**Lecture 10: Mechanics of Materials I**

- Announcements:
- Module 7 on "Mechanics of Materials" online
- HW#2 online and due Tuesday, 2/28, at 10 a.m.
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- Reading: Senturia Chpt. 3, Jaeger Chpt. 11, Handouts: "Bulk Micromachining of Silicon"
- Lecture Topics:
  - Bulk Micromachining
  - Anisotropic Etching of Silicon
  - Boron-Doped Etch Stop
  - Electrochemical Etch Stop
  - Isotropic Etching of Silicon
  - Deep Reactive Ion Etching (DRIE)
  - Wafer Bonding
- ----------------------------------
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
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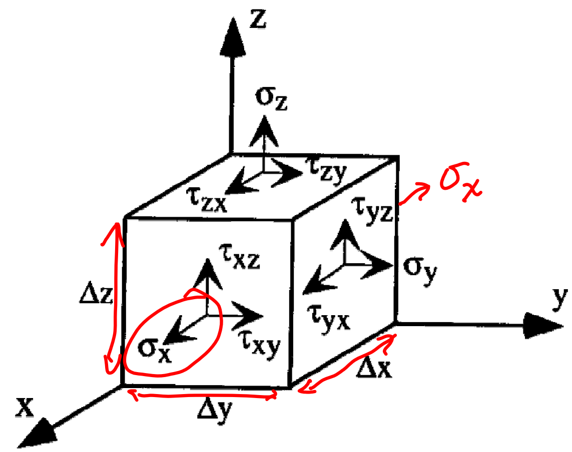
- Last Time: Going thru Module 6 … finish this
- Move on to Module 7

Example. Exercise the "terms"

⇒ Determine the volume change $\Delta V$ resulting from a uniaxial stress $\sigma_x$ (along the x-direction)




Upon application of $\sigma_x$, what is $\Delta V$?

$$\Delta x \rightarrow \Delta x(1+\epsilon_x)$$
$$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$$
$$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$$

} assuming isotropic material → same $\nu$ along y & z

The resulting change in volume: $\Delta V$

$$\Delta V = \underbrace{\Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2}_{\text{volume after application of } \sigma_x} - \Delta x \Delta y \Delta z$$

$$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains] $\Rightarrow (1+mx)^n \approx 1+nmx$

$$\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$$

$$\boxed{\Delta V = \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

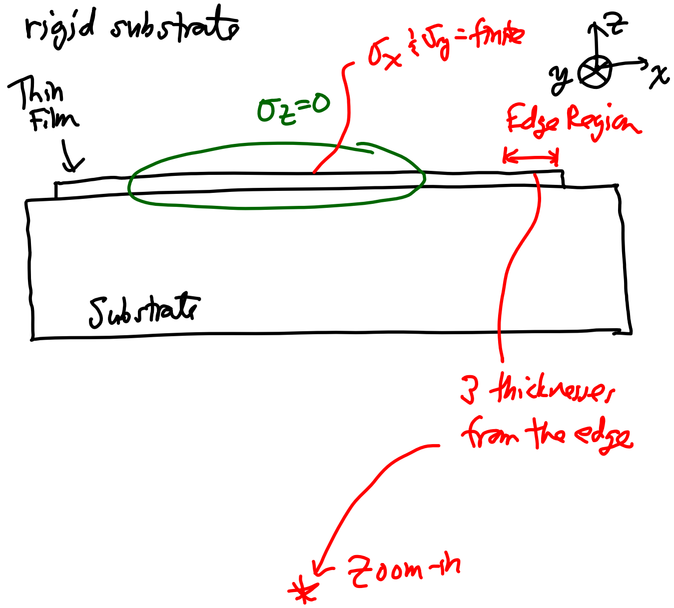
For $\nu = 0.5$ (rubber) $\rightarrow$ no $\Delta V$!

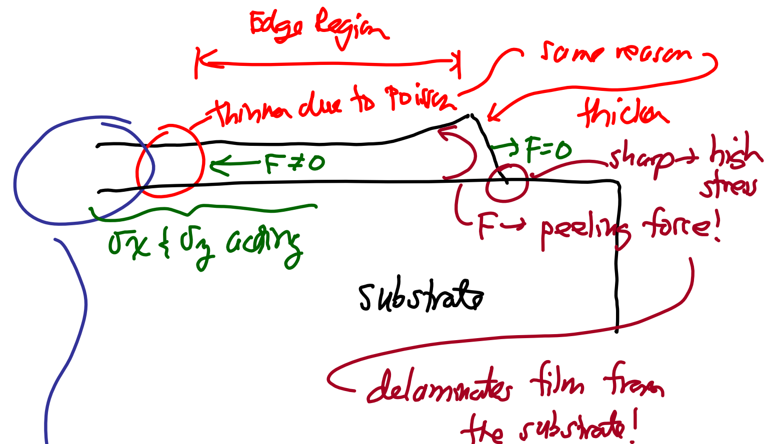
$\nu < 0.5 \rightarrow$ finite $\Delta V$

For isotropic materials $\rightarrow$ Module 7, pg. 13

**Important Case: Plane Stress**

$\Rightarrow$ common case for a thin-film coating on a rigid substrate




Take a closer look @ this region: $\sigma_z = 0$

Get two components of stress:

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + 0)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + 0)\right]$$

Assume: Plane stress $\rightarrow$ isotropic $\rightarrow \sigma_x = \sigma_y = \sigma$
(symmetry in the xy-plane) $\qquad \downarrow$ $\epsilon_x = \epsilon_y = \epsilon$

$$\epsilon_x = \frac{1}{E}\left[\sigma - \nu\sigma\right] = \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} \rightarrow \epsilon_x = \frac{\sigma}{E'}$$

where $\boxed{E' = \frac{E}{1-\nu} \triangleq \text{Biaxial Modulus}}$

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition. linear thermal expansion coefficient

$$\boxed{\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Exp. Coefficient}\end{array}\right\} \triangleq \alpha_T = \frac{d\varepsilon_x}{dT} \quad [\text{Kelvin}^{-1}]}$$

Remarks.

① $\alpha_T$ values tend to be in the $10^{-6}$ to $10^{-7}$ range

② $10^{-6}K^{-1} = 1\mu$strain/K

③ In 3D, get volume thermal exp. coefficient:

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

④ For moderate $\Delta T$'s → $\alpha_T \approx$ constant

for larger $\Delta T$, then $\alpha_T = f(T)$