



EE C247B - ME C218
Introduction to MEMS Design
Spring 2016

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Lecture Module 14: Sensing Circuits

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Lecture Outline

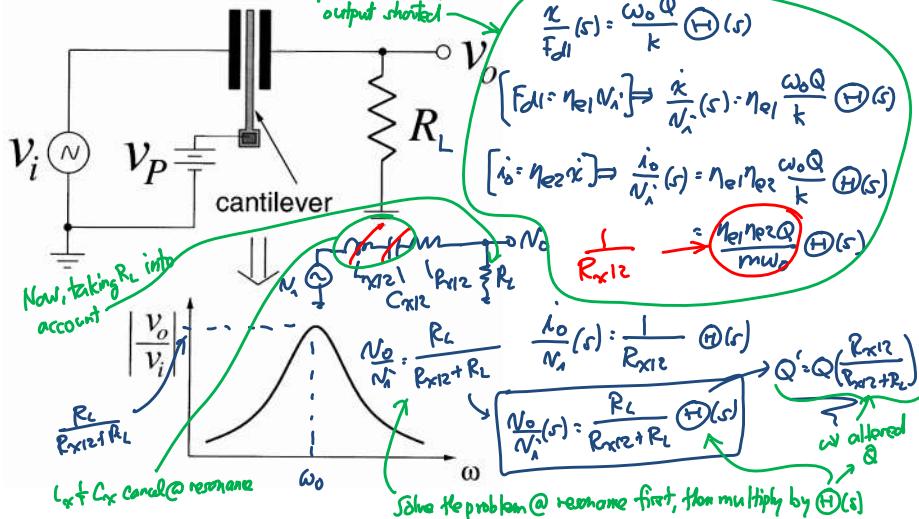
- Reading: Senturia, Chpt. 14
- Lecture Topics:
 - ↳ Detection Circuits
 - Velocity Sensing
 - Position Sensing

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Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, w



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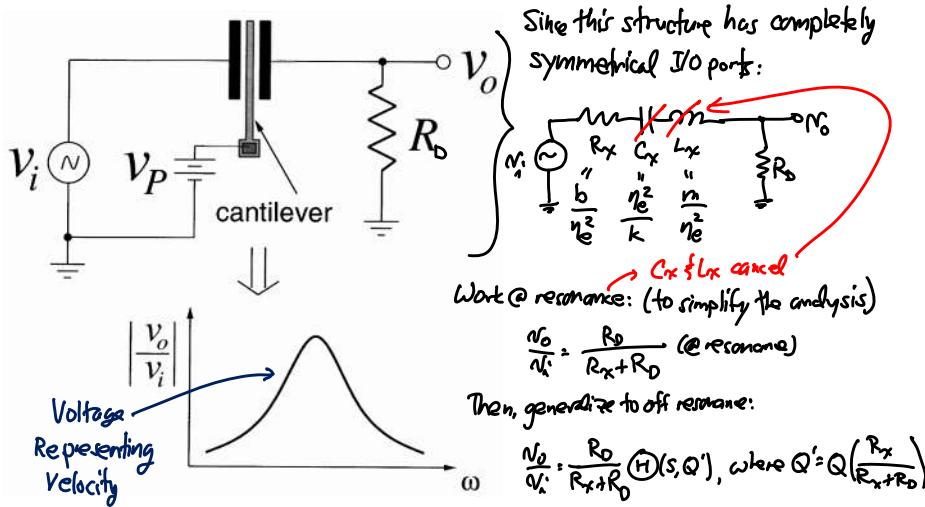
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Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



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Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$Q := \frac{\omega_0 L_x}{R_x} \rightarrow Q' := \frac{\omega_0 L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_0}{Q'}$$

Brute force approach:

$$\frac{N_0(s)}{V_i} := \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_x C_x}{sR_x C_x + 1 + s^2 L_x C_x + sR_D C_x} = \frac{s \frac{R_D}{L_x}}{s^2 + s \frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$$= \frac{R_D}{R_x + R_D} \frac{s \left(\frac{R_x + R_D}{L_x} \right)}{s^2 + s \left(\frac{R_x + R_D}{L_x} \right) + \frac{1}{L_x C_x}}$$

$$\Rightarrow \frac{N_0(s)}{V_i} = \frac{R_D}{R_x + R_D} \frac{s (\omega_0 / Q')}{s^2 + s (\omega_0 / Q') + \omega_0^2} = \frac{R_D}{R_x + R_D} H(s, Q')$$

$Q' = Q \left(\frac{R_x}{R_x + R_D} \right)$

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Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

Again, here port-to-port I/O symmetry:

Brute force approach:

$$\frac{N_0(s)}{V_i} = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

$$\Rightarrow \frac{N_0(s)}{V_i} = \frac{\frac{1}{sC_D}}{sR_x C_x + 1 + s^2 L_x C_x + \frac{1}{sC_D}} = \frac{\frac{1}{sC_D}}{1 + \frac{sR_x C_x}{1 + C_x/C_D} + s^2 \frac{L_x C_x}{1 + C_x/C_D}}$$

$$\Rightarrow \frac{N_0(s)}{V_i} = \frac{\frac{1}{sC_D}}{1 + \frac{sR_x C_x}{1 + C_x/C_D} + s^2 \frac{L_x C_x}{1 + C_x/C_D}}$$

$$= \frac{C_x/C_D}{1 + C_x/C_D} \frac{\frac{1}{sC_D}}{s^2 + s \left(\frac{R_x}{L_x} \right) + \frac{(1 + C_x/C_D)}{L_x C_x}}$$

$$\left[\begin{array}{l} \omega_0^2 \cdot \frac{1}{L_x C_x} \rightarrow (\omega_0')^2 = \omega_0^2 (1 + C_x/C_D) \\ Q' = \frac{\omega_0' L_x}{R_x} = \frac{R_x}{L_x} \cdot \frac{\omega_0'}{Q'}, Q' = Q \sqrt{1 + C_x/C_D} \end{array} \right] \text{ over } \rightarrow$$

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Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

$$\frac{V_o}{V_i}(s) = \frac{C_x/C_D}{1 + C_x/C_D} \cdot \frac{(w_0')^2}{s^2 + (w_0'/Q')s + (w_0')^2}$$

DC Gain Term Low-Pass Biquad
 To maximize gain $\rightarrow 1$, need $C_x \ll C_D$.
 (must minimize C_D)

Note: Can we similar short-cut to the R case.

- Get DC response $\rightarrow C$'s dominate.
- Then:

$$\frac{V_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, w_0', Q') \cdot w_0' Q'$$

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Velocity Sensing Circuits

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Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

Work @ resonance: (to simplify the analysis)

$$\frac{N_O}{N_i} = \frac{R_D}{R_x + R_D} \quad (@\text{resonance})$$

Then, generalize to off resonance:

$$\frac{N_O}{N_i} = \frac{R_D}{R_x + R_D} H(s, Q'), \text{ where } Q' = Q \left(\frac{R_x}{R_x + R_D} \right)$$

Voltage Representing Velocity

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Problems With Purely Resistive Sensing

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Now, we get: (approximately)

$$\frac{N_O}{N_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot H(s, \omega_b, Q')$$

$\omega_p = \frac{1}{(R_x || R_D)C_p}$

Depend on both R_D & C_p .

Impact depends on where ω_p is relative to ω_0 .

Not Good

Okay

ω_c ω_b ω_D

Includes C_o , line C , bond pad C , and next stage C

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Problems With Purely Resistive Sensing

- In general, the sensor output must be connected to the inputs of further signal conditioning circuits \rightarrow input R_i of these circuits can load R_D

These change w/ hook-up \rightarrow not good.

Problem: need a sensing circuit that is immune to parasitics or loading. → **Soln:** use op amps.

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The TransR Amplifier Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p and R_i
- The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground
 \Rightarrow No voltage across C_p
 $\hookrightarrow C_p$ effectively isn't there!

$$V_0 = -\frac{R_2}{R_x} H(s) V_i$$

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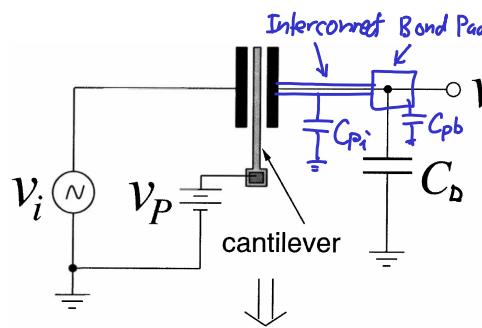
Position Sensing Circuits

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Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load



$$\frac{V_o}{V_i} = \frac{C_x/C_D}{1+C_x/C_D} \cdot \frac{1}{s} \cdot H(s, \omega_0, Q') \cdot \omega_0 \alpha$$

Integration yields displacement.

To maximize gain, minimize C_D .
 ⇒ Problem: parasitic capacitance
 $C_D \rightarrow C_D + C_{Pi} + C_{Pb}$

⇒ DC Gain: $\frac{C_x/(C_D + C_{Pi} + C_{Pb})}{1 + C_x/(C_D + C_{Pi} + C_{Pb})}$

Output will get smaller!

Remedy: Suppress C_D via use of op amps.

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The Op Amp Integrator Advantage

The diagram shows a MEMS mass-spring system. A mass m is attached to a spring with stiffness k , which is fixed to a wall. Two electrodes, "Electrode 1" and "Electrode 2", are positioned facing the mass. The distance between Electrode 1 and the mass is d_1 , and the distance between Electrode 2 and the mass is d_2 . A force F_{d1} acts on Electrode 1. The mass m has a position x relative to the electrodes. The system includes parasitic capacitances C_1 and C_2 between the electrodes and ground, and a parasitic capacitance C_p between the electrodes.

The output current i_o is measured at the center of the mass. The circuit on the right shows an operational amplifier (op-amp) configured as an integrator. The non-inverting input is connected to ground through a resistor R_2 and a capacitor C_2 . The inverting input is connected to the output v_o through a resistor $R_o = 0\Omega$. The output voltage v_o is given by the equation:

$$v_o = -i_o \left(R_2 / (sC_2) \right)$$

Handwritten notes explain that the virtual ground provided by the op-amp eliminates the parasitic capacitance C_p . The output voltage is well defined and good. The transfer function is given as:

$$\frac{v_o}{v_s} (s) = -\frac{1}{R_x C_2 s} H(s)$$

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Differential Position Sensing

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Differential Position Sensing

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- Example: ADXL-50

Tethers with fixed ends

Proof Mass

Sense Finger

Applied Acceleration

Fixed Electrodes

Suspension Beam in Tension

$V_o = -V_p + \frac{(2V_p)}{C_1+C_2}$

$$= -\frac{V_p C_1 - V_p C_2 + 2V_p C_1}{C_1+C_2} = \frac{V_p (C_1 \cdot C_2)}{C_1+C_2} = V_o$$

Issue: Parasitic Capacitance

$$V_o = \frac{C_1 \cdot C_2}{C_1+C_2+C_p} V_p$$

As before, C_p reduces gain → Soln: Use op amp!

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Buffer-Bootstrapped Position Sensing

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+V_P

-V_P

V_0

C_p

C_{gd}

Unity Gain Buffer

C_{gd} = gate-to-drain capacitance of the input MOS transistor

Includes capacitance from interconnects, bond pads, and C_{gs} of the op amp

- Bootstrap the ground lines around the interconnect and bond pads
 - ↳ No voltage across C_p
 - ↳ It's effectively not there!

Interconnect

Ground Plane

1x

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Effect of Finite Op Amp Gain

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Total ADXL-50 Sense $C \sim 100\text{fF}$

$$+V_p \quad -V_p \quad N_o = A_o(N_+ - N_-) = A_o(V_i - V_o) \rightarrow N_o(1 + A_o) = A_o N_i \rightarrow \frac{N_o}{N_i} = \frac{A_o}{1 + A_o}$$

$$\text{Get } Z_a = \frac{V_o}{i_o}: \quad i_o = (N_i - N_o) sC_p = N_i \left(1 - \frac{A_o}{1 + A_o}\right) sC_p = N_i \frac{1}{1 + A_o} sC_p$$

$$\therefore \frac{N_i}{i_o} = Z_a = \frac{1}{s \left[\frac{C_p}{1 + A_o} \right]} \quad \boxed{C_{\text{eff}} = \frac{C_p}{1 + A_o}}$$

No longer zero!

*Ex: $A_o = 100, C_p = 2\text{pF}$
 $\Rightarrow C_{\text{eff}} = \frac{2\text{pF}}{101} = 20\text{fF}$
Not negligible compared w/ ADXL-50 $C_{\text{tot}} \sim 100\text{fF}$!*

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Integrator-Based Diff. Position Sensing

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+V_p i_1 $\frac{1}{sC_1}$
-V_p i_2 $\frac{1}{sC_2}$

$$i_o = i_1 - i_2 = N_p(sC_1) - N_p(sC_2) = V_p s(C_1 - C_2)$$

$$\therefore N_o = -i_o \left(\frac{1}{sC_F} \right) = -N_p \left(\frac{C_1 - C_2}{C_F} \right)$$

$$\boxed{\frac{N_o}{V_p} = -\frac{C_1 - C_2}{C_F}}$$

Can drive next stage's R_1 w/o interference to transfer function!

A seemingly perfect differential Sensor/amplifier output! ...but only when the op amp is ideal...

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