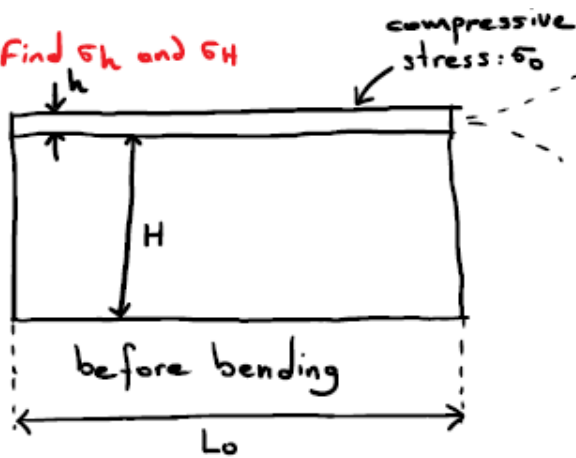


EE247B SPRING 2017 / Homework #3 Problem #1 & #3 Note

--- How to derive Senturia Equation 9.89 and 9.90 ---

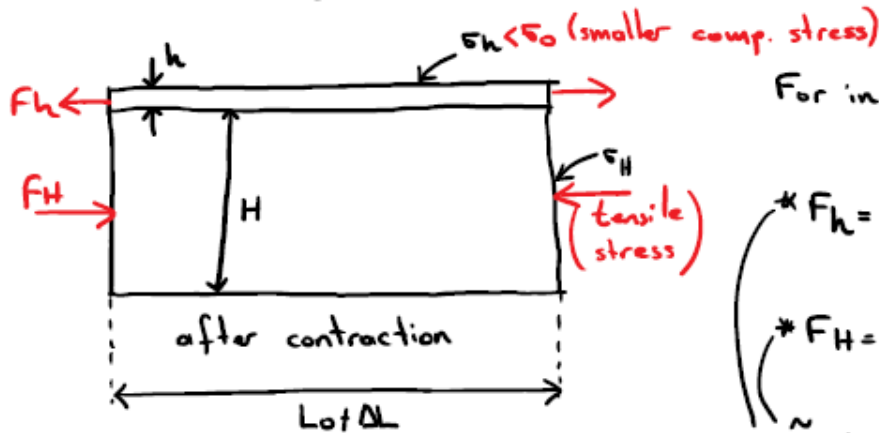
1st Step: Find σ_h and σ_H



thin film w/ no stress
assume the stress is somehow released
(no stress condition)

$$\tilde{E}_h W h \frac{\Delta L_0}{L_0 + \Delta L_0} = \sigma_0 W h \Rightarrow \Delta L_0 = L_0 \frac{\sigma_0}{\tilde{E}_h}$$

neglect ($\Delta L_0 \ll L_0$)



For in-plane stress equilibrium: $F_h = F_H$

$$* F_h = \tilde{E}_h W h \frac{\Delta L_0 - \Delta L}{L_0 + \Delta L_0} \approx \tilde{E}_h W h \frac{L_0 \frac{\sigma_0}{\tilde{E}_h} - \Delta L}{L_0}$$

neglect

$$* F_H = \tilde{E}_H W H \frac{\Delta L}{L_0}$$

$$\tilde{E}_h h \left\{ -\frac{\Delta L}{L_0} + \frac{\sigma_0}{\tilde{E}_h} \right\} = \tilde{E}_H H \left\{ \frac{\Delta L}{L_0} \right\}$$

$$\sigma_0 h = \frac{\Delta L}{L_0} \left\{ \tilde{E}_H H + \tilde{E}_h h \right\} \Rightarrow \frac{\Delta L}{L_0} = \frac{\sigma_0 h}{\tilde{E}_H H + \tilde{E}_h h}$$

Now, substitute $\Delta L/L_0$ in σ_h and σ_H :

$$\sigma_h = \tilde{E}_h \frac{\Delta L_0 - \Delta L}{L_0} = \frac{\sigma_0 \tilde{E}_H H}{\tilde{E}_H H + \tilde{E}_h h} \quad \text{and} \quad \sigma_H = \tilde{E}_H (\Delta L/L_0) = \frac{\sigma_0 \tilde{E}_h h}{\tilde{E}_H H + \tilde{E}_h h}$$