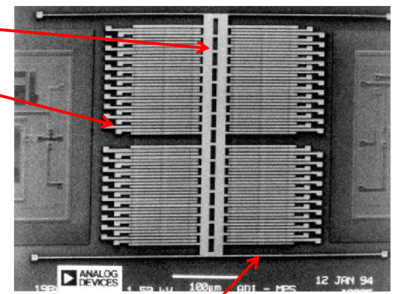


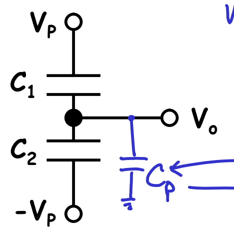
Differential Position Sensing

Differential Position Sensing

- Example: ADXL-50







capacitive divider

$$V_o = -V_p + (2V_p)\frac{C_1}{C_1+C_2}$$

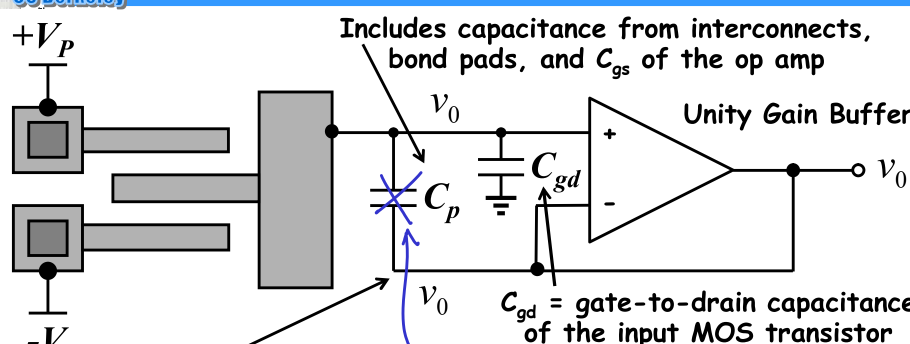
$$= \frac{-V_pC_1 - V_pC_2 + 2V_pC_1}{C_1+C_2} = V_p\left(\frac{C_1-C_2}{C_1+C_2}\right) = V_o$$

Issue: Parasitic Capacitance

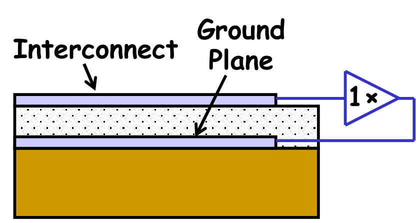
$$V_o = \frac{C_1-C_2}{C_1+C_2+C_p}V_p$$

As before, C_p reduces gain → Soln: use op amp!

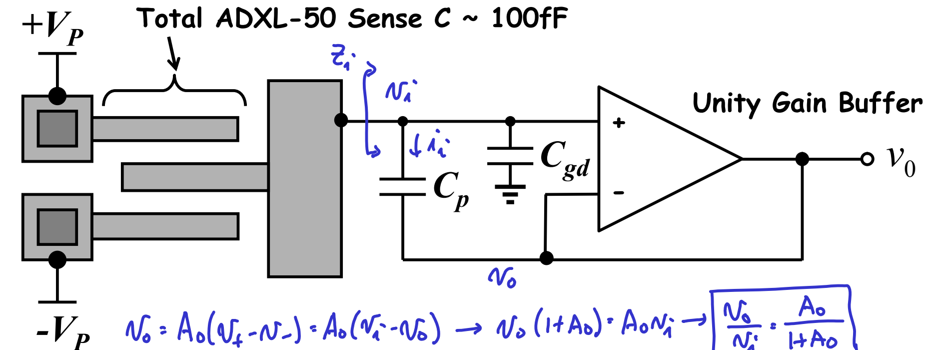
Buffer-Bootstrapped Position Sensing



- Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across C_p
  - It's effectively not there!



Effect of Finite Op Amp Gain



$v_o = A_o(v_+ - v_-) = A_o(v_i - v_o) \rightarrow v_o(1+A_o) = A_o v_i \rightarrow \boxed{\dfrac{v_o}{v_i} = \dfrac{A_o}{1+A_o}}$

Get $z_i = \dfrac{v_i}{i_i}$: $\quad i_i = (v_i - v_o)sC_p = v_i\left(1 - \dfrac{A_o}{1+A_o}\right)sC_p = v_i \dfrac{1}{1+A_o} sC_p$

$\therefore \dfrac{v_i}{i_i} = z_i = \dfrac{1}{s\left[\dfrac{C_p}{1+A_o}\right]}$ $\quad \boxed{C_{eff} = \dfrac{C_p}{1+A_o}}$

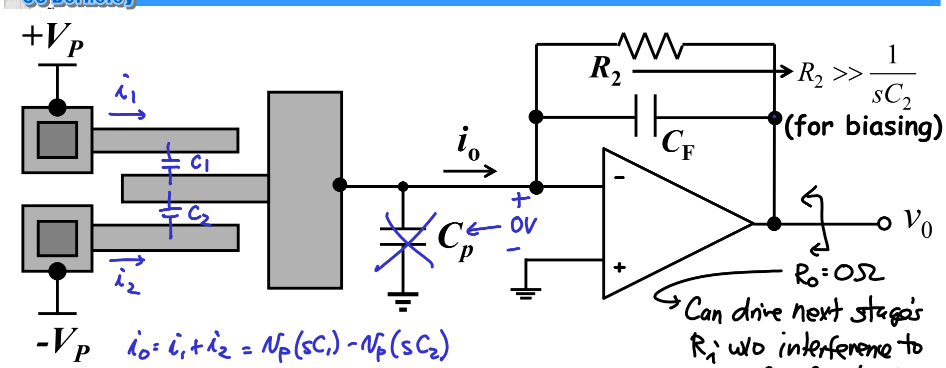
No longer zero!

Ex: $A_o = 100$, $C_p = 2pF$

$\Rightarrow C_{eff} = \dfrac{2p}{101} = 20fF$

Not negligible compared w/ ADXL-50 $C_{tot} \sim 100fF$!

Integrator-Based Diff. Position Sensing



$i_o = i_1 + i_2 = v_p(sC_1) - v_p(sC_2)$

$= v_p s(C_1 - C_2)$

$\therefore v_o = -i_o\left(\dfrac{1}{sC_F}\right) = -v_p\left(\dfrac{C_1 - C_2}{C_F}\right)$

$\boxed{\dfrac{v_o}{v_p} = -\dfrac{C_1 - C_2}{C_F}}$ $\Rightarrow$ A seemingly perfect differential sensor/amplifier output! . . . but only when the op amp is ideal . . .