Lecture 21: Electrostatic Comb Drive

- Announcements:
- Module 12 on Capacitive Transducers online
- HW#5 online and due Thursday, April 13
- Project Slide Set #1 due Friday, April 14
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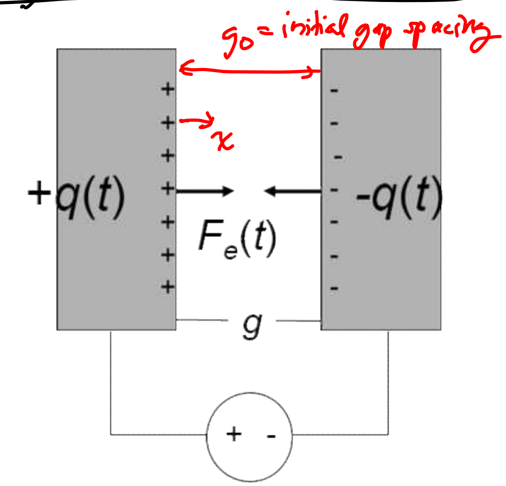
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- -----------------------------------
- Last Time:
- In the midst of deriving electrical stiffness

over

Disadvantages of Electrostatic Actuators:

- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale

$\boxed{\text{Linearizing the Voltage-to-Force Transfer Fcn}}$



$$v(t) = \underbrace{V_P}_{\text{DC-Bias}} + \underbrace{v_i(t)}_{\text{signal (AC)}}$$

$$F_e(t) = \frac{\partial W'}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{2}C[v(t)]^2\right]$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}[v(t)]^2 = \frac{1}{2}\frac{\partial C}{\partial x}[V_P + v_i(t)]^2$$

$$= \frac{1}{2}\left[V_P^2 + 2V_P v_i(t) + \cancel{[v_i(t)]^2}\right]\frac{\partial C}{\partial x}$$

$$[V_P \gg v_i(t)] \Rightarrow \underbrace{\frac{1}{2}V_P^2\frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_P\frac{\partial C}{\partial x}v_i(t)}_{\text{AC Drive Signal}}$$

$$C_o = \frac{\epsilon A}{g_o} \rightarrow C(x) = \frac{\epsilon A}{g_o - x} = C_o\left(1-\frac{x}{g_o}\right)^{-1}$$

binomial theorem

$$[x \ll g_o] \Rightarrow \approx C_o\left(1+\frac{x}{g_o}\right)$$

$$\therefore \frac{\partial C}{\partial x} = \frac{C_o}{g_o} = \frac{\epsilon A}{g_o^2}$$

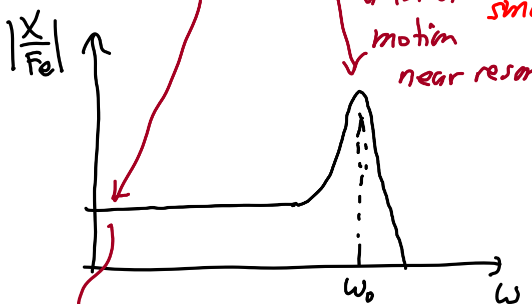
$$\Rightarrow F_e(t) = \underbrace{\frac{1}{2}\frac{C_o}{g_o}V_P^2}_{\text{DC offset}} + V_P\frac{C_o}{g_o}v_i(t)$$

~constant for small amplitudes

∴ this is a linear dependence!

↓ only holds for small amplitudes

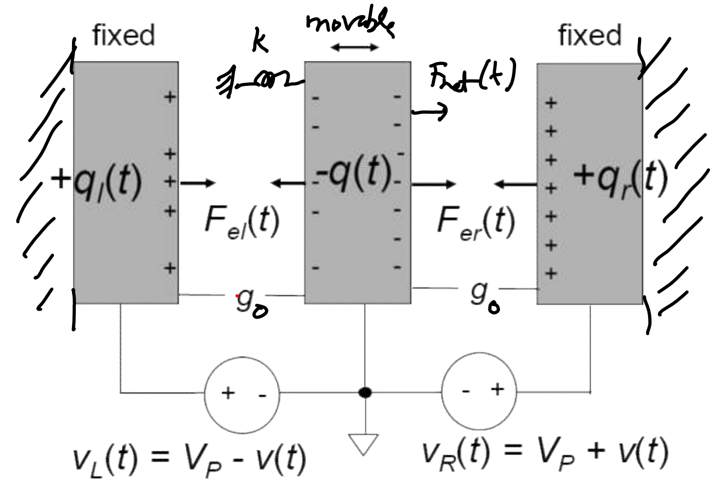
$v_i \ll V_P$ can still instigate a lot of motion near resonance!



very small response

↳ But still must worry about pull-in, V_{PI}!

Can Cancel the DC Offset Via Differential Symmetry



$$F_{net}(t) = F_{er}(t) - F_{el}(t)$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{[v_R(t)]^2 - [v_L(t)]^2\right\}$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{\cancel{V_P^2} + 2V_P v(t) + \cancel{[v(t)]^2} - \left(\cancel{V_P^2} - 2V_P v(t) + \cancel{[v(t)]^2}\right)\right\}$$

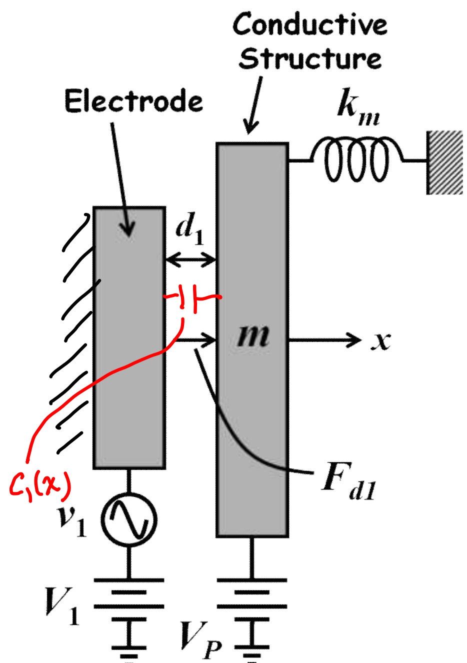
$$\therefore \boxed{F_{net}(t) = 2V_P\frac{\partial C}{\partial x}v(t) = 2V_P\frac{C_o}{g_o}v(t)}$$

↳ quite linear w/ $v(t)$!

V_P^2 force term goes away as long as the electrodes on either side are matched → how good is matching? 10%? (still must worry about V_P!

still an approximation for $\frac{\partial C}{\partial x}$!

Nonlinearity Still Effects Us (even though we're using small signals)



More Complete Expressions

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_{01}\left(1 + \frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1 + \frac{x}{d_1}\right)^{-2}$$

[Expand into Taylor Series]

$$\frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1 + \underline{A_1 x} + A_2 x^2 + A_3 x^3 + \cdots\right)$$

where $A_1 = -\frac{2}{d_1}$, $A_2 = \frac{3}{d_1^2}$, $A_3 = -\frac{4}{d_1^3}$, ...

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(\underbrace{V_P - V_1}_{V_{P1} = V_P - V_1} - v_1)^2 = \frac{1}{2}\frac{\partial C}{\partial x}(V_{P1} - v_1)^2$$

[small displacements: $x \ll d_1$]

$$F_{d1} = \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)(1 + A_1 x)(V_{P1}^2 + 2V_{P1}v_1 + v_1^2)$$

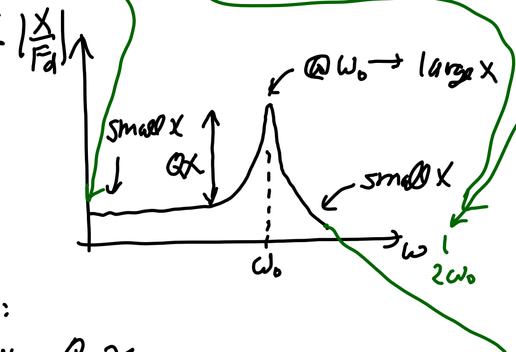
$v_1 = |v_1| \cos \omega_o t$

$\cos^2 \omega_o t = \frac{1}{2}(1 + \cos 2\omega_o t)$

$$= \frac{1}{2}\left(-\frac{C_0}{d_0}\right)\left\{V_{P1}^2 + 2V_{P1}v_1 + v_1^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x v_1 + A_1 x v_1^2\right\}$$

$\rightarrow \omega_o$

Resonance: $\left|\frac{X}{F_d}\right|$



@ resonance:

$$x = \frac{Q F_{d1}}{jk} = \frac{Q}{jk}\underbrace{\frac{\partial C}{\partial x} V_{P1} v_1}_{\text{Force}}$$

↑ 90° phase shift

$$v_1 = |v_1| \cos \omega_o t \longrightarrow x = |x| \sin \omega_o t$$

↑ 90° phase-shifted

Force terms @ ω_0

$$F_{d1}\Big|_{\omega_0} = \underbrace{V_{P1}\frac{C_{01}}{d_1}|v_i|\cos\omega_0 t}_{\text{drive force term}} + \overbrace{V_{P1}^2\frac{C_{01}}{d_1^2}}^{k_e \rightarrow \text{ electrical stiffness}}|x|\sin\omega_0 t$$

proportional to x

90° phase-shifted f_1

∴ in phase w/ displacement!

∴ it's a stiffness!

Electrical Stiffness:

① A negative spring constant!

② Derives from V_P:

$$\boxed{k_e = V_{P1}^2\frac{C_{01}}{d_1^2} = V_{P1}^2\frac{\epsilon A}{d_1^3}}$$

overlap area of C

DC Bias

3rd power dependence on gap!

$k_e \rightarrow$ can affect resonance freq., f_0!

$\omega_0 \triangleq$ radian resonance freq. w/ no V_P applied (i.e., $V_P = 0V$) $\rightarrow$ $\omega_0 = \sqrt{\frac{k_m}{m}}$ ← mechanical stiffness

↓ apply V_P

$$\omega_0' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$

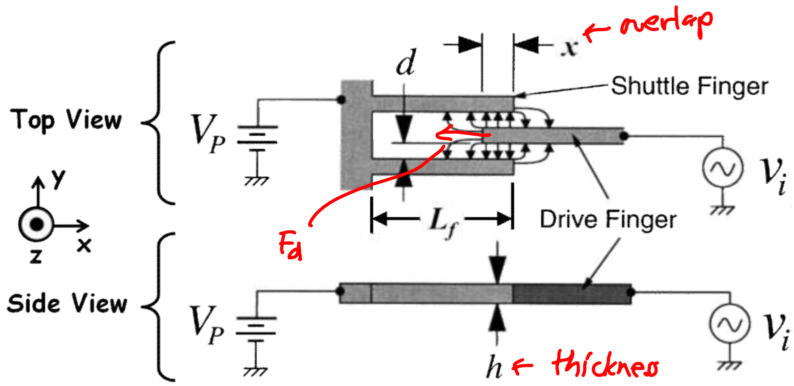
$$= \sqrt{\frac{k_m}{m}}\left(1 - \frac{k_e}{k_m}\right)^{1/2}$$

$$\boxed{\omega_0' = \omega_0\left[1 - \frac{V_{P1}^2}{k_m}\frac{\epsilon A}{d_1^3}\right]^{1/2}}$$

now a fcn of dc-bias voltage! (voltage-controllable!)

- Go through Module 12 slides 26-35

Electrostatic Comb-Drive



$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2$$ Need C(x).

$$C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d}$$ → not a fcn of x! (ideally)

$$F_d = \frac{1}{\cancel{2}}\frac{\cancel{2}\epsilon_0 h}{d}\left(\cancel{V_P^2} - 2V_P v_i + \cancel{v_i^2}\right)$$

can balance out using symmetrically placed electrodes

$v_i << V_P$ (on balance)

$$\boxed{F_d = -2V_P\frac{\epsilon_0 h}{d}v_i}$$

∴ no electrical stiffness! (no k_e!)

- Go through remaining comb-drive slides in Module 12