(a) First express the stress gradient as a function of $z$:

$$
\sigma_x(z) = -\sigma_o + \frac{2\sigma_1}{H} z \quad \text{(from the plot in Fig. P544)}
$$

$dM$ is the bending moment of this portion around the origin:

$$
dM = \sigma_x(z) z W d\tau
$$

$$
= \left\{ -\sigma_o + \frac{2\sigma_1}{H} z \right\} z W d\tau
$$

$$
M = \int_{-H/2}^{H/2} -\sigma_o + \frac{2\sigma_1}{H} z W d\tau
$$

$$
= -\sigma_o W \left[ \frac{z^2}{2} \right]_{-H/2}^{H/2} + \frac{2\sigma_1 W}{3H} \left[ \frac{z^3}{3} \right]_{-H/2}^{H/2}
$$

$$
= \frac{\sigma_o WH^2}{6}
$$
The deflection equation is given as:

\[ E I \frac{\partial^2 w}{\partial x^2} = M \text{ bending moment already found in part (a)} \]

\[ E \left( \frac{1}{12} WH^3 \right) \frac{\partial^2 w}{\partial x^2} = \frac{1}{6} E I W H^2 \Rightarrow \frac{\partial^2 w}{\partial x^2} = \frac{2EI}{EH} \]

Take the integral of this equation with respect to \( x \):

\[ \frac{\partial w}{\partial x} = \frac{2EI}{EH} x + c \text{ where } c \text{ is a constant} \]

for a cantilever \( \partial w/\partial x = 0 \) at \( x = 0 \) (anchoring point). Thus, \( c = 0 \). Integrate this equation once more:

\[ w(x) = \frac{6EI}{EH} x^2 + d \text{ where } d \text{ is yet another constant} \]

for a cantilever \( w = 0 \) at \( x = 0 \) (anchoring point). Thus, \( d = 0 \). Finally:

\[ w(x) = \frac{6EI}{EH} x^2 \]

The tip deflection \( w_{tip} \) occurs at \( x = L \):

\[ w_{tip} = \frac{6EI}{EH} \frac{L^2}{2} \]

The deflection equation is given as:

\[ E I \frac{\partial^2 w}{\partial x^2} = M - F(L-x) \]

\[ E \left( \frac{1}{12} WH^3 \right) \frac{\partial^2 w}{\partial x^2} = \frac{1}{6} E I W H^2 - F(L-x) \]

\[ \frac{\partial^2 w}{\partial x^2} = \frac{12}{EWH^3} \left( \frac{1}{6} E I W H^2 - F(L-x) \right) = \frac{2EI}{EH} - \frac{12F}{EWH^3} (L-x) \]

\[ \frac{\partial w}{\partial x} = \frac{2EI}{EH} x - \frac{12F}{EWH^3} \left( Lx - \frac{x^2}{2} \right) + c \leftarrow \frac{\partial w}{\partial x} = 0 \text{ at } x = 0 \Rightarrow c = 0 \]

\[ w = \frac{EI x^2}{EH} - \frac{12F}{EWH^3} \left( \frac{Lx^2}{2} - \frac{x^3}{6} \right) + d \leftarrow w = 0 \text{ at } x = 0 \Rightarrow d = 0 \]

\[ w = \frac{EI x^2}{EH} - \frac{6F}{EWH^3} \left( L - \frac{x}{3} \right) x^2 \]
\[ \omega = \frac{6}{E} \frac{x^2}{H^3} \left( \frac{L-x}{3} \right)^2 \]

at the tip location \( x=L \) \( \Rightarrow \omega = 0 \)

\[ \frac{\sigma_{ij}}{E} = \frac{6}{E} \frac{F}{WH^3} \left( \frac{L-L}{3} \right)^2 \]

\[ F = \frac{\sigma_{ij}}{E} \frac{EWH^3}{6} \frac{3}{2L} = \frac{3\sigma_{ij}WH^2}{12L} = \frac{\sigma_{ij}WH^2}{4L} \]

Substitute this in \( \omega(x) \) equation:

\[ \omega(x) = \frac{6}{E} \frac{x^2}{H^3} - \frac{6}{E} \frac{\sigma_{ij}WH^2}{4L} \left( \frac{L-x}{3} \right)^2 \]

\[ = \frac{6}{E} \frac{x^2}{H} - \frac{3\sigma_{ij}x^2}{2EHL} \left( \frac{L-x}{3} \right) \]

not perfectly flat!

(d) As in previous parts:

\[ \sigma_{ij}(x) = \begin{cases} 
-\sigma_0 + \frac{2\sigma_1}{H} x & \text{if } -0.50H < x < 0.25H \\
-\sigma_0 & \text{if } 0.25H < x < 0.30H \\
-\sigma_0 + \frac{2\sigma_1}{H} x & \text{if } 0.30H < x < 0.50H 
\end{cases} \]

\[ dM = \sigma_{ij}(x) W_z dx \]

where \( \sigma_{ij}(x) = \begin{cases} 
-\sigma_0 + \frac{2\sigma_1}{H} x & \text{if } -0.50H < x < 0.25H \\
-\sigma_0 & \text{if } 0.25H < x < 0.30H \\
-\sigma_0 + \frac{2\sigma_1}{H} x & \text{if } 0.30H < x < 0.50H 
\end{cases} \]

\[ M = \int \left( -\sigma_0 + \frac{2\sigma_1}{H} x \right) W_z dx + \int \left( -\sigma_0 \right) W_z dx + \int \left( -\sigma_0 + \frac{2\sigma_1}{H} x \right) W_z dx \]

\[ = -\sigma_0 W \left[ \frac{x^2}{2} \right]_{-0.50H}^{0.25H} + \frac{2\sigma_1 W}{H} \left[ \frac{x^3}{3} \right]_{-0.50H}^{0.25H} - \sigma_0 W \left[ \frac{x^2}{2} \right]_{-0.50H}^{0.30H} - \sigma_0 W \left[ \frac{x^2}{2} \right]_{0.25H}^{0.30H} + \frac{2\sigma_1 W}{H} \left[ \frac{x^3}{3} \right]_{0.25H}^{0.30H} \]
\[ M = \frac{5}{2} WH^2 \left\{ \frac{0.25 - 0.50^2}{2} - \frac{0.50^2 - 0.30^2}{2} \right\} + 6,1 WH^2 \left\{ \frac{2}{3} (0.25^3 + 0.50^3) + \frac{2}{3} (0.50^3 - 0.30^3) \right\} - 6,1 WH^2 \left\{ \frac{0.30^2 - 0.25^2}{2} \right\} \]

\[ = 0.01375 \frac{5}{2} WH^2 + 0.159 \frac{6,1}{6} WH^2 - 0.01375 \frac{5}{2} WH^2 \]

\[ = 0.01375 (\sigma_o - \sigma_i) WH^2 + 0.159 \frac{6,1}{6} WH^2 \]

The deflection equation:

\[ EI \frac{d^4w}{dx^4} = M \Rightarrow \omega(x) = \frac{M}{2EI} x^2 \]

Substitute \( M \) and \( I \) in \( \omega(x) \) expression:

\[ \omega(x) = \frac{0.01375 (\sigma_o - \sigma_i) WH^2 + 0.159 \frac{6,1}{6} WH^2}{\frac{1}{6} EWH^3} \cdot x^2 = \frac{0.0825 (\sigma_o - \sigma_i) + 0.95461}{EH} \cdot x^2 \]

Tip deflection \( w_{tip} \) is at \( x = L \):

\[ w_{tip} = \frac{0.0825 (\sigma_o - \sigma_i) + 0.95461}{EH} \cdot \frac{L^2}{1} \]

Equate \( w_{tip} = 0 \):

\[ 0.0825 (\sigma_o - \sigma_i) + 0.95461 = 0 \]

\[ \Rightarrow \sigma_o - \sigma_i = 11.569 \]

Yes, it is perfectly flat now.

(e) First, let's examine an infinitesimally small two-layer structure at an elevated temp:

\[ \sigma_c = \frac{F_c}{A_c} \]

\[ = \frac{E_c (\alpha_c - \alpha_p) \Delta T}{L} \]

\[ F = \frac{E_c L (\alpha_c - \alpha_p) \Delta T}{L} \]

\[ \sigma = \frac{F}{A} = \frac{E_c (\alpha_c - \alpha_p) \Delta T}{A} \]

Compressive stress due to temp. mismatch.
New stress gradient:

\[
\sigma_x(z) = \begin{cases} 
-\sigma_0 + \frac{25}{H} z, & \text{if } -H/2 \leq z \leq H/2 \\
-E_c(\alpha_c - \alpha_p) \Delta T, & \text{if } H/2 < z \leq H/2 + H_c
\end{cases}
\]

The total moment:

\[
M = \int_{-H/2}^{H/2} \left[ -\sigma_0 + \frac{25}{H} z \right] W_d z \, dz + \int_{H/2}^{H/2 + H_c} -E_c(\alpha_c - \alpha_p) \Delta T W_d z \, dz
\]

\[
= -\sigma_0 W \left[ \frac{z^2}{2} \right]_{-H/2}^{H/2} + \frac{25}{H} W \left[ \frac{z^3}{3} \right]_{-H/2}^{H/2} - E_c(\alpha_c - \alpha_p) \Delta T W \left[ \frac{z^2}{2} \right]_{H/2}^{H/2 + H_c}
\]

\[
= \frac{51 WH^2}{6} - E_c(\alpha_c - \alpha_p) \Delta T W \left\{ (H/2 + H_c)^2 - \frac{H^2}{4} \right\}
\]

from previous part:

\[
\omega(x) = \frac{M}{2EI_x} \Rightarrow \text{for } \omega_{tip} = 0 \text{ we need } M = 0
\]

\[
\frac{51 WH^2}{6} = E_c(\alpha_c - \alpha_p) \Delta T W \left\{ (H + H_c)^2 \right\}
\]

\[
\Delta T = \frac{51 H^2}{3E_c(\alpha_c - \alpha_p) H_c (H + H_c)} \Rightarrow \Delta T = T - T_0
\]

\[
T = T_0 + \frac{51 H^2}{3E_c(\alpha_c - \alpha_p) H_c (H + H_c)}
\]

Yes, the cantilever is perfectly flat.
(a) An approximate solution is as follows:

![Diagram](image)

\[ t_{etch} = \frac{10 \text{ mm}}{3 \text{ mm/min}} = 3.33 \text{ min} \]  
but this does not fully answer this question.

A more exact solution is as follows:

\[ (20 - d)^2 + 5^2 = d^2 \]
\[ 400 - 40d + d^2 + 25 = d^2 \]  \( \Rightarrow \)  \( d = 10.625 \text{ mm} \)

\[ t_{etch} = \frac{10.625 \text{ mm}}{3 \text{ mm/min}} = 3.54 \text{ min} \]  
still close to the approximate solution.

at \( t = 3.54 \text{ min} \)

\[ t_{etch2} = t_{etch} + \frac{d_2}{ER} \]

\[ = 3.54 \text{ min} + \frac{2 \sqrt{2}}{3 \text{ mm/min}} \]

\[ = 3.816 \text{ min} \]

\[ t_{etch3} = \frac{50 \text{ mm}}{3 \text{ mm/min}} = 16.67 \text{ min} \]
(b) \[ R_{cr} = h \left( \frac{2.575}{60} \right)^{1/2} = 2\mu m \left( \frac{2.575}{0.0025} \right)^{1/2} = 64.2\mu m \]

(ii) The beams will buckle OUT OF PLANE because the width, \( W = 10\mu m \) is greater than the thickness, \( H = 2\mu m \).

(c) This device will function in a similar fashion with \( L_1 = 71\mu m \), \( L_2 = 33\mu m \).

These values will detect a strain of 0.25%.

To determine the total stiffness, it's easiest to break the structure into smaller pieces and combine them in series or parallel.

Each beam consists of two cantilevers in series. Thus, for each beam:

\[ k_b = k_c \frac{k_c}{k} \]

4 beams in series and there are 2 of these in parallel.
As indicated in the figure, there are two groups of series connected beams in parallel. For each series connected group of 4:

\[ k_{4\text{group}} = \frac{k_b}{4} = \frac{k_c}{4} \]

Thus, the total x-direction stiffness contribution from the bottom part:

\[ k_{xb} = 2 \cdot k_{4\text{group}} = 2 \left( \frac{k_c}{4} \right) \Rightarrow k_{xb} = \frac{k_c}{2} \Rightarrow k_{xb} = \frac{EH}{L^2} \left( \frac{W}{L} \right)^3 \]

For a cantilever of length 4/2:

\[ k_c = \frac{24EIz}{L^3} = 2EH \left( \frac{W}{L} \right)^3 \]

Plugging in numbers:

\[ k_{xb} = \left( \frac{150G}{2} \right) \left( \frac{2\mu}{100\mu} \right) = 1.2 \text{ N/m} \]

For the top part:

\[ k_{xt} = \left( \frac{k_c}{2} \right) \left( \frac{k_c}{2} \right) = \frac{k_c}{4} \times 2 = \frac{k_c}{2} \]

\[ k_{xt} = \frac{EH}{L^2} \left( \frac{W}{L} \right)^3 = 2.4 \text{ N/m} \]

\[ k_x = k_{xb} + k_{xt} \text{ (they're in parallel)} \]

\[ = 1.2 \text{ N/m} + 2.4 \text{ N/m} \]

\[ = 3.6 \text{ N/m} \]

(b) Since the beam width, W, and thickness, H are the same, i.e. 2\mu m, the z-direction stiffness is the same as x-direction stiffness:

\[ k_z = k_x = 3.6 \text{ N/m} \]