# EE C247B – ME C218
# Introduction to MEMS Design
# Fall 2016

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**Module 17**: Noise & MDS



## Lecture Outline

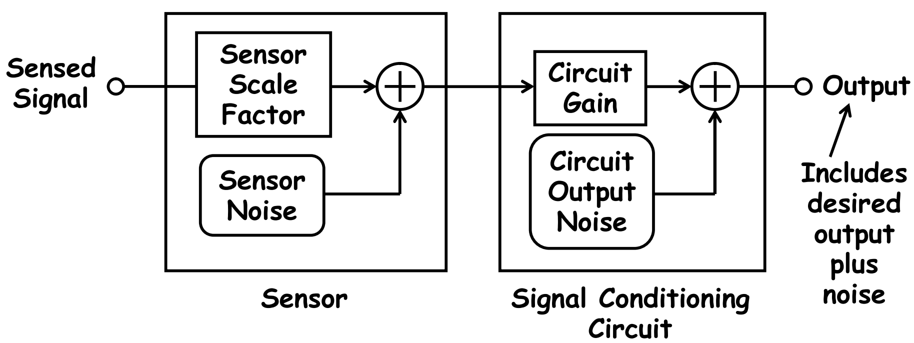
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination

## Determining Sensor Resolution



## Minimum Detectable Signal (MDS)

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity



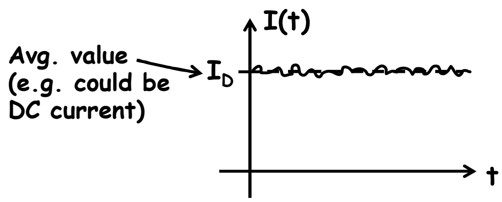

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

# Noise



## Noise

- Noise: Random fluctuation of a given parameter I(t)
- In addition, a noise waveform has a zero average value



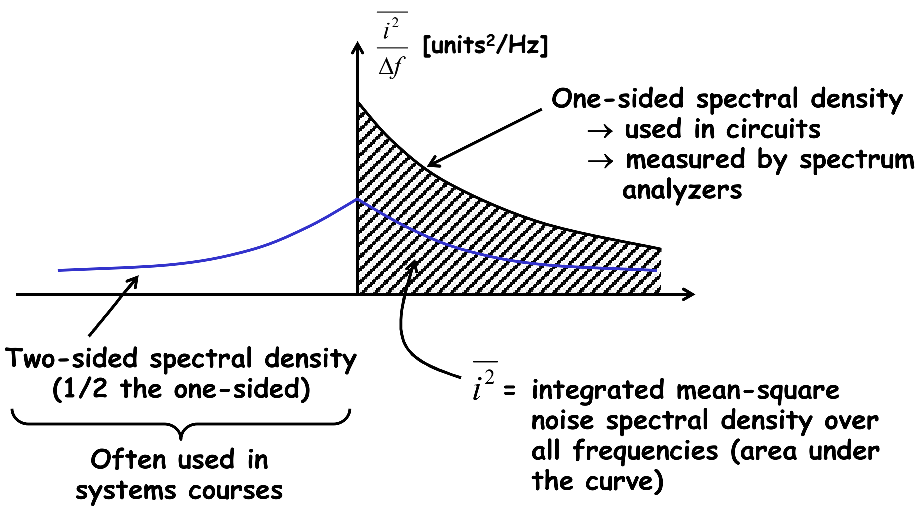

- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

Then $$\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T |I - I_D|^2\, dt$$

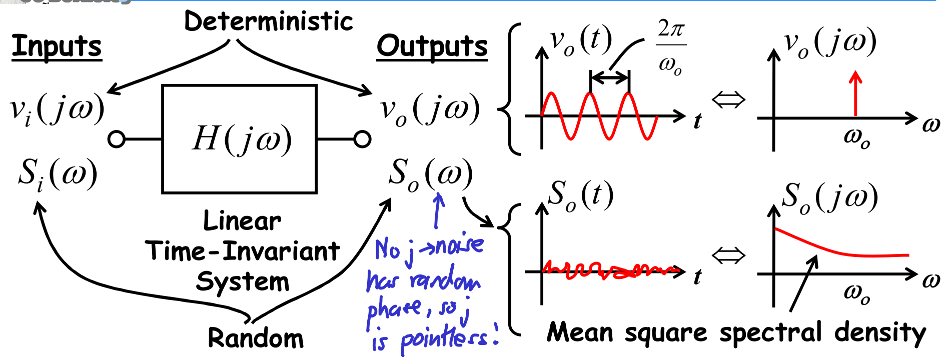
## Noise Spectral Density

- We can plot the spectral density of this mean-square value:






## Circuit Noise Calculations



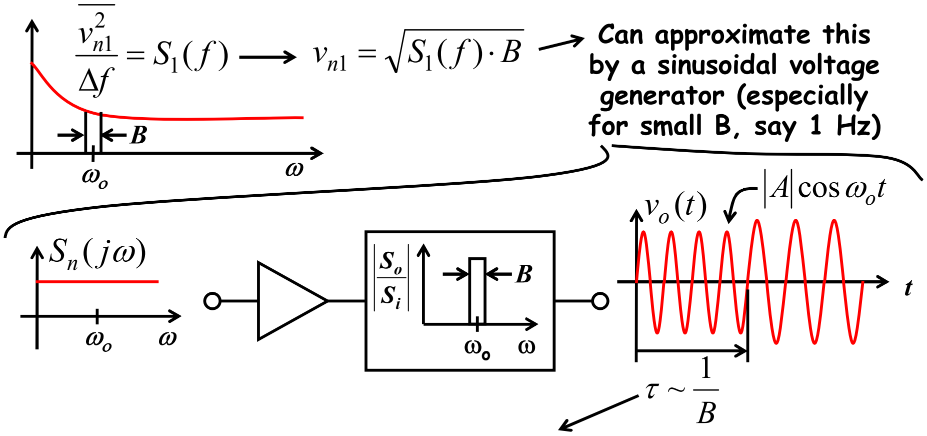

- **Deterministic:** $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- **Random:** $S_o(\omega) = \left[H(j\omega)H^*(j\omega)\right]S_i(\omega) = \left|H(j\omega)\right|^2 S_i(\omega)$

$$\sqrt{S_o(\omega)} = \left|H(j\omega)\right|\sqrt{S_i(\omega)} \longrightarrow \text{How is it we can do this?}$$

Root mean square amplitudes

## Handling Noise Deterministically

- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

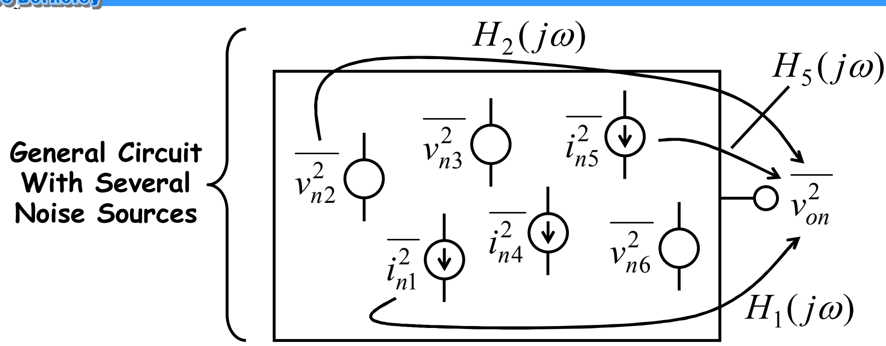



[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter]

Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.



## Systematic Noise Calculation Procedure




- Assume noise sources are uncorrelated
  1. For $\overline{i_{n1}^2}$, replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f} \cdot (1\,\text{Hz})}$$

## Systematic Noise Calculation Procedure

2. Calculate $v_{on1}(\omega) = i_{n1}(\omega)H(j\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H(j\omega)|^2$
4. Repeat for each noise source: $\overline{i_{n1}^2}$, $\overline{v_{n2}^2}$, $\overline{v_{n3}^2}$
5. Add noise power (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots}$$

Total rms value



## Noise Sources

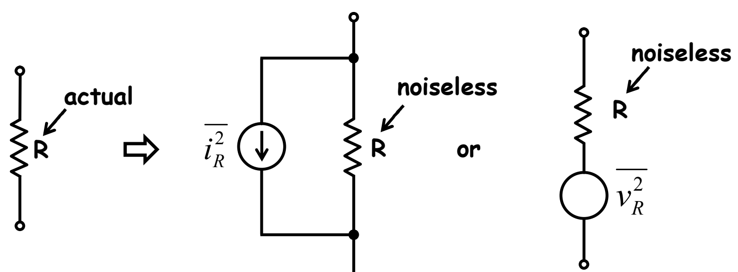
## Thermal Noise

- Thermal Noise in Electronics: (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free $e^-$'s in a conducting medium
  - Path of $e^-$'s randomly oriented due to collisions
- Thermal Noise in Mechanics: (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.
- Properties:
  - Thermal noise is white (i.e., constant w/ frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor



## Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R^2}$ or a shunt current generator $\overline{i_R^2}$




$$\boxed{\frac{\overline{i_R^2}}{\Delta f} = \frac{4kT}{R}} \qquad \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR}$$

where $4kT = 1.66x10^{-20} V \cdot C$
and where these are spectral densities.

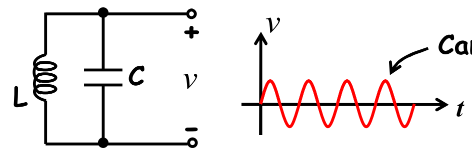
Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.



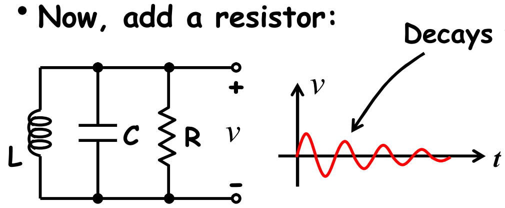
 

## Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?




- Now, add a resistor:




But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature

Need to add a forcing function, like a noise voltage $\overline{v_R^2}$ to keep the motion going → and this noise source is associated with R



## Why 4kTR?

- Why is $\overline{v_R^2} = 4kTR\Delta f$ (a heuristic argument)
- The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy (1/2)kT associated w/ each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom:
  - ✎ Current, i, and voltage, v
  - ✎ Thus, we can write:

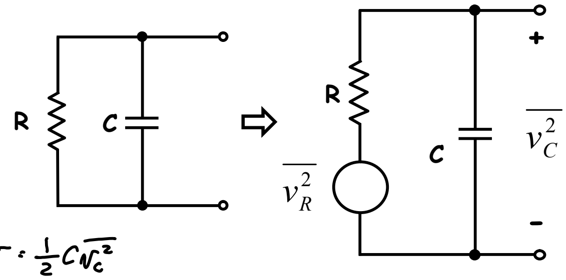
$$\frac{1}{2}L\overline{i^2} = \frac{1}{2}k_BT \quad , \quad \overbrace{\frac{1}{2}C\overline{v^2}}^{\text{Energy}} = \frac{1}{2}k_BT$$

- Similar expressions can be written for mechanical systems
  - ✎ For example: for displacement, x

$$\frac{1}{2}\underbrace{k}_{\text{Spring constant}}\overline{x^2} = \frac{1}{2}k_BT$$

## Why 4kTR? (cont)

- Why is $\overline{v_R^2} = 4kTR\Delta f$ ? (a heuristic argument)
- Consider an RC circuit:




$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_c^2}$

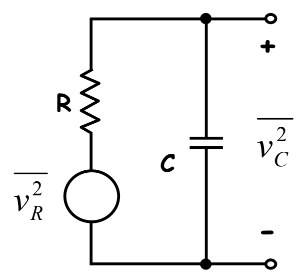
$\therefore \overline{v_c^2} = \frac{kT}{C}$ ← integrated noise over all freqs.
(total mean square voltage integrated over all freqs.)

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ (assuming white noise) gives us this?



## Why 4kTR? (cont)

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ (assuming white noise) gives us $\overline{v_c^2} = \frac{kT}{C}$?




$\overline{v_c^2} = \int_0^\infty \left|\frac{1}{1+j\omega RC}\right|^2 \frac{\overline{v_R^2}}{\Delta f}\, df$

[noise is white] [$\omega_b = \frac{1}{RC}$] $= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega^2}\, d\omega$

$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$

$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\frac{\omega_b^2}{\omega_b}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Big|_0^\infty = \frac{1}{2\cancel{\pi}}\frac{\overline{v_R^2}}{\Delta f}\left(\frac{\cancel{\pi}}{2}\omega_b - 0\right)$

$= \frac{1}{4}\omega_b\frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C} \rightarrow \frac{\overline{v_R^2}}{\Delta f} = 4kT\left(\frac{\omega_b}{C}\right) \Rightarrow \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR}$ ✓

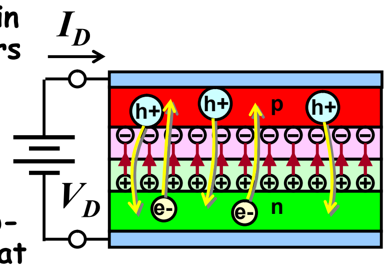
$\uparrow$
R



## Shot Noise

- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which $e^-$'s and $h^+$'s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of $h^+$'s from the p-region and $e^-$'s from the n-region that have sufficient energy to overcome the potential barrier at the junction → noise process should be proportional to DC current
- Attributes:
  - Related to DC current over a barrier
  - Independent of temperature
  - White (i.e., const. w/ frequency)
  - Noise power ~ $I_D$ & bandwidth




pn-junction

$$\boxed{\frac{\overline{i_n^2}}{\Delta f} = 2qI_D}$$

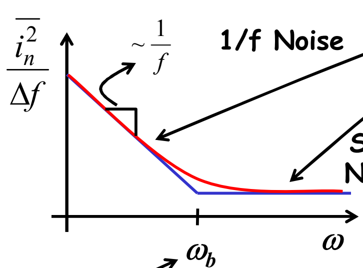
Charge on an $e^-$ (=$1.6\times10^{-19}$C)

DC Current



## Flicker (1/f) Noise

- In general, associated w/ random trapping & release of carriers from "slow" states
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:




1/f Noise

Shot Noise

1/f Noise Corner Frequency

$$\boxed{\frac{\overline{i_n^2}}{\Delta f} = 2qI_D + K\left(\frac{I_D^a}{f^b}\right)}$$

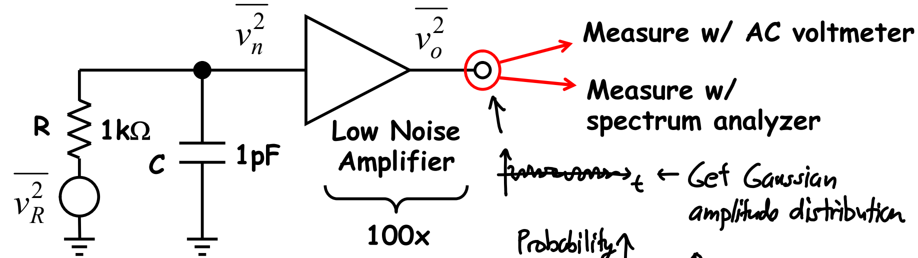
$I_D$ = DC current
K = const. for a particular device
a = 0.5 → 2
b ~ 1

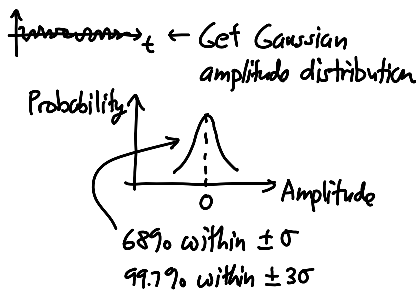


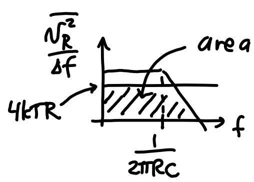
 

## Example: Typical Noise Numbers

- Hookup the circuit below and make some measurements



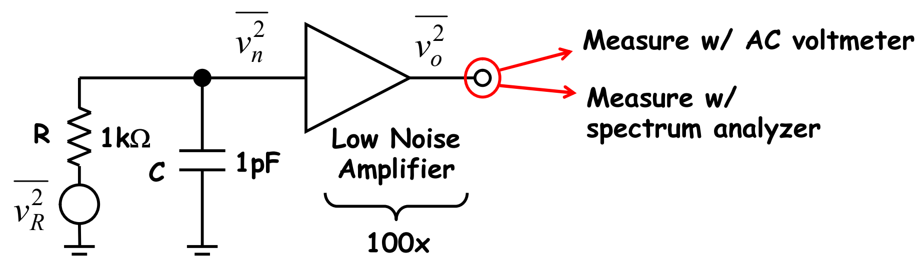

$\sqrt{(1.66\times10^{-20})(1k)}$ → 1kΩ: 4nV/√Hz (for every 1K of R)

1pF: $\sqrt{\frac{kT}{C}}$ = 64 μV rms



## Example: Typical Noise Numbers

- Hookup the circuit below and make some measurements

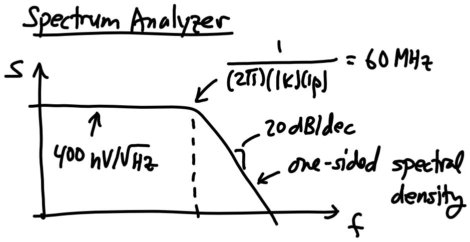



AC Voltmeter

$\sqrt{\overline{v_o^2}}$ = (100)(64 μV rms)

= 6.4 mV rms

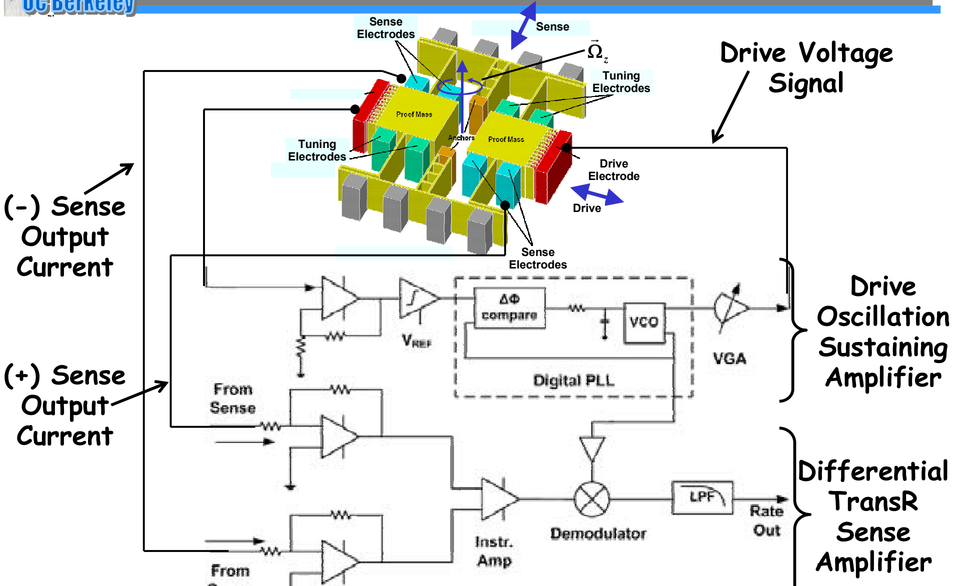
Spectrum Analyzer



## Back to Determining Sensor Resolution

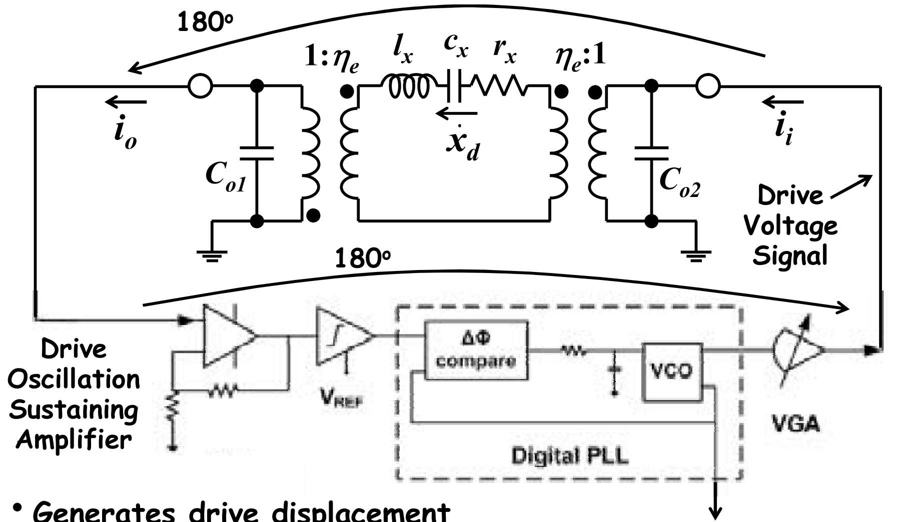


## MEMS-Based Tuning Fork Gyroscope




[Zaman, Ayazi, et al, MEMS'06]

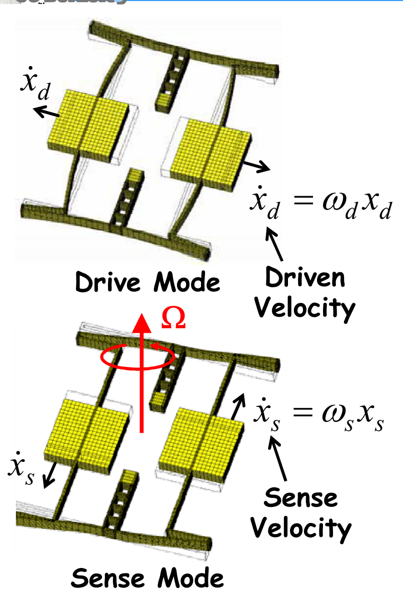
## Drive Axis Equivalent Circuit



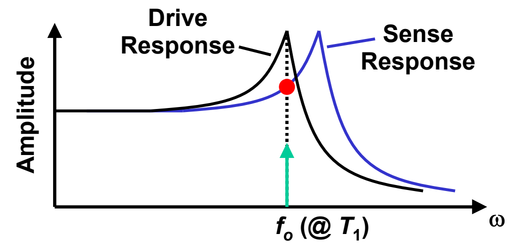

- Generates drive displacement velocity $\dot{x}_d$ to which the Coriolis force is proportional



## Drive-to-Sense Transfer Function




**Drive/Sense Response Spectra:**




Rotation-Induced Coriolis Force:

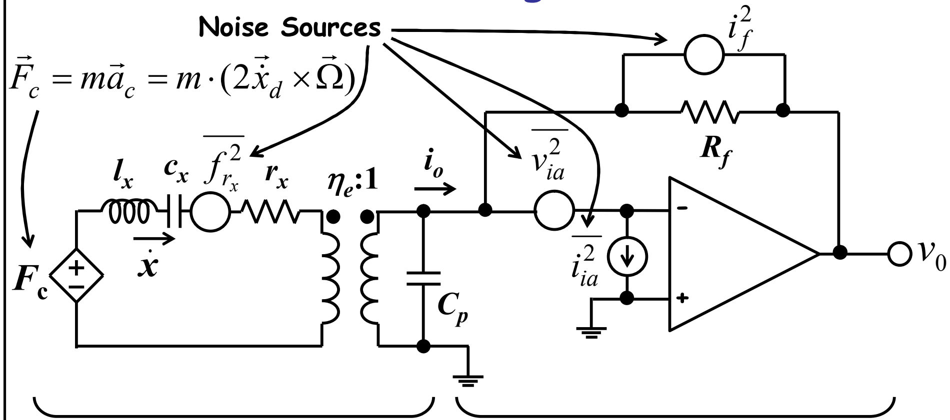
$\vec{a}_c = 2\vec{\dot{x}}_d \times \vec{\Omega} = 2\omega_d \vec{x}_d \times \vec{\Omega}$

$\Rightarrow$ Acts in the sense mode direction

$a_s = 2\omega_d x_d \Omega \sin 90°$

$\boxed{a_s = 2\omega_d x_d \Omega}$

## Gyro Readout Equivalent Circuit (for a single tine)




Gyro Sense Element Output Circuit

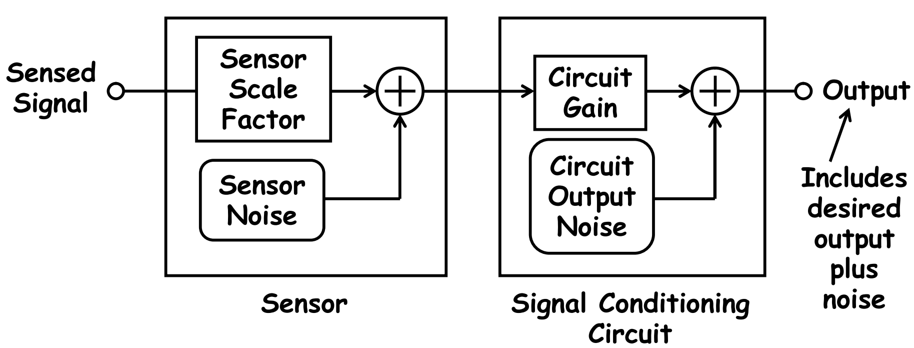
Signal Conditioning Circuit (Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node



## Minimum Detectable Signal (MDS)

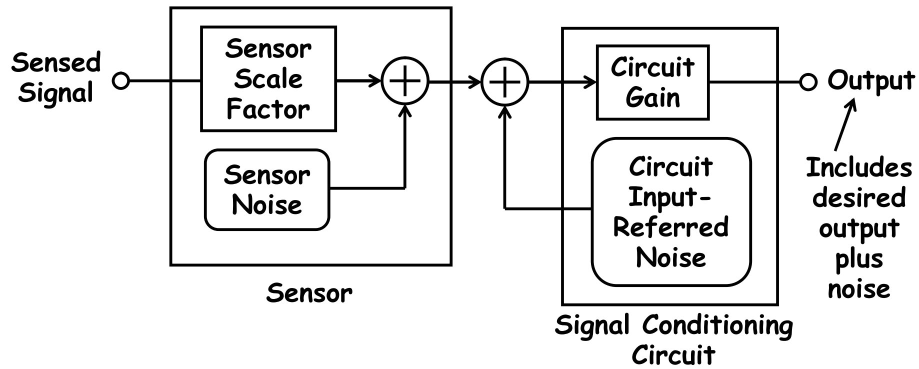
- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity




- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

## Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

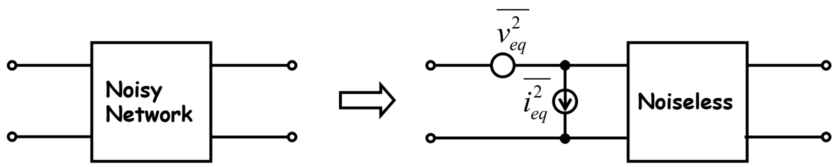





## Equivalent Input-Referred Voltage and Current Noise Sources

## Equivalent Input v, i Noise Generators

- Take a noisy 2-port network and represent it by a noiseless network with input *v* and *i* noise generators that generate the same total output noise



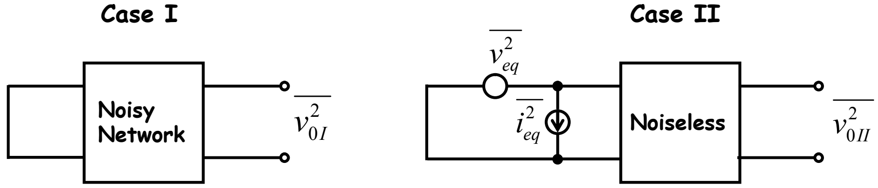

- **Remarks**:
  1. Works for linear time-invariant networks
  2. $v_{eq}$ and $i_{eq}$ are generally correlated (since they are derived from the same sources)
  3. In many practical circuits, one of $v_{eq}$ and $i_{eq}$ dominates, which removes the need to address correlation
  4. If correlation is important $\rightarrow$ easier to return to original network with internal noise sources



## Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$

a) To get $\overline{v_{eq}^2}$ for a two-port:

Case I Case II



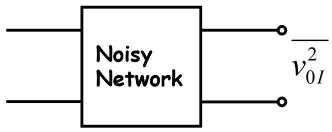

1) Short input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$ )
2) For eq. network, short input, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$ )

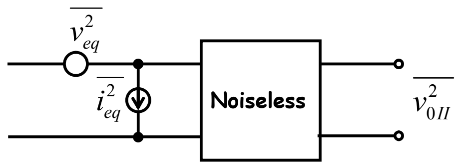
$$\overline{v_{0II}^2} = f\left(\overline{v_{eq}^2}\right) \qquad \overline{i_{0II}^2} = f\left(\overline{v_{eq}^2}\right)$$

3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}$ $\rightarrow$ solve for $\overline{v_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}$ )

## Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

b) To get $\overline{i_{eq}^2}$ for a 2-port:




1) Open input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$ )
2) Open input for eq. circuit, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}\left(\overline{i_{eq}^2}\right)$ → solve for $\overline{i_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}\left(\overline{i_{eq}^2}\right)$)

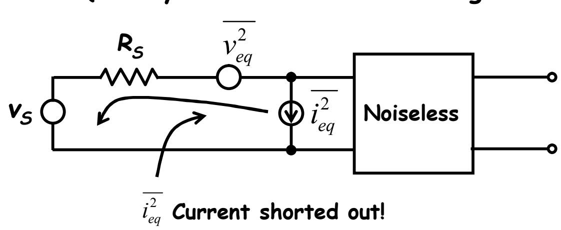
- Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated



## Cases Where Correlation Is Not Important

- There are two common cases where correlation can be ignored:
  1. Source resistance $R_s$ is small compared to input resistance $R_i$ → i.e., voltage source input
  2. Source resistance $R_s$ is large compared to input resistance $R_i$ → i.e., current source input

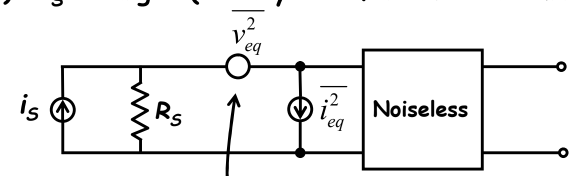
1) $R_S$ = small (ideally = 0 for an ideal voltage source):



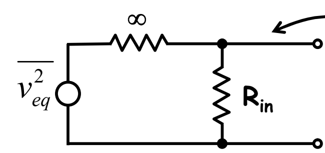

∴ For $R_S$= small, $\overline{i_{eq}^2}$ can be neglected → only $\overline{v_{eq}^2}$ is important!
(Thus, we need not deal with correlation)

## Cases Where Correlation Is Not Important

2) $R_S$ = large (Ideally = ∞ for an ideal current source)




Voltage $\overline{v_{eq}^2}$ effectively "opened" out!




$$v_i = \frac{R_{in}}{\infty + R_{in}} v_{eq} = 0!$$

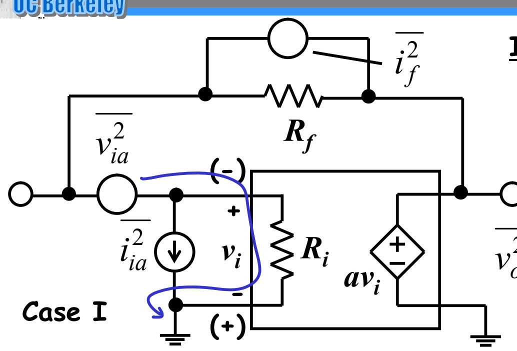
∴ For $R_S$= large, $\overline{v_{eq}^2}$ can be neglected!

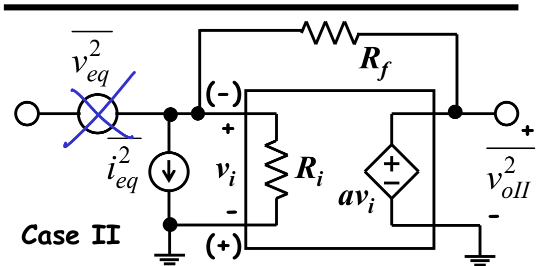
→ only $\overline{i_{eq}^2}$ is important!

(… and again, we need not deal with correlation)



## Example: TransR Amplifier Noise




Input-referred current noise:

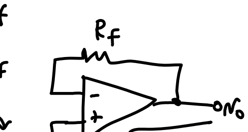
Open inputs; equate output voltage noise.

Case I:

$v_{oI1} = i_{ia} R_f$

$v_{oI2} = i_f R_f$

$v_{oI3} = v_{ia}$




Have $v_{ia}$ through $R_{in}$ to

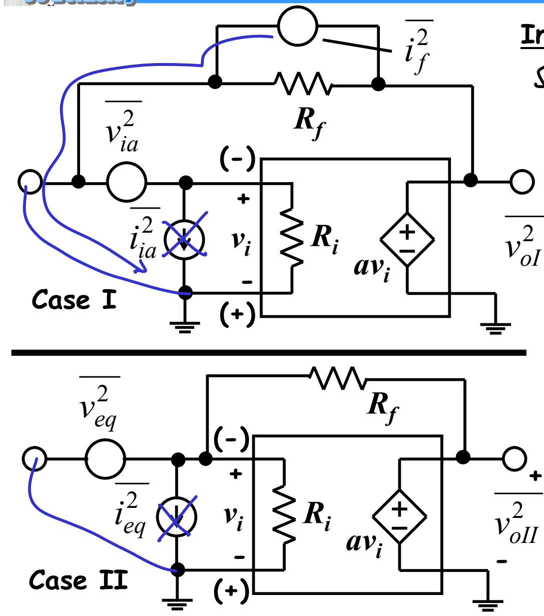
This is unity gain!

$$\therefore \overline{v_{oI}^2} = \overline{i_{ia}^2} R_f^2 + \overline{i_f^2} R_f^2 + \overline{v_{ia}^2}$$

Case II: $\overline{v_{oII}^2} = \overline{i_{eq}^2} R_f^2$

$$\therefore \boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}}$$

## Example: TransR Amplifier Noise (cont)




**Input-referred voltage noise:**

Short inputs; equate output voltage noise

Case I:

$\overline{v_{oI}^2} = \overline{v_{ia}^2}a^2$

(Both $\overline{i_{ia}^2}$ & $\overline{i_f^2}$ are shorted out.)

Case II:

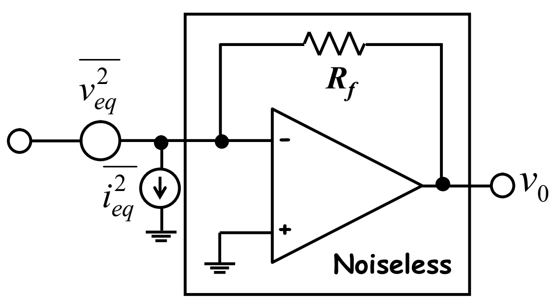
$\overline{v_{oII}^2} = \overline{v_{eq}^2}a^2$

$\therefore \boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$



## Example: TransR Amplifier Noise (cont)

- To summarize, for a transresistance amplifier, the equivalent input-referred current and voltage noise generators are given by:



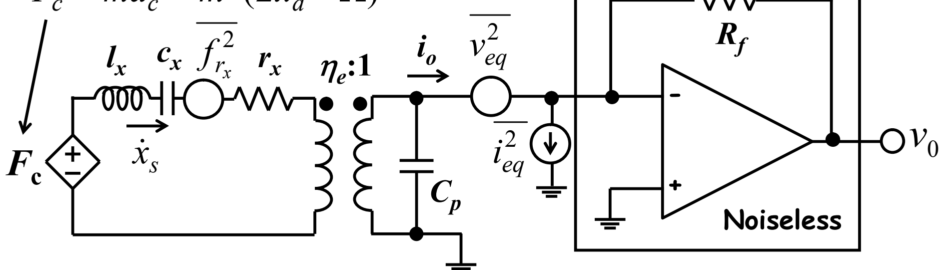

$$\boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}} \qquad \boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$$

# Back to Gyro Noise & MDS



## Example: Gyro MDS Calculation

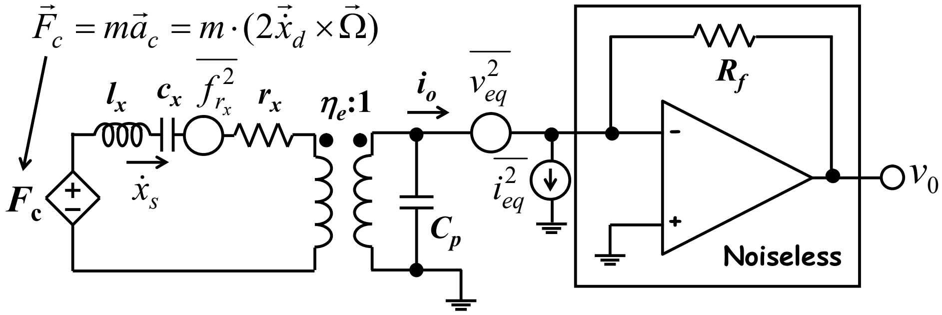
$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$




- The gyro sense presents a large effective source impedance
  - Currents are the important variable; voltages are "opened" out
  - Must compare $i_o$ with the total current noise $i_{eqTOT}$ going into the amplifier circuit

## Example: Gyro MDS Calculation (cont)

$$\vec{F}_c = m\vec{a}_c = m\cdot(2\vec{\dot{x}}_d \times \vec{\Omega})$$




- First, find the rotation to $i_o$ transfer function:

$$\dot{x}_s = \frac{\omega_s Q_s}{k_s}\Theta_s(j\omega_d) F_s = \frac{\omega_s Q_s}{k_s}\cdot 2\omega_d x_d \Omega m \cdot \Theta_s(j\omega_d)$$

$$\frac{m}{k_s} = \frac{1}{\omega_s^2}$$

$$[F_s = F_c = 2\omega_d x_d \Omega m]$$

$$\dot{x}_s = 2\frac{\omega_d}{\omega_s} Q_s x_d \Theta_s(j\omega_d)\cdot\Omega$$



## Example: Gyro MDS Calculation (cont)

$$i_o = \eta_e \dot{x}_s = \underbrace{2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d)}_{A \triangleq \text{scale factor}}\cdot\Omega \rightarrow \boxed{\begin{array}{l} i_o = A\Omega \\ \text{Where } A = 2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d) \end{array}}$$

When $\Omega = \Omega_{min} \triangleq$ MDS, $i_o = i_{eqTOT}$ ← input-referred noise current entering the sense amplifier → in pA/$\sqrt{Hz}$

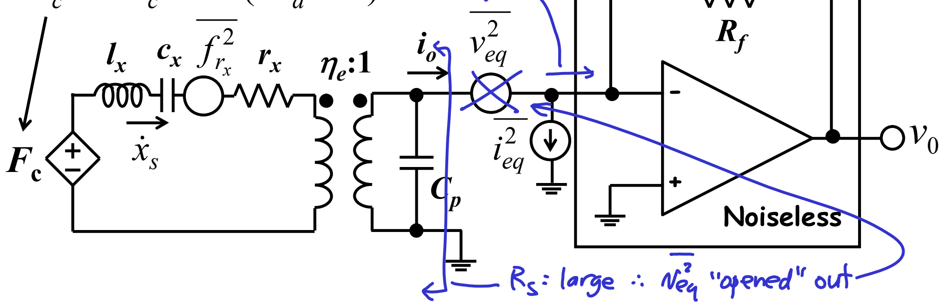
$$\therefore\ i_{eqTOT} = A\Omega_{min} \rightarrow \boxed{\Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180°}{\pi}\right)\ [(°/hr)/\sqrt{Hz}]}$$

$$\boxed{\text{Angle Random Walk} = \text{ARW} = \frac{1}{60}\Omega_{min}\quad [°/\sqrt{hr}]}$$

Easier to determine directional error as a function of elapsed time.

## Example: Gyro MDS Calculation (cont)

$$\vec{F}_c = m\vec{a}_c = m\cdot(2\dot{\vec{x}}_d \times \vec{\Omega})$$



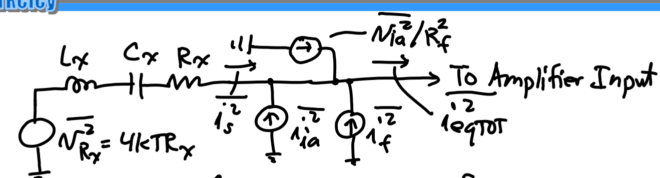

- Now, find the $i_{eqTOT}$ entering the amplifier input:

$$i_{eqTOT} = i_s + i_{eq} \rightarrow \overline{i_{eqTOT}^2} = \overline{i_s^2} + \overline{i_f^2} + \overline{i_{ia}^2} + \frac{\overline{v_{ia}^2}}{R_f^2} \qquad \frac{\overline{f_{r_x}^2}}{\Delta f} = 4kTr_x$$

Brownion motion noise of the sense element → determined entirely by the noise in $r_x$ → $\overline{f_{r_x}^2}$
↳ easiest to convert to an all electrical equiv. ckt.



## Example: Gyro MDS Calculation (cont)




where $L_x = \frac{l_x}{\eta_e^2}$, $C_x = \eta_e^2 c_x$, $R_x = \frac{r_x}{\eta_e^2}$

$$\therefore\ i_s = v_{R_x}\left(\frac{1}{R_x}\right)\Theta_s(j\omega_d) \rightarrow \frac{\overline{i_s^2}}{\Delta f} = 4kTR_x\left(\frac{1}{R_x^2}\right)|\Theta_s(j\omega_d)|^2$$

$$\Rightarrow \boxed{\frac{\overline{i_s^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2}$$

Thus:

$$\boxed{\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)}$$

learn to get these from EE240.
↳ or just get them from a data sheet...

# LF356 Op Amp Data Sheet

UC Berkeley

## LF155/LF156/LF256/LF257/LF355/LF356/LF357
## JFET Input Operational Amplifiers

### General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits

**Common Features**

- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: $0.01\ pA/\sqrt{Hz}$
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

$\sqrt{\frac{\overline{i_{ia}^2}}{\Delta f}} = 0.01\ pA/\sqrt{Hz}$

### Features

**Advantages**

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

### Uncommon Features

| | LF155/ LF355 | LF156/ LF256/ LF356 | LF257/ LF357 ($A_V$=5) | Units |
|---|---|---|---|---|
| Extremely fast settling time to 0.01% | 4 | 1.5 | 1.5 | μs |
| Fast slew rate | 5 | 12 | 50 | V/μs |
| Wide gain bandwidth | 2.5 | 5 | 20 | MHz |
| Low input noise voltage | 20 | 12 | 12 | $nV/\sqrt{Hz}$ |

$\sqrt{\frac{\overline{v_{ia}^2}}{\Delta f}} = 12\,nV/\sqrt{Hz}$

### Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers



# Example ARW Calculation

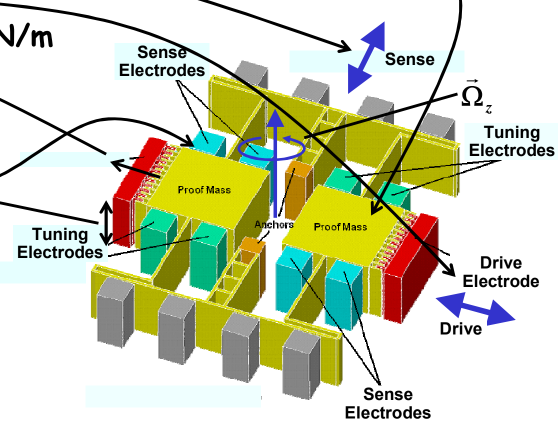
UC Berkeley

- Example Design:
  - Sensor Element:

$m = (100\mu m)(100\mu m)(20\mu m)(2300kg/m^3) = 4.6\times10^{-10}kg$
$\omega_s = 2\pi(15kHz)$
$\omega_d = 2\pi(10kHz)$
$k_s = \omega_s^2 m = 4.09\ N/m$
$x_d = 20\ \mu m$
$Q_s = 50{,}000$
$V_P = 5V$
$h = 20\ \mu m$
$d = 1\ \mu m$

  - Sensing Circuitry:

$R_f = 100k\Omega$
$i_{ia} = 0.01\ pA/\sqrt{Hz}$
$v_{ia} = 12\ nV/\sqrt{Hz}$





## Example ARW Calculation (cont)

Get rotation rate to output current scale factor:

$$A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e |\Theta_s(j\omega_d)| = 2\left(\frac{10k}{15k}\right)(50k)(20\mu)(5)(2000\epsilon_o)(0.000024) = \underline{2.83\times10^{-12}C}$$

$$\Theta_s(j\omega_d) = \frac{(j\omega_d)(\omega_s/Q_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10k)(15k)/(50k)}{(15k)^2-(10k)^2+\frac{j(10k)(15k)}{50k}} = \frac{j(3k)}{1.25\times10^8 + j(3k)}$$

$$\rightarrow |\Theta_s(j\omega_d)| = \frac{3k}{\sqrt{(1.25\times10^8)^2+(3k)^2}} = 0.000024$$

$$\frac{\partial C}{\partial x} = \frac{C_o}{d} = \frac{\epsilon_o h W_p}{d} = \frac{\epsilon_o(20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_o \rightarrow \eta_e = V_P\frac{\partial C}{\partial x} = \overbrace{5(2000\epsilon_o)}^{8.854\times10^{-8}F/m}$$

Assume electrode covers the whole sidewall.

$8.854\times10^{-12}$ F/m

Then, get noise:

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)$$

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## Example ARW Calculation (cont)

$$R_x = \frac{\omega_s m}{Q_s\eta_e^2} = \frac{2\pi(15k)(4.6\times10^{-10})}{(50k)(8.854\times10^{-8})^2} = 110.6k\Omega$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6k)}(0.000024)^2}_{8.64\times10^{-35}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

sensor element noise insignificant

Noise from $R_f$ dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.68\times10^{-26}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 1.30\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{1.30\times10^{-13}}{2.83\times10^{-12}}(3600)\left(\frac{180}{\pi}\right) = \underline{9448\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(9448) = 157\ ^\circ/\sqrt{hr} = ARW$$

→ Almost turned around in 1 hour!

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## What if $\omega_d = \omega_s$?

If $\omega_d = \omega_s = 15KHz$, then $|\Theta_s(j\omega_d)| = 1$ and

$$A = 2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e |\Theta_s(j\omega_d)| = 2Q_s x_d \eta_e = 2(50K)(20\mu)(5)(2000\epsilon_0) = \underline{1.77\times10^{-7}C}$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6K)}(1)^2}_{1.51\times10^{-25}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

Now, the sensor element dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.67\times10^{-25}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 4.08\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{4.08\times10^{-13}}{1.77\times10^{-7}}(3600)\left(\frac{180}{\pi}\right) = \underline{0.476\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(0.476) = 0.0079\ ^\circ/\sqrt{hr} = ARW$$

⇒ Navigation grade!

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