Lecture 11: Mechanics of Materials II

- **Announcements**:
- HW#2 online and due Tuesday, 2/28, at 10 a.m.
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- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
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- **Last Time**:
- Defined the linear thermal expansion coefficient
- Now, continue with this

over

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition: linear thermal expansion coefficient

$$\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Exp. Coefficient}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{Kelvin}^{-1}]$$

Remarks:

① α_T values tend to be in the 10^{-6} to 10^{-7} range

② $10^{-6} K^{-1} = 1\mu\text{strain}/K$

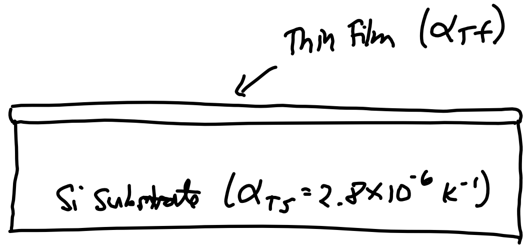
③ In 3D, get volume thermal exp. coefficient:

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

④ For moderate ΔT's → $\alpha_T \approx$ constant

↘ for larger ΔT, then $\alpha_T = f(T)$

Ex. Thin-Film Thermal Stress



Assume:

1. Substrate is much thicker than the film.
2. Film is deposited stress free @ T_d ← deposition temperature
3. The whole thing after deposition is cooled to room temperature, T_r.

Thermal Strain of the Substrate: (in one plane dimension)

$\epsilon_s = -\alpha_{Ts}\Delta T$, where $\Delta T = T_d - T_r$

If the film were not attached to the substrate:

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But the film is attached to the substrate

→ thickness of substrate >> thickness of film
∴ substrate wins!

→ therefore the actual strain experienced by the film is that of the substrate:

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

Thermal Mismatch Strain: $\epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T$

→ Note this is biaxial strain (assuming the film deposits isotropically onto the substrate)

$$\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{E'} \epsilon_{f,mismatch}$$

Ex. Thin film is polyimide → $\alpha_{Tf} = 70\times10^{-6}K^{-1}$, $E' = 4GPa$

deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225K$

$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$

[$\mu = 10^{-6}$, $m = 10^{-3}$, $k = 10^3$, $G = 10^9$]

$\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = 60.5\ MPa$

($G = 10^9$)

stress is (+) → tensile

[(−) would be compressive] ← SiO_2