Lecture 14: Beam Combos I

- **Announcements:**
- HW#3 online, due Thursday, next week, 10 a.m.
  - shorter time span than before
- Midterm Exam less than 3 weeks away, Tuesday, March 21, 3:30-5 p.m., 521 Cory (right here)
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- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
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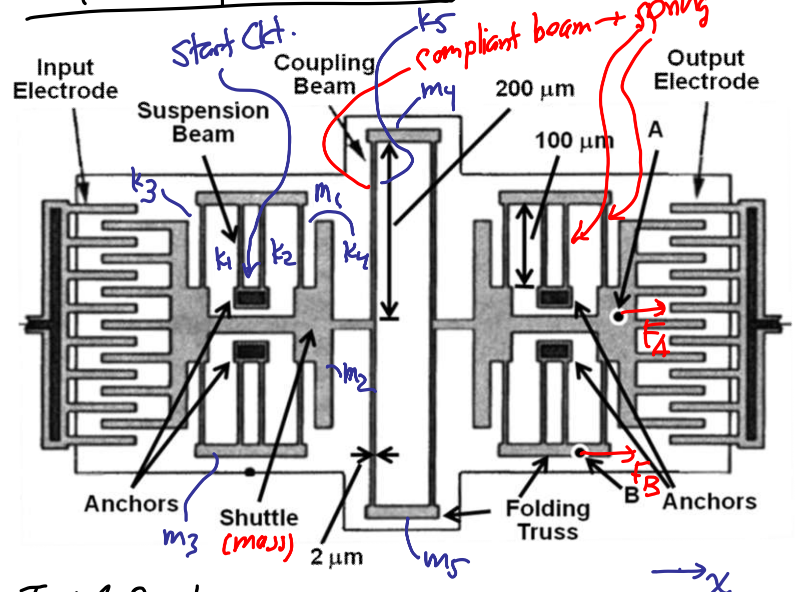
- **Last Time:**
- Finished beam bending due to stress gradient
- Started beam combos
- Continue with this

over

The Problem: How to analyze an interconnected ensemble of beams (springs) & masses



Typical Questions:

① How does the structure move in response to a force at a specific location.

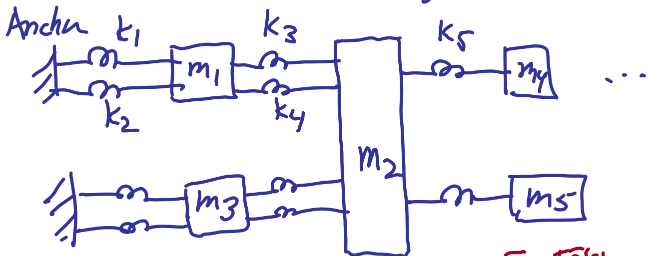
② What is the frequency response to an AC force applied at a specific location.

③ Noise?

④ Response to environmental stimuli? (e.g., rotation)

Procedure:

① Build the ckt: (Extract the ckt.) (in the x-direction)



② Analyze to get $x = f(F)$
displacement, x ← ↑ → force, F

For EE's: C_1 C_2 0V

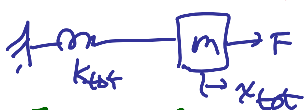
(a) Case 1: Series

$F = kx \rightarrow x = \frac{F}{k} = \left(\frac{1}{k}\right)F$

thru variable (I)

series → must go through both k_1 & k_2 to get from the anchor to the forcing pt.

x_{tot} ← want this → so need across variable (V) k_{tot}



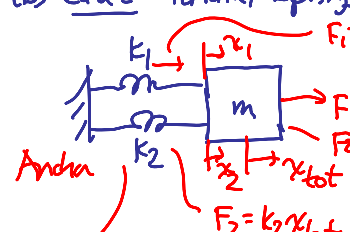
$\boxed{k_{tot} = k_1 \| k_2}$

$x_1 = \frac{F}{k_1}$ $\quad x_2 = \frac{F}{k_2}$

$x_{tot} = x_1 + x_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{F}{(k_1 \| k_2)} = \frac{F}{k_{tot}}$

["||" operator: $A \| B = \frac{1}{\frac{1}{A} + \frac{1}{B}} = \frac{AB}{A+B}$]

(b) Case 2: Parallel Springs



$F = k_{tot} x_{tot}$
↑
want k_{tot}.

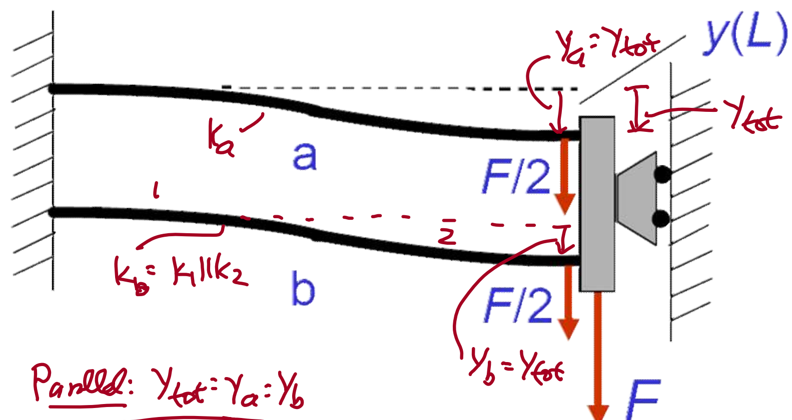
$F = F_1 + F_2 = \underbrace{(k_1 + k_2)}_{k_{tot}} x_{tot} \rightarrow \boxed{k_{tot} = k_1 + k_2}$

in parallel when I need go through only one of the springs to get from the anchor to the forcing pt.

⇒ get the k_{tot} @ the forcing pt. → use this to predict structure behavior

Parallel Combination of Beams

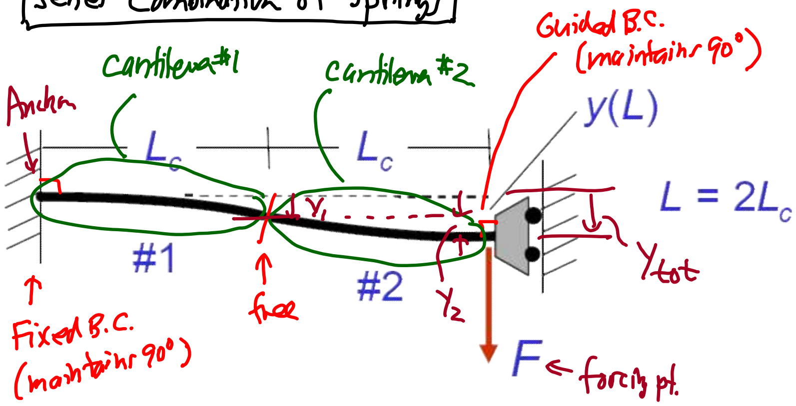
For EE's: $V \rightarrow x$, $I \rightarrow F$



Parallel: $y_{tot} = y_a = y_b$

$k_{tot} = k_a + k_b$

Series Combination of Springs

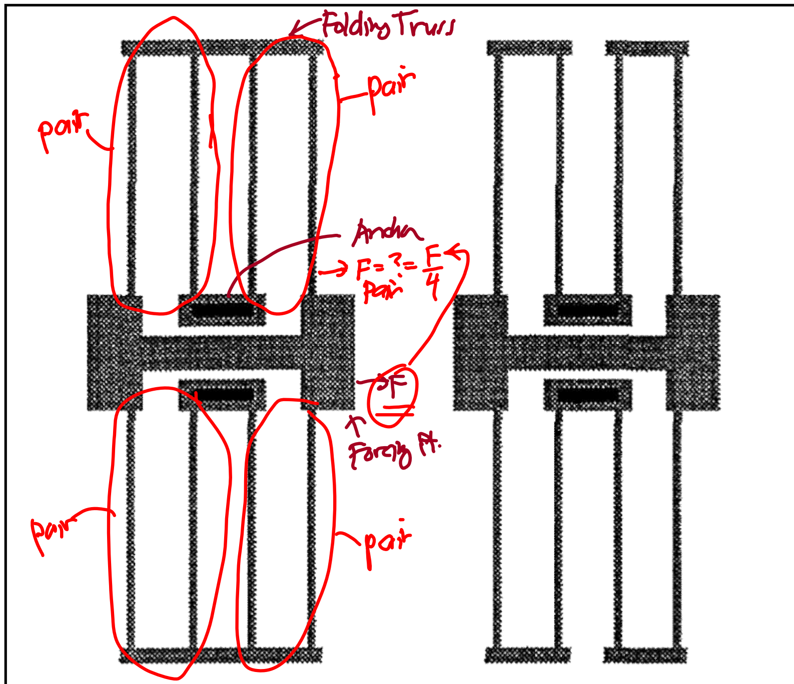


Series: $y_{tot} = y_1 + y_2$ (from anchor pt. to forcing pt.)

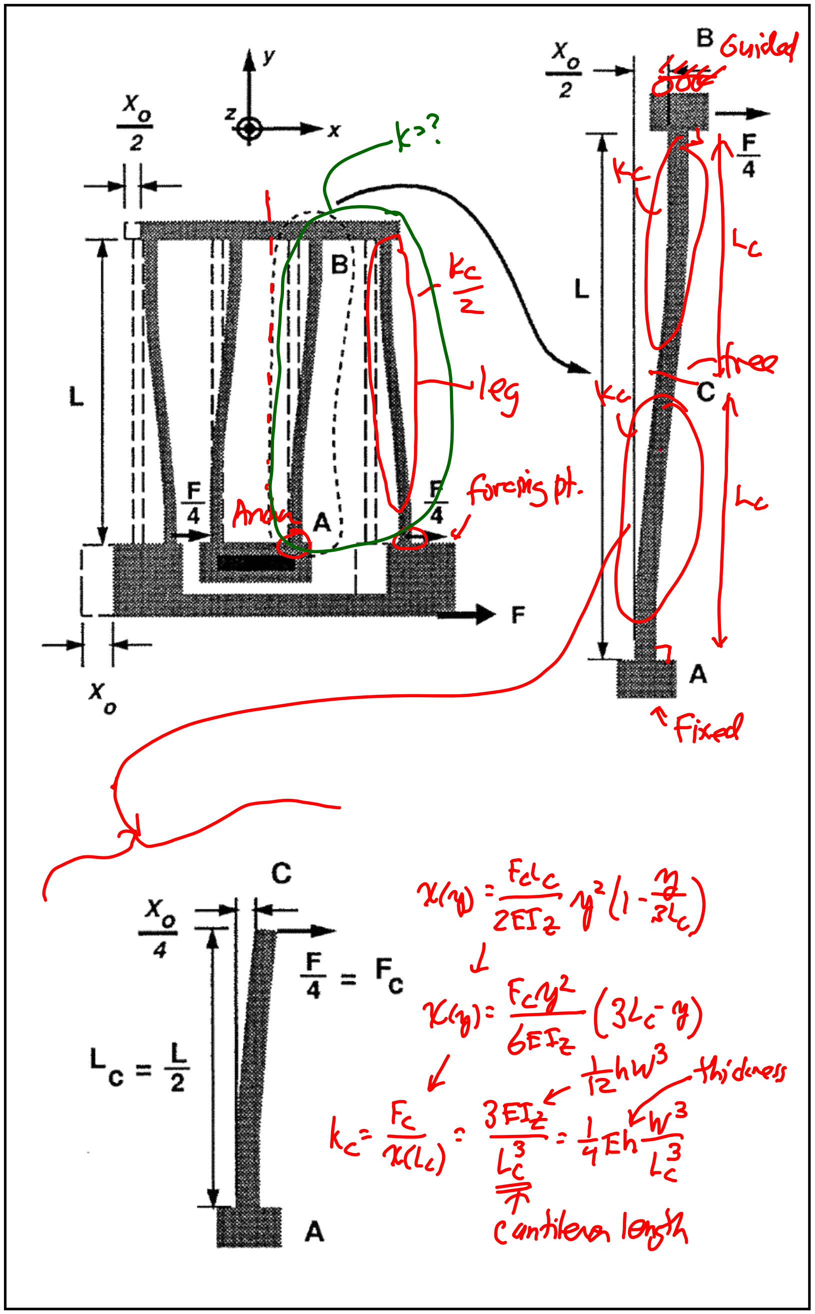
or $y_{tot} > y_1$ & $y_{tot} > y_2$

$$k_{tot} = k_1 \| k_2 = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \frac{k_1 k_2}{k_1 + k_2}$$

$\{k_1 = k_2 = k_c\} \rightarrow k_{tot} = \frac{k_c}{2}$



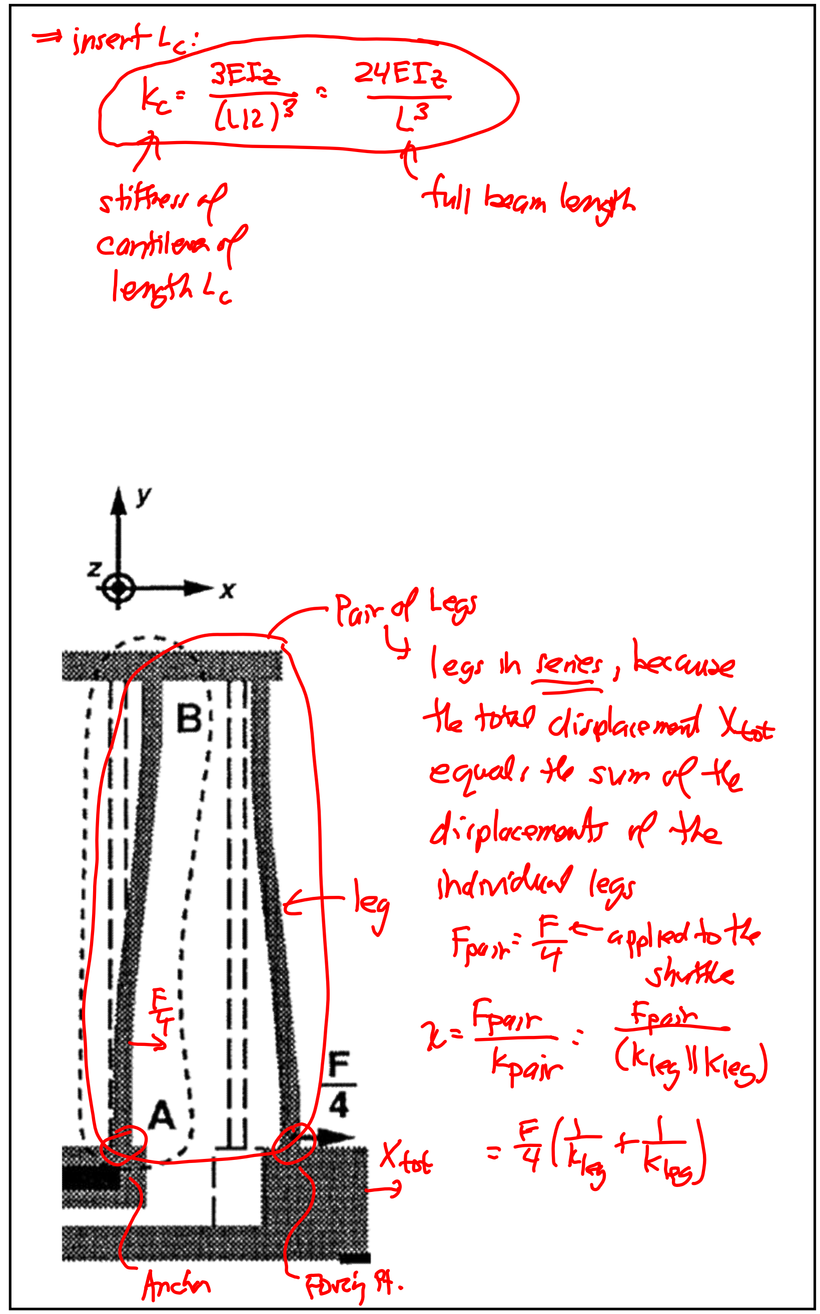
(a) Inner fold, continuous truss (b) Inner fold, discontinuous truss



$$x(y) = \frac{F_c L_c}{2EI_z} y^2 \left(1 - \frac{y}{3L_c}\right)$$

$$x(y) = \frac{F_c y^2}{6EI_z}\left(3L_c - y\right)$$

$$k_c = \frac{F_c}{x(L_c)} = \frac{3EI_z}{L_c^3} = \frac{1}{4}Eh\frac{w^3}{L_c^3}, \quad I_z = \frac{1}{12}hw^3$$



⇒ insert L_c:

$$k_c = \frac{3EI_z}{(L/2)^3} = \frac{24EI_z}{L^3}$$

stiffness of cantilever of length L_c

full beam length

legs in series, because the total displacement X_{tot} equals the sum of the displacements of the individual legs

$F_{pair} = \frac{F}{4}$ ← applied to the shuttle

$$x = \frac{F_{pair}}{k_{pair}} = \frac{F_{pair}}{(k_{leg} \parallel k_{leg})} = \frac{F}{4}\left(\frac{1}{k_{leg}} + \frac{1}{k_{leg}}\right)$$

From before: $k_{leg} = k_c \| k_c = \frac{k_c}{2}$

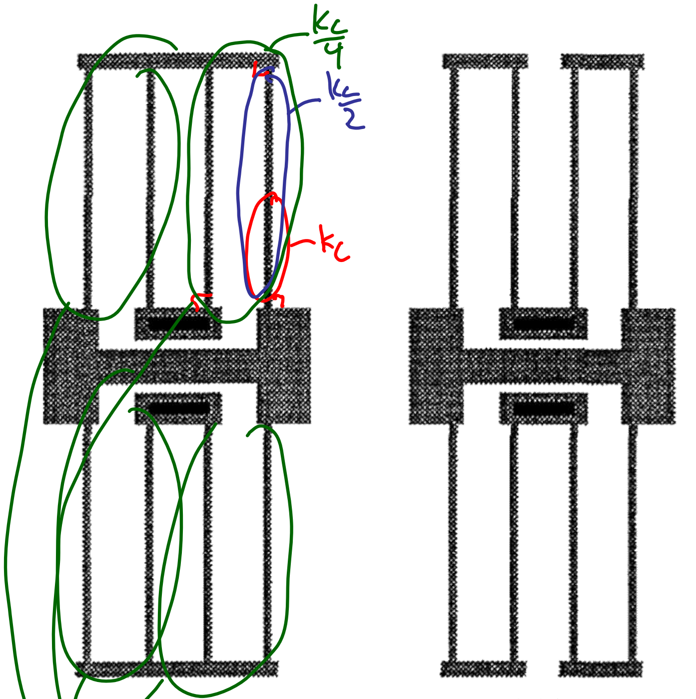
Thus:

$$x = \left(\frac{F}{4}\right)\left(\frac{2}{k_c}+\frac{2}{k_c}\right) = \frac{F}{k_c} = \frac{F}{k_{tot}}$$

$$\boxed{k_{tot} = k_c = \frac{24EI_z}{L^3}}$$

Better Way to Do It → By Inspection

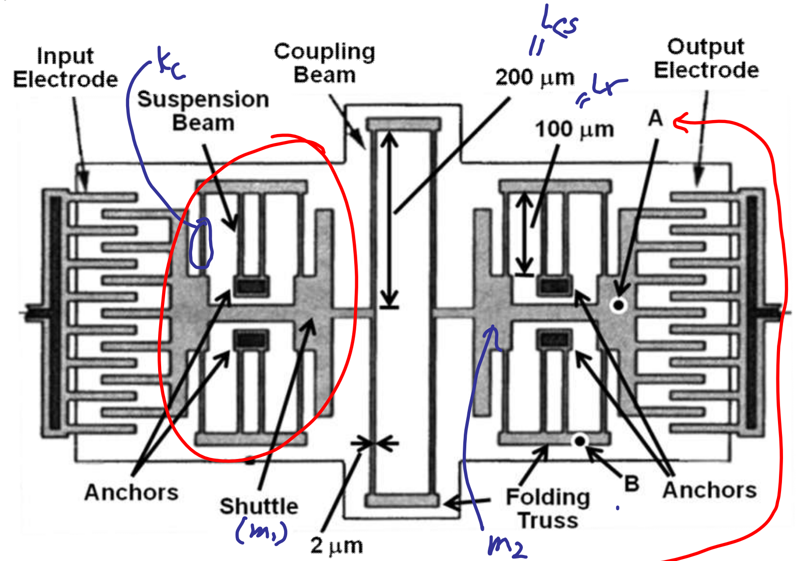
⇒ just consider stiffness



(a) Inner fold, continuous truss (b) Inner fold, discontinuous truss

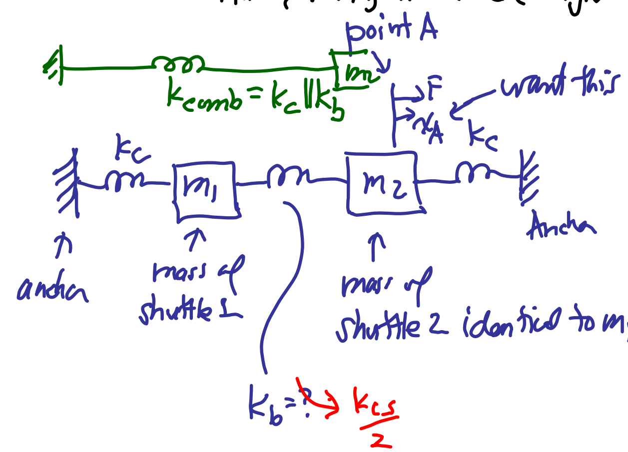
Parallel: $k_{tot} = 4\left(\frac{k_c}{4}\right) = k_c$ ✓

Micromechanical Filter

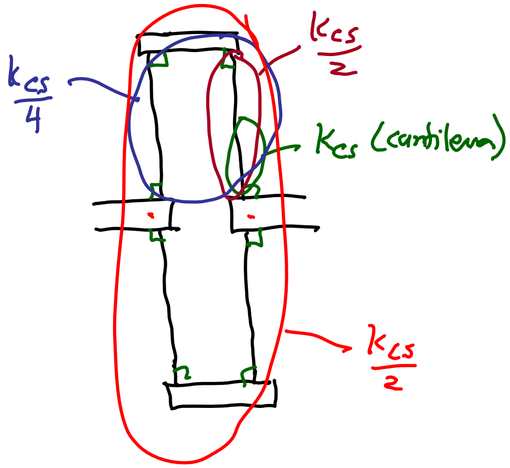


⇒ Find the stiffness at point A.

Assume shuttle & folding trusses are rigid.



Get k_L:



$\therefore k_A = k_c + k_{comb}$

$$k_A = k_c + k_c \| \frac{k_{cs}}{2} \quad \text{where} \quad k_c = \frac{24EI_z}{L_r^3}$$

$$k_{cs} = \frac{24EI_z}{L_{cs}^3}$$