# EE C247B – ME C218
# Introduction to MEMS Design
# Spring 2016

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 7: Mechanics of Materials

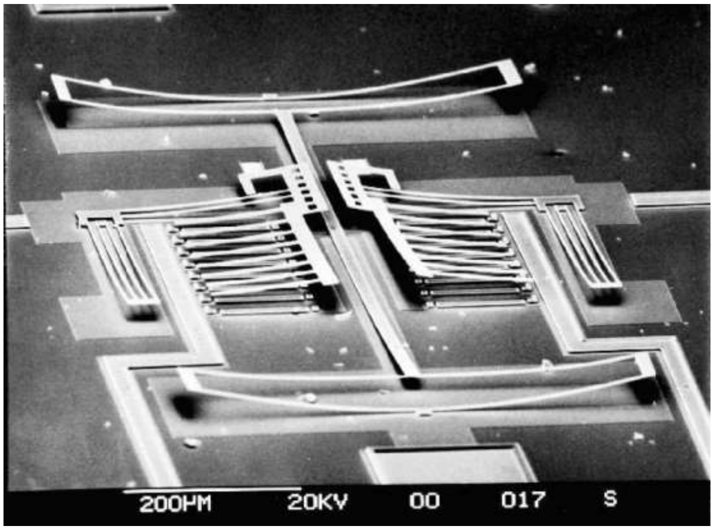


## Outline

- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics

## Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction



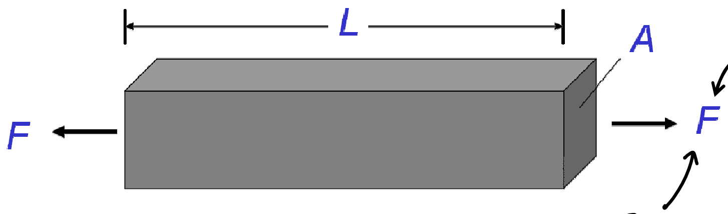



## Elasticity



## Normal Stress (1D)

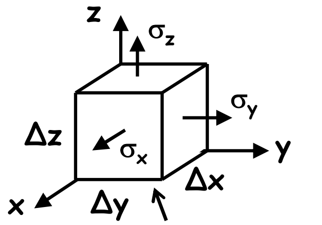



If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

$$\text{Stress} = \begin{Bmatrix}\text{Force per}\\ \text{unit area}\end{Bmatrix} = \sigma = \frac{F}{A} \quad [N/m^2 = Pa]$$

↖ standard mks unit



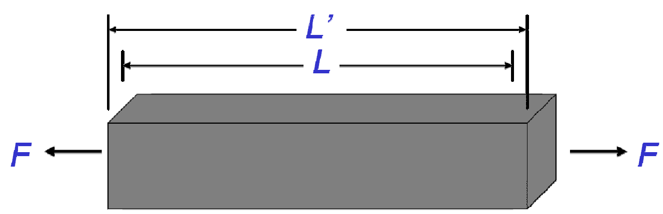

Differential volume element

⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point



## Strain (1D)

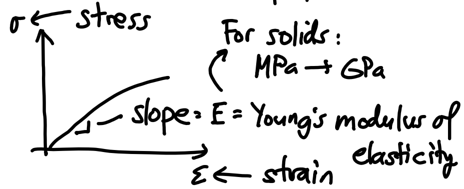



Sometimes a unit called the "microstrain" is used, where

$1\mu\varepsilon = \frac{\Delta L}{L}$ of 1 part in $10^6$

$$\text{Strain} = \begin{Bmatrix}\text{Fractional Change}\\ \text{in Length}\end{Bmatrix} = \varepsilon = \frac{L'-L}{L} = \frac{\Delta L}{L} \quad [\text{unitless}]$$

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress




For solids: MPa → GPa

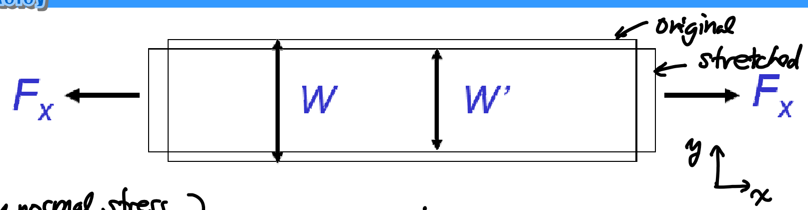
$\sigma = \varepsilon E \rightarrow$ $\boxed{\varepsilon = \frac{\sigma}{E} \quad [\text{unitless}]}$

Thus, the units of E are the same as σ → Pa



# The Poisson Ratio




Apply normal stress to a free-standing object → uniaxial strain
→ but also get contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (−) strain:

$$\varepsilon_y = \frac{W' - W}{W} = \frac{\Delta W}{W} = -\nu \varepsilon_x$$

$\nu$ = Poisson ratio [unitless]

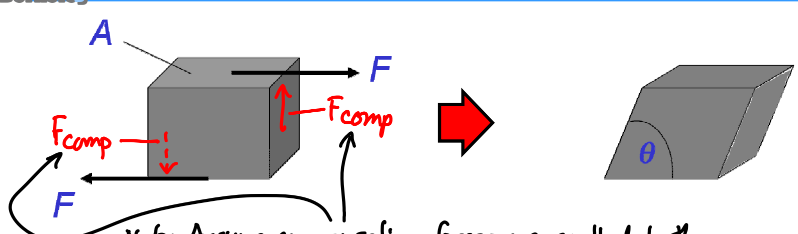
Typical values: $0 \rightarrow 0.5$

⇒ inorganic solids: $0.2 \rightarrow 0.3$

⇒ elastomers (e.g., rubber): $\sim 0.5$



# Shear Stress & Strain (1D)




Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)

Shear Stress = {Force Per Unit Area Parallel to the Surfaces} $= \tau = \frac{F}{A}$ [Pa]

→ Generates a shear strain:

$$\text{Shear Strain} = \theta = \frac{\tau}{G}$$

← $G \triangleq$ shear modulus

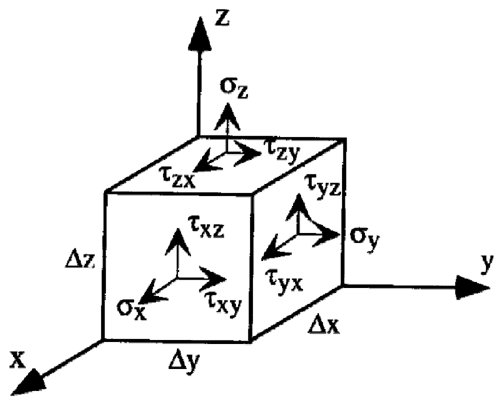
$$G = \frac{E}{2(1+\nu)}$$



## 2D and 3D Considerations

- Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:



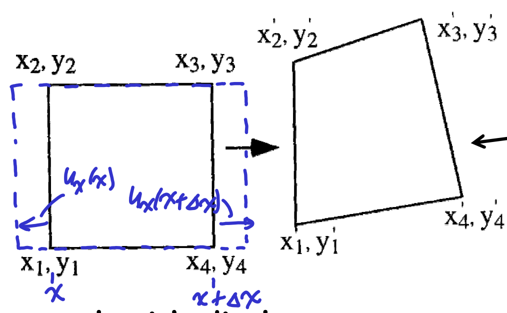

Stresses acting on a differential volume element

$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy}$$



## 2D Strain

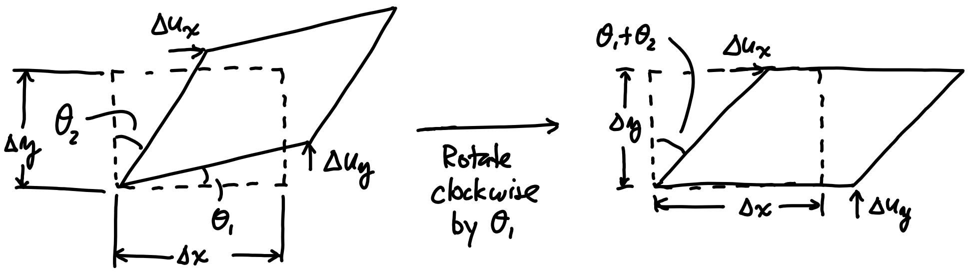
- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation




Area element experiences both displacement and deformation

- Must work with displacement vectors
- Differential definition of axial strain: ⟶ $\varepsilon_x = \dfrac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \dfrac{\partial u_x}{\partial x}$

# 2D Shear Strain




⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation

↳ use a symmetric definition of shear strain:

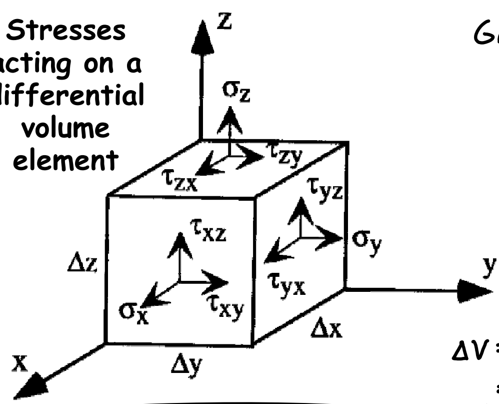
$$\gamma_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

↑ For small amplitude deformations.



# Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element




Given an x-directed uniaxial stress, $\sigma_x$:

$\Delta x \rightarrow \Delta x(1+\epsilon_x)$

$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$

↓ The resulting change in volume $\Delta V$

$$\Delta V = \Delta x\Delta y\Delta z(1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x\Delta y\Delta z$$
$$= \Delta x\Delta y\Delta z\left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

{Assume small strains} ⇒ $\Delta V = \Delta x\Delta y\Delta z\left[(1+\epsilon_x)(1-2\nu\epsilon_x)-1\right]$

$[(1+mx)^n \approx 1+nmx]$ ⇒ $\approx \Delta x\Delta y\Delta z\left[\cancel{1}+\epsilon_x - 2\nu\epsilon_x - \cancel{2\nu\epsilon_x^2} - \cancel{1}\right]$ (negl.)

$$\boxed{\Delta V \approx \Delta x\Delta y\Delta z(1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber) → no $\Delta V$!

$\nu < 0.5$ → finite $\Delta V$

## Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

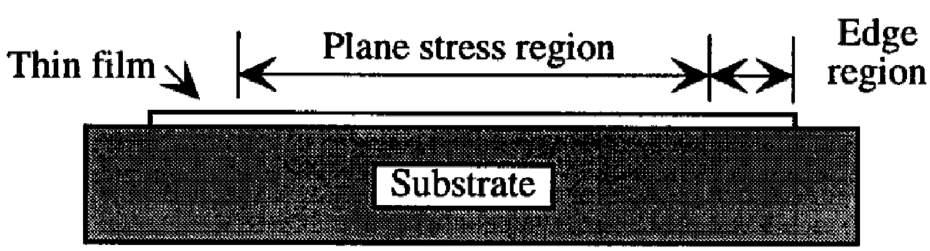
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \underbrace{\nu\left(\sigma_x + \sigma_y\right)}\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Basically, add in off-axis strains from normal stresses in other directions



## Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)




- At regions more than 3 thicknesses from edges, the top surface is stress-free → $\sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$

$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$



## Important Case: Plane Stress (cont.)

- Symmetry in the xy-plane → $\sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$
  where

$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$



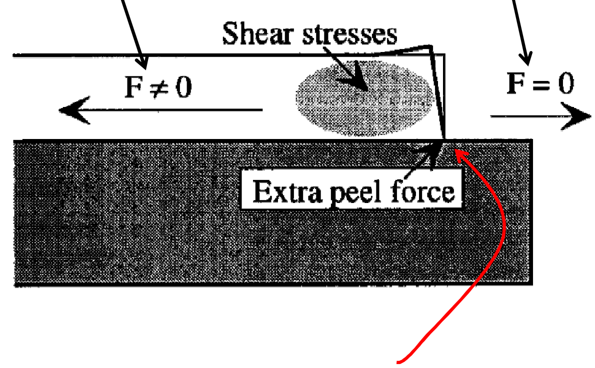
## Edge Region of a Tensile ($\sigma$>0) Film

Net non-zero in-plane force (that we just analyzed)

At free edge, in-plane force must be zero

Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here




Discontinuity of stress at the attached corner → stress concentration

Peel forces that can peel the film off the surface



# Linear Thermal Expansion

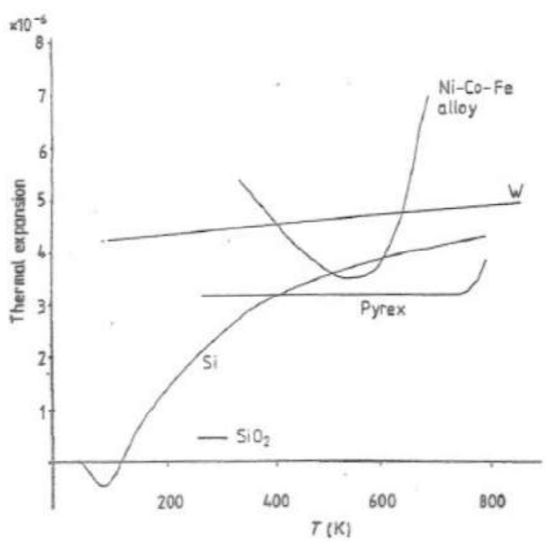
- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

Linear thermal expansion coefficient $\triangleq \alpha_T = \dfrac{d\varepsilon_x}{dT}$ [Kelvin$^{-1}$]

Remarks:
- $\alpha_T$ values tend to be in the $10^{-6}$ to $10^{-7}$ range
- Can capture the $10^{-6}$ by using dimensions of μstrain/K, where $10^{-6}$ $K^{-1}$ = 1 μstrain/K
- In 3D, get volume thermal expansion coefficient ⟶ $\dfrac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, $\alpha_T$ can be treated as a constant of the material, but in actuality, it is a function of temperature



# $\alpha_T$ As a Function of Temperature



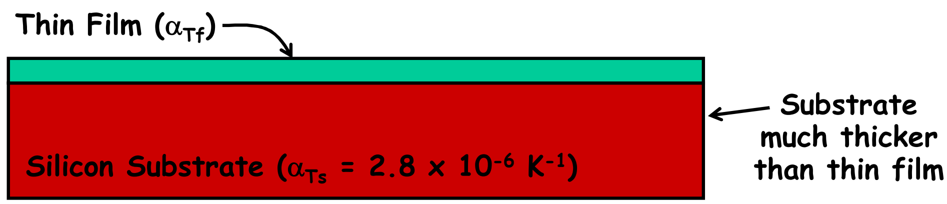

[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction



## Thin-Film Thermal Stress




- Assume film is deposited stress-free at a temperature $T_r$, then the whole thing is cooled to room temperature $T_r$
- Substrate much thicker than thin film → substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T \quad , \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate: $\epsilon_{f,free} = -\alpha_{Tf}\Delta T$

↪ over



## Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film is the same as that in the substrate.

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

↳ Note that this is biaxial strain

↳ it can only be developed by an in-plane biaxial stress:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\ K^{-1}$, $E' = 4GPa$ (e.g., $SiO_2$)
deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225K$

$$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$$

↳ $\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = \underline{\underline{60.5MPa}}$ ← stress is (+), ∴ tensile [(-) would be compressive]



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# MEMS Material Properties



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# Material Properties for MEMS

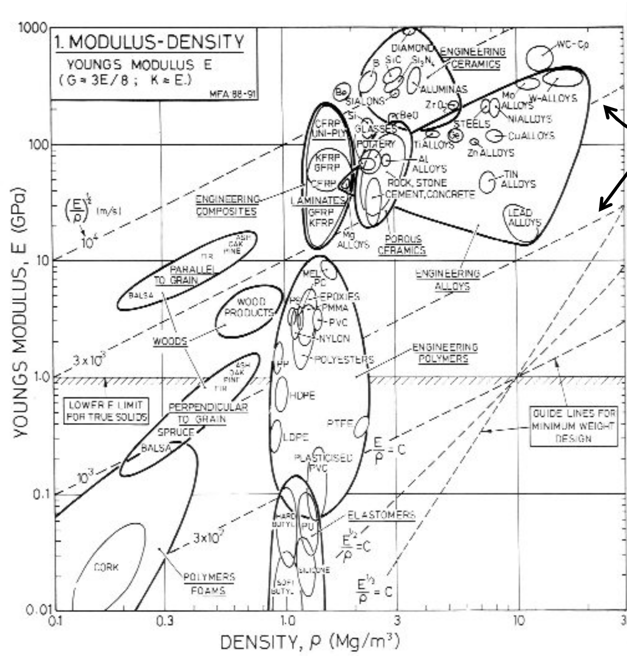
| Material | Density, ρ, Kg/m³ | Modulus, E, GPa | E/ ρ GN/kg-m |
|---|---|---|---|
| Silicon | 2330 | 165 | 72 |
| Silicon Oxide | 2200 | 73 | 36 |
| Silicon Nitride | 3300 | 304 | 92 |
| Nickel | 8900 | 207 | 23 |
| Aluminum | 2710 | 69 | 25 |
| Aluminum Oxide | 3970 | 393 | 99 |
| Silicon Carbide | 3300 | 430 | 130 |
| Diamond | 3510 | 1035 | 295 |

Units: $(m/s)^2$

$\sqrt{(E/\rho)}$ is acoustic velocity

[Mark Spearing, MIT]

## Young's Modulus Versus Density




Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]



## Yield Strength

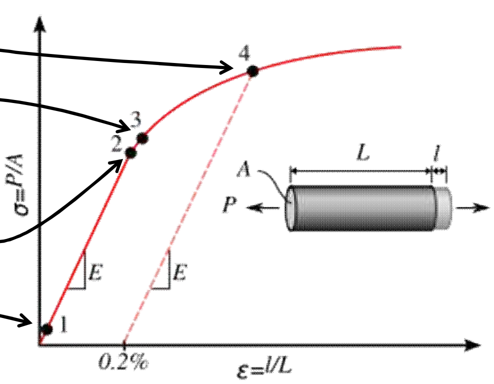
- Definition: the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
- Below the yield point: material deforms elastically → returns to its original shape when the applied stress is removed
- Beyond the yield point: some fraction of the deformation is permanent and non-reversible

Yield Strength: defined at 0.2% offset pt.

Elastic Limit: stress at which permanent deformation begins

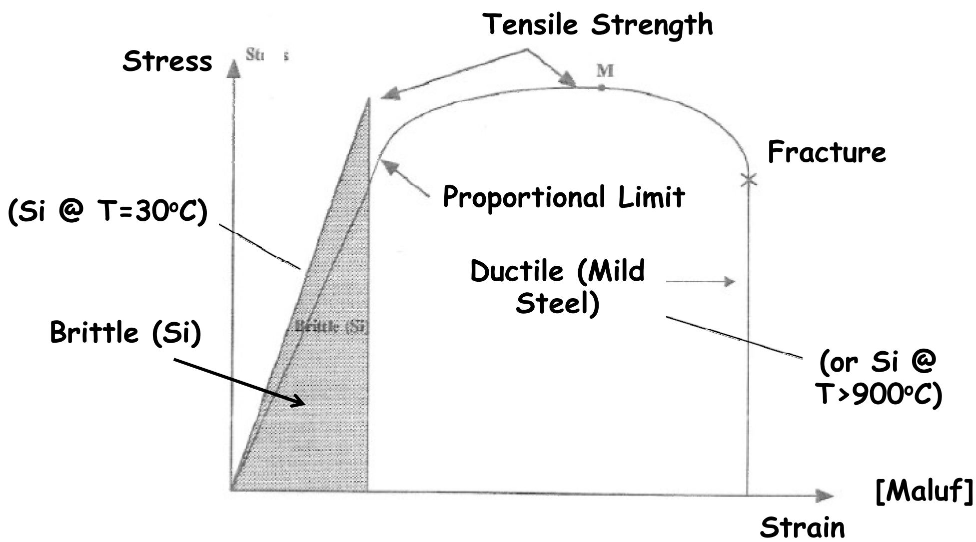
Proportionality Limit: point at which curve goes nonlinear

True Elastic Limit: lowest stress at which dislocations move



## Yield Strength (cont.)

- Below: typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g. steel) materials





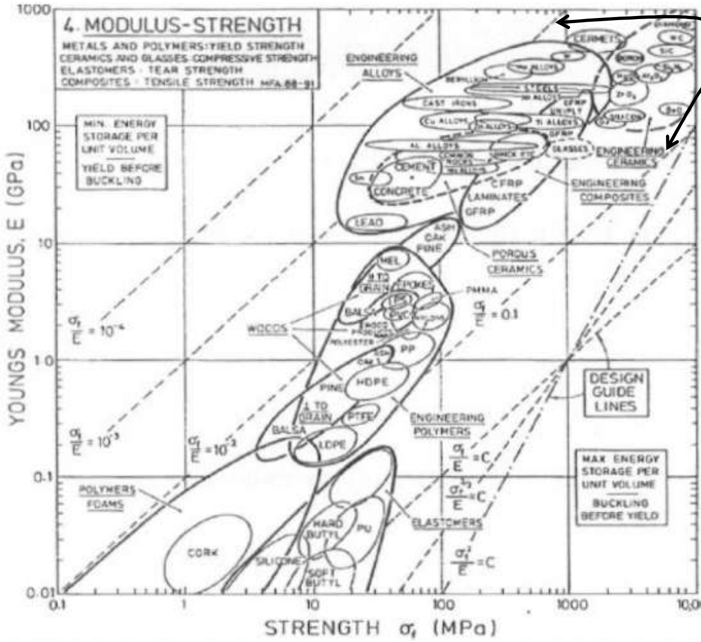

## Young's Modulus and Useful Strength

Stored mechanical energy

| Material | Modulus, E, GPa | Useful Strength*, $\sigma_f$, MPa | $\frac{\sigma_f}{E}$ (-) x $10^{-3}$ | $\frac{\sigma_f^2}{E}$ MJ/m³ |
|---|---|---|---|---|
| Silicon | 165 | 4000 | 24 | 97 |
| Silicon Oxide | 73 | 1000 | 13 | 14 |
| Silicon Nitride | 304 | 1000 | 3 | 4 |
| Nickel | 207 | 500 | 2 | 1.2 |
| Aluminum | 69 | 300 | 4 | 1.3 |
| Aluminum Oxide | 393 | 2000 | 5 | 10 |
| Silicon Carbide | 430 | 2000 | 4 | 9.3 |
| Diamond | 1035 | 1000 | 1 | 0.9 |

From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

## Young's Modulus Versus Strength



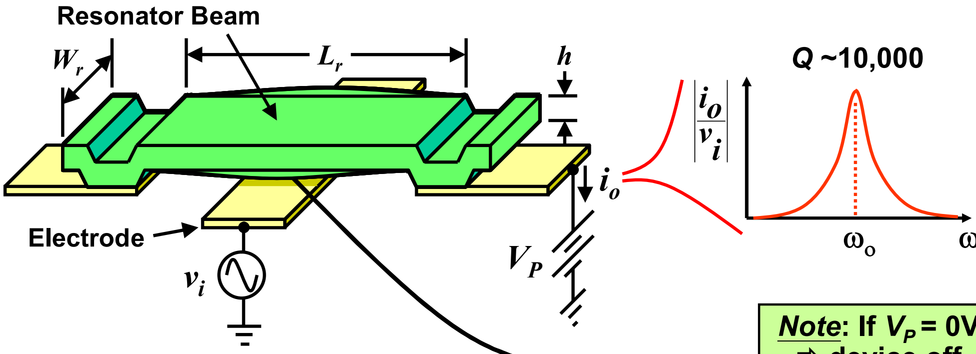

Lines of constant maximum strain

[Ashby, Mechanics of Materials, Pergamon, 1992]



## Quality Factor (or Q)

## Clamped-Clamped Beam μResonator

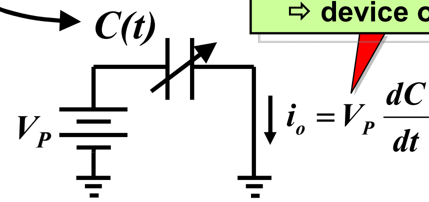



*Frequency*:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

Stiffness; Young's Modulus; Mass; Density

Mass ➡ (e.g., $m_r = 10^{-13}$ kg) ➡ Smaller mass ⇨ higher freq. range and lower series $R_x$



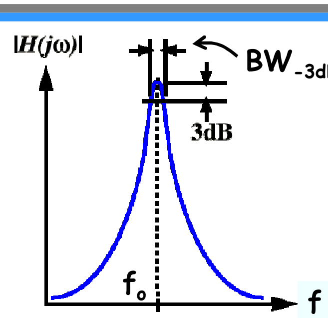

*Note*: If $V_P$ = 0V ⇨ device off



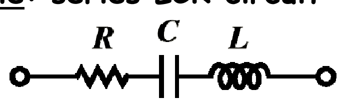
## Quality Factor (or Q)

- Measure of the frequency selectivity of a tuned circuit
- Definition:

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

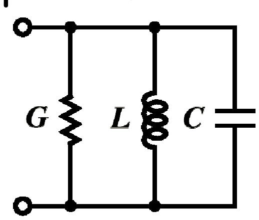



- Example: series LCR circuit




$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

- Example: parallel LCR circuit

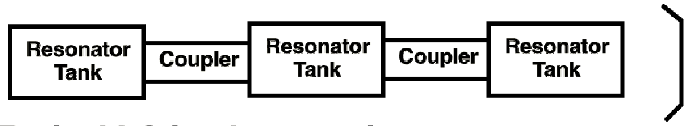



$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$



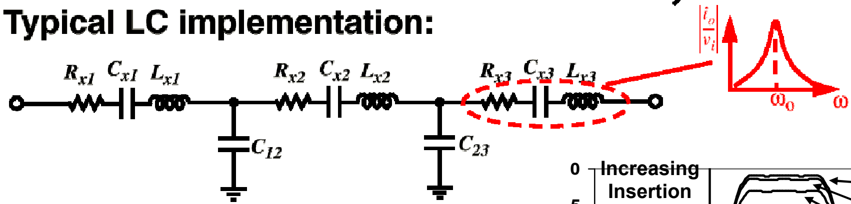
 

## Selective Low-Loss Filters: Need Q

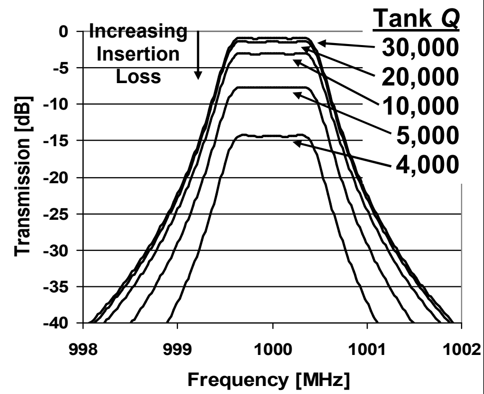



General BPF Implementation

Typical LC implementation:



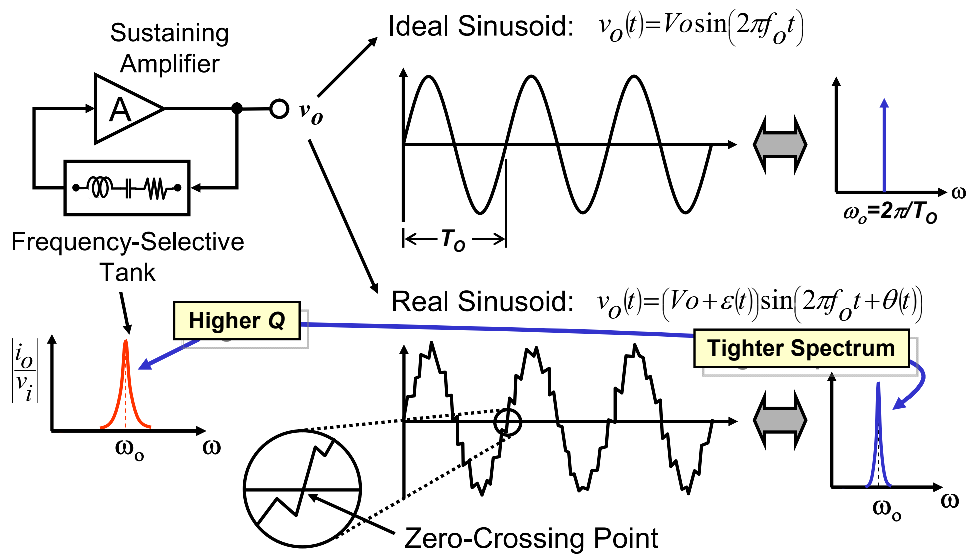

- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - ↳ heavy insertion loss for resonator Q < 10,000






## Oscillator: Need for High Q

- Main Function: provide a stable output frequency
- Difficulty: superposed noise degrades frequency stability

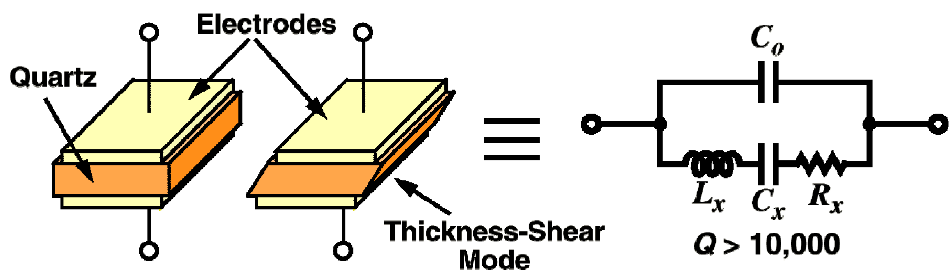




## Attaining High Q

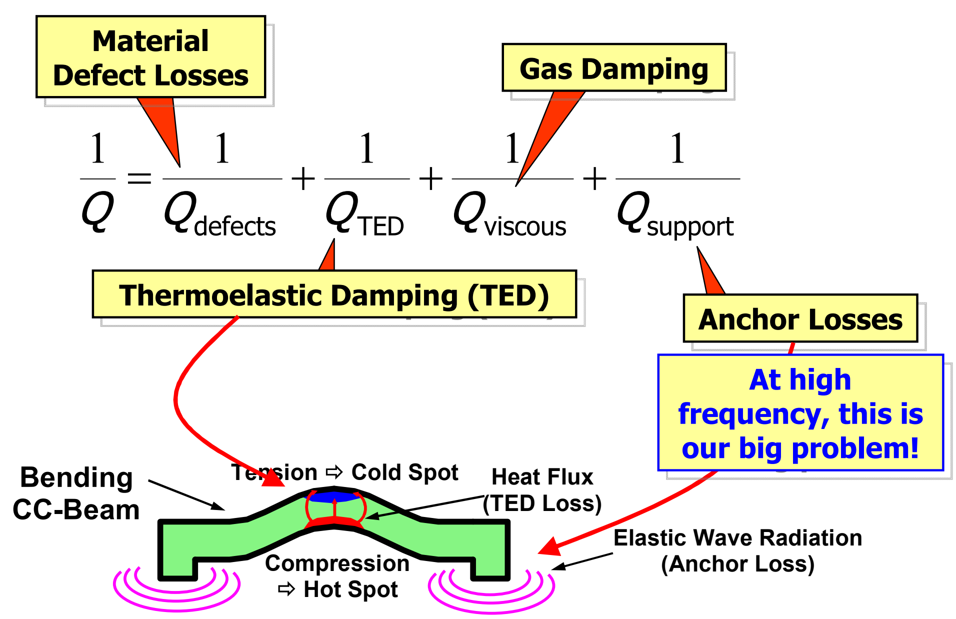
- Problem: IC's cannot achieve Q's in the thousands
  - transistors ⇨ consume too much power to get Q
  - on-chip spiral inductors ⇨ Q's no higher than ~10
  - off-chip inductors ⇨ Q's in the range of 100's
- Observation: vibrating mechanical resonances ⇨ Q > 1,000
- Example: quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q's ~ 10,000 or higher (Q ~ $10^6$ possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode






## Energy Dissipation and Resonator Q

$$\frac{1}{Q} = \frac{1}{Q_{\text{defects}}} + \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{viscous}}} + \frac{1}{Q_{\text{support}}}$$





# Thermoelastic Damping (TED)

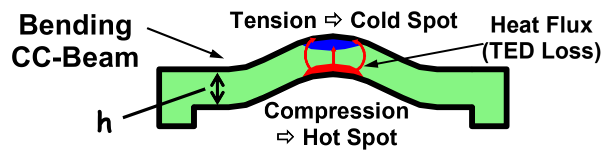
- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss

$$\varsigma = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2\left[\frac{f_{TED} f}{f_{TED}^2 + f^2}\right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$




$\zeta$ = thermoelastic damping factor
$\alpha$ = thermal expansion coefficient
T = beam temperature
E = elastic modulus
$\rho$ = material density
$C_p$ = heat capacity at const. pressure
K = thermal conductivity
f = beam frequency
h = beam thickness
$f_{TED}$ = characteristic TED frequency



# TED Characteristic Frequency

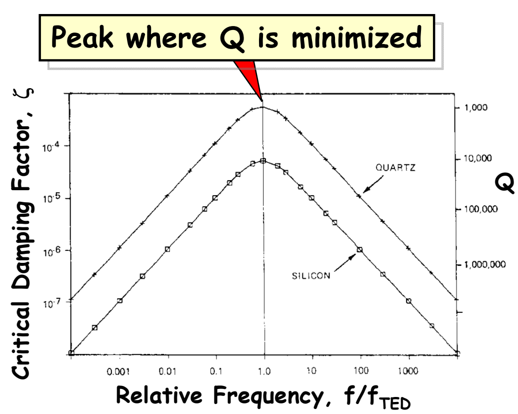
$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

$\rho$ = material density
$C_p$ = heat capacity at const. pressure
K = thermal conductivity
h = beam thickness
$f_{TED}$ = characteristic TED frequency

- Governed by
  - Resonator dimensions
  - Material properties

TABLE 1. MATERIAL PROPERTIES

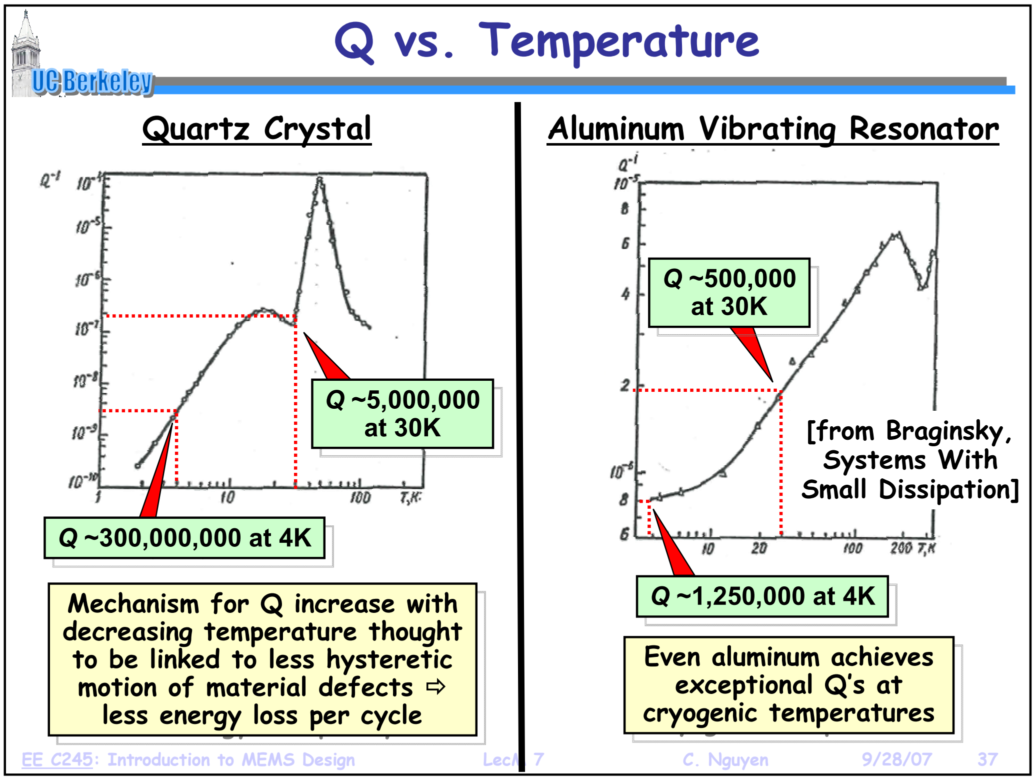
| Property | Silicon | Quartz | Units |
|---|---|---|---|
| Thermal expansion | 2.60 | 13.70 | ppm/°K |
| Elastic modulus | 1.70 | 0.78 | $10^{12}$ dyne/cm$^2$ |
| Material density | 2.33 | 2.60 | g/cm$^3$ |
| Heat capacity | 0.70 | 0.75 | J/g/°K |
| Thermal conductivity | 1.50 | 0.10 | $10^7$ dyne/°K/s |
| Peak damping @ 300°K | 1.06 | 11.34 | $10^{-4}$ |

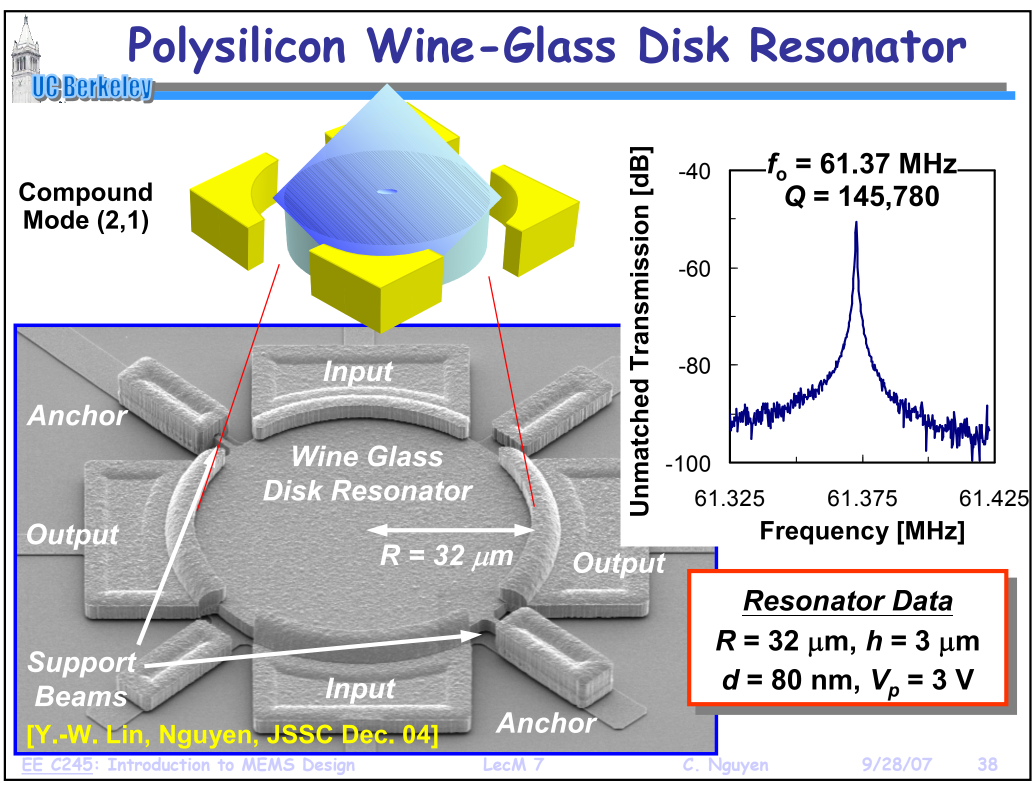



[from Roszhart, Hilton Head 1990]




Q vs. Temperature
UC Berkeley
Quartz Crystal
Aluminum Vibrating Resonator
Q ~5,000,000 at 30K
Q ~300,000,000 at 4K
Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇨ less energy loss per cycle
Q ~500,000 at 30K
[from Braginsky, Systems With Small Dissipation]
Q ~1,250,000 at 4K
Even aluminum achieves exceptional Q's at cryogenic temperatures
EE C245: Introduction to MEMS Design LecM 7 C. Nguyen 9/28/07 37


Polysilicon Wine-Glass Disk Resonator
UC Berkeley
Compound Mode (2,1)
Input
Anchor
Wine Glass Disk Resonator
Output
R = 32 μm
Output
Support Beams
Input
Anchor
[Y.-W. Lin, Nguyen, JSSC Dec. 04]
fo = 61.37 MHz
Q = 145,780
Unmatched Transmission [dB]
-40
-60
-80
-100
61.325
61.375
61.425
Frequency [MHz]
Resonator Data
R = 32 μm, h = 3 μm
d = 80 nm, Vp = 3 V
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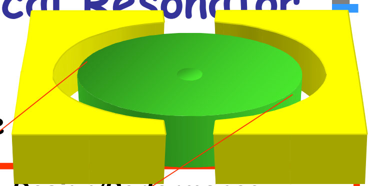
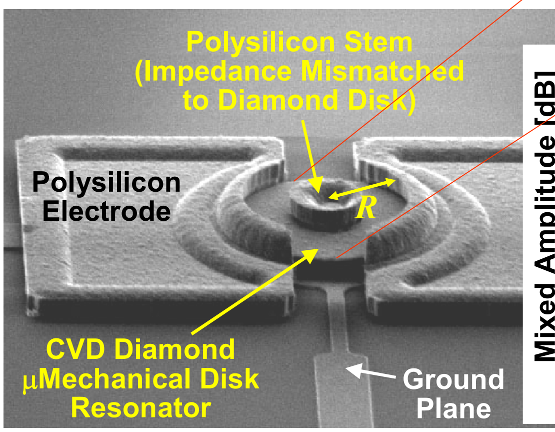
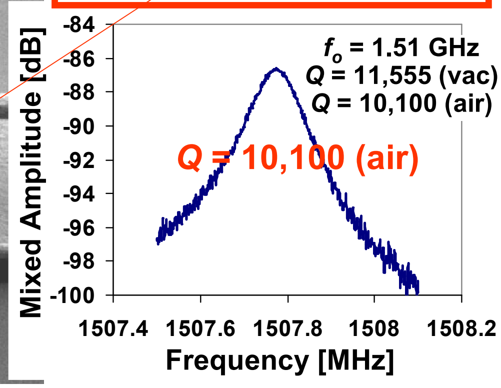
## 1.51-GHz, Q=11,555 Nanocrystalline Diamond Disk μMechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- Q ~11,555 (vacuum); Q ~10,100 (air)
- *Below*: 20 μm diameter disk



**Design/Performance:**
$R$=10μm, $t$=2.2μm, $d$=800Å, $V_P$=7V
$f_o$=1.51 GHz (2nd mode), $Q$=11,555

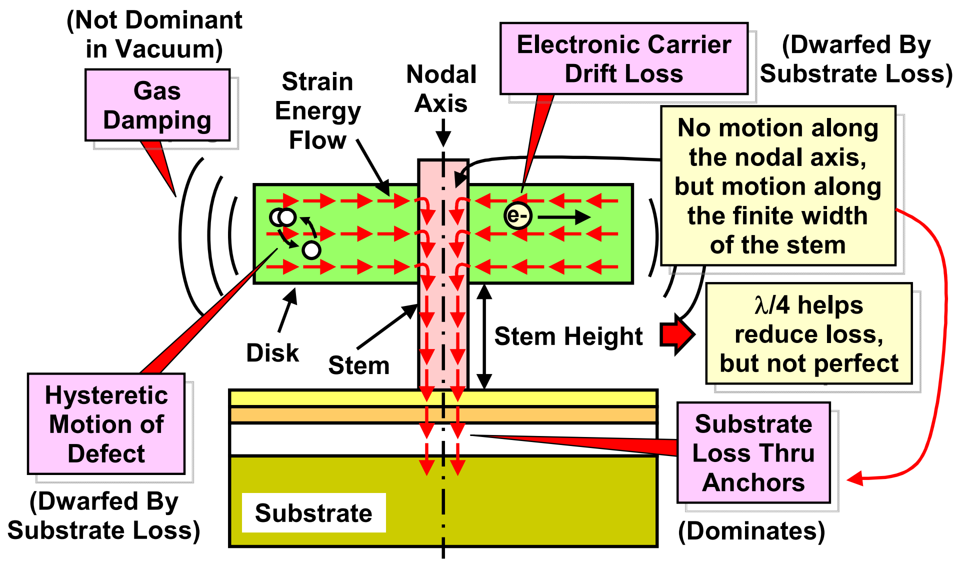



[Wang, Butler, Nguyen MEMS'04]



## Disk Resonator Loss Mechanisms





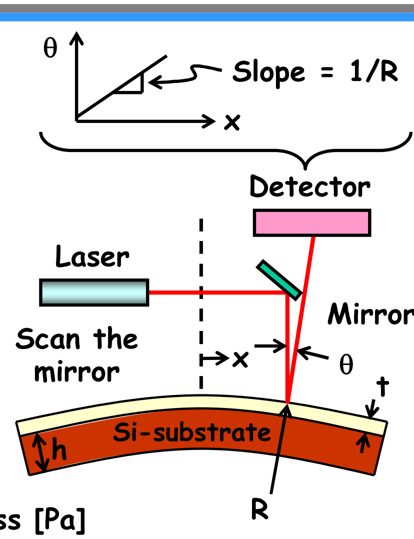

 

# MEMS Material Property Test Structures



## Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:




$$\sigma = \frac{E'h^2}{6Rt}$$

σ = film stress [Pa]
E' = E/(1-ν) = biaxial elastic modulus [Pa]
h = substrate thickness [m]
t = film thickness
R = substrate radius of curvature [m]



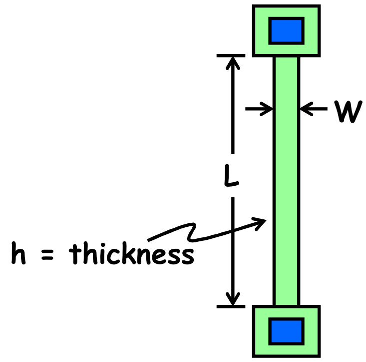
## MEMS Stress Test Structure

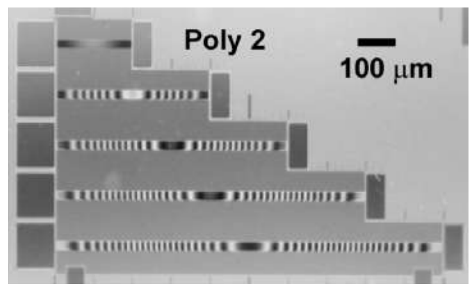
- Simple Approach: use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where

$$\sigma_{critical} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}$$

E = Young's modulus [Pa]
I = (1/12)Wh$^3$ = moment of inertia
L, W, h indicated in the figure

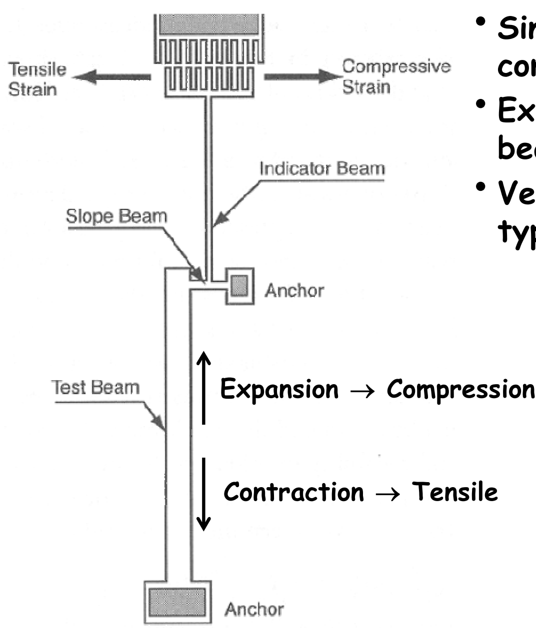
  - Limitation: Only compressive stress is measurable



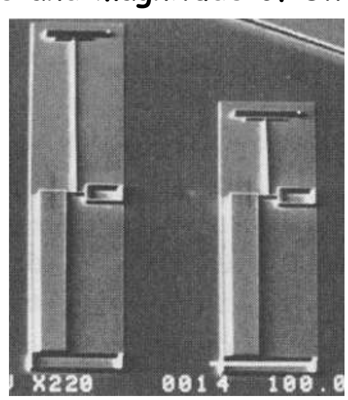





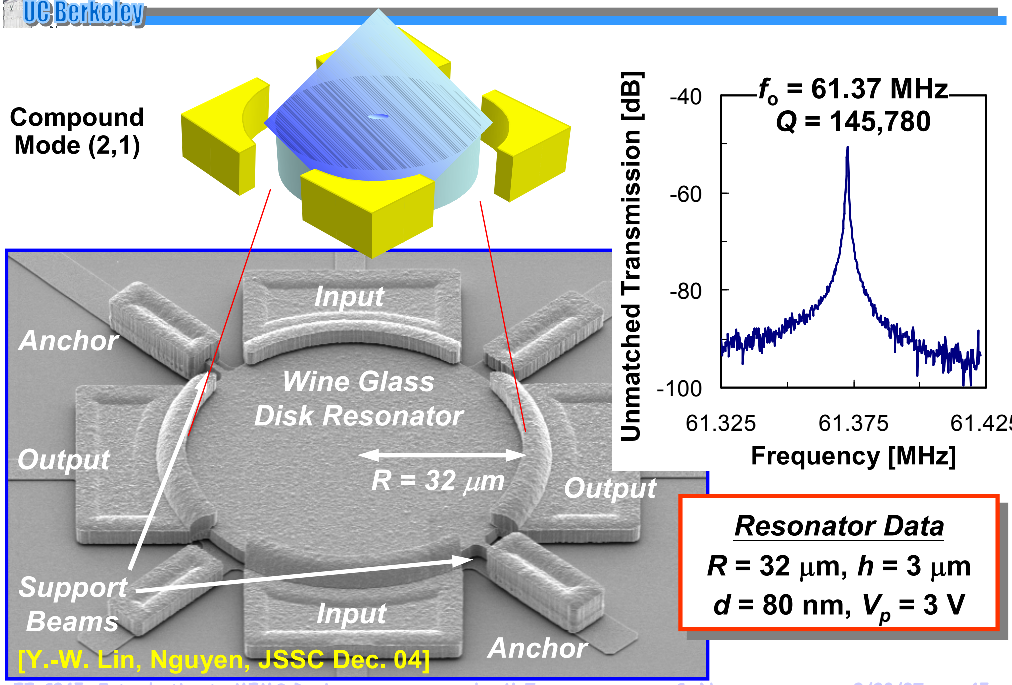

## More Effective Stress Diagnostic




- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress



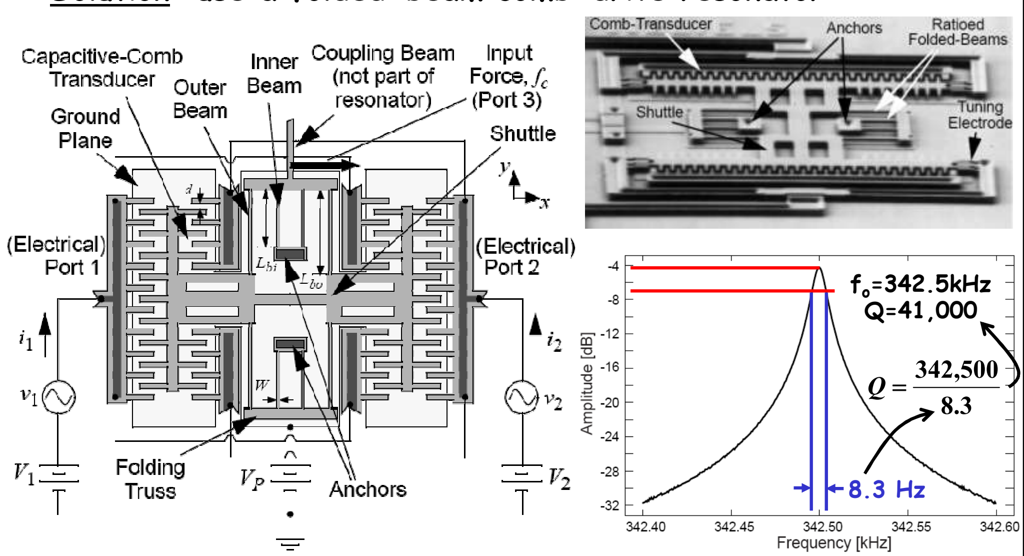
## Q Measurement Using Resonators






## Folded-Beam Comb-Drive Resonator

- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator

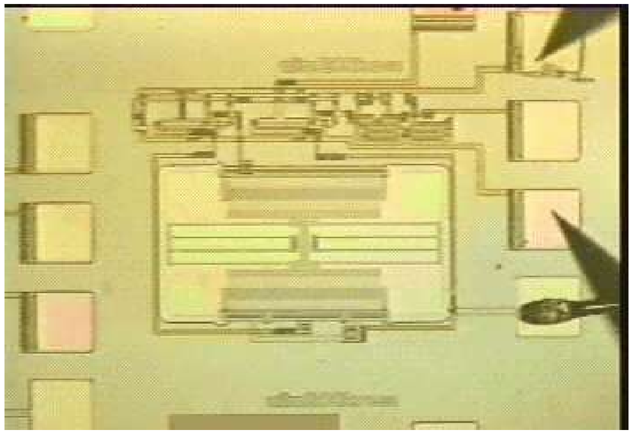




## Comb-Drive Resonator in Action

- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach





## Folded-Beam Comb-Drive Resonator

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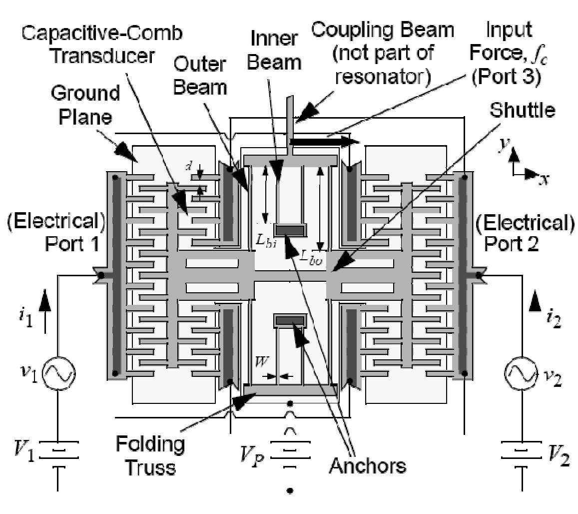
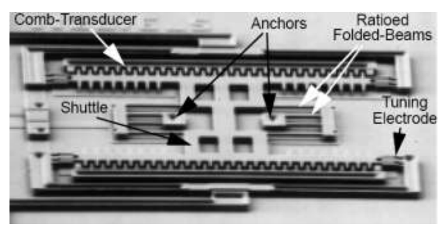
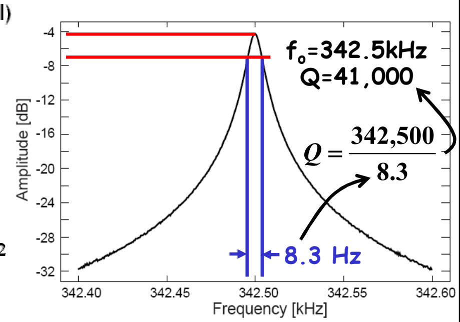




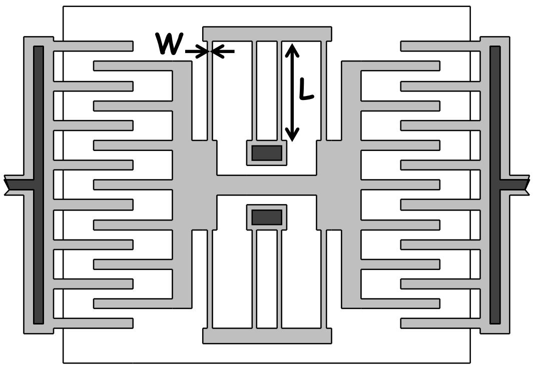
## Measurement of Young's Modulus

- Use micromechanical resonators
  - Resonance frequency depends on E
  - For a folded-beam resonator:

Resonance Frequency = $f_o = \left[ \dfrac{4Eh(W/L)^3}{M_{eq}} \right]^{1/2}$

(E: Young's modulus; $M_{eq}$: Equivalent mass)

h = thickness




- Extract E from measured frequency $f_o$
- Measure $f_o$ for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters



## Anisotropic Materials

## Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}$$

Stresses — Stiffness Coefficients — Strains



## Stiffness Coefficients of Silicon

- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

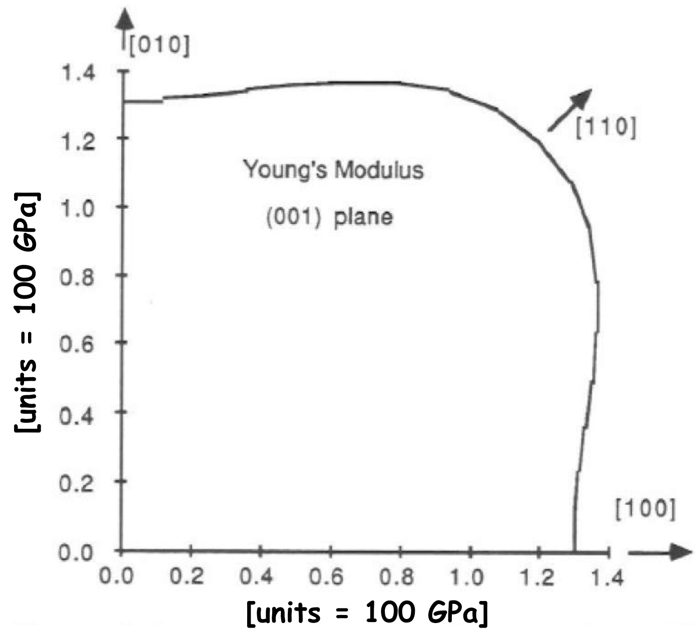
$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}$$

where $\begin{cases} C_{11} = 165.7 \text{ GPa} \\ C_{12} = 63.9 \text{ GPa} \\ C_{44} = 79.6 \text{ GPa} \end{cases}$



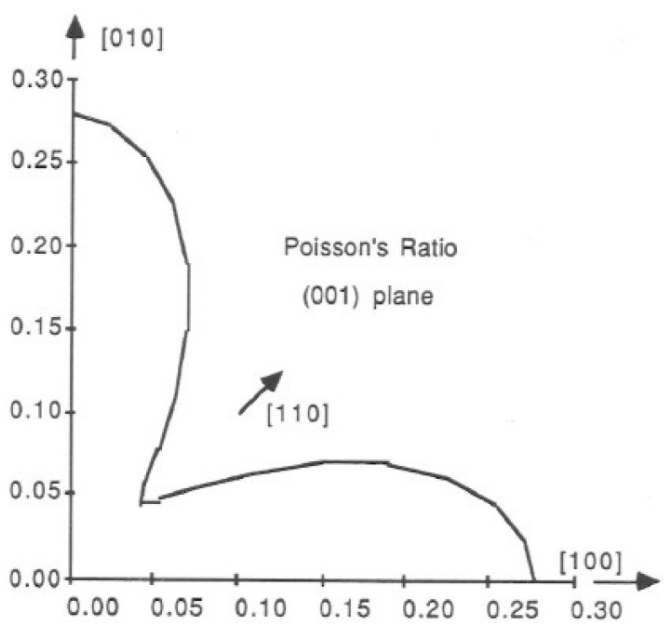
 

## Young's Modulus in the (001) Plane






## Poisson Ratio in (001) Plane

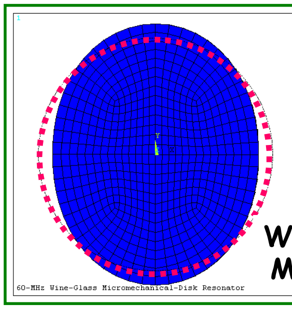




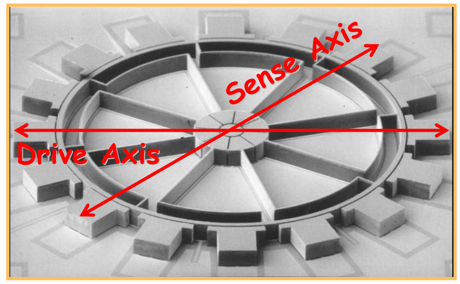

 

# Anisotropic Design Implications

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
  - Mode matching is required, where frequencies along different axes of a ring must be the same
  - Not okay to ignore anisotropic variations, here



Wine-Glass Mode Disk




Ring Gyroscope

