EE C247B – ME C218
Introduction to MEMS Design
Spring 2017

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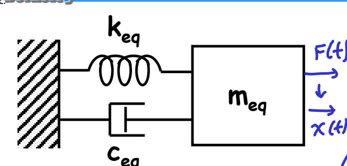
Lecture Module 13: Equivalent Circuits II

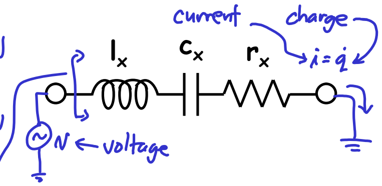
Lecture Outline

- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions

Input Modeling

Electromechanical Analogies





$F(t) = F\cos(\omega t) \rightarrow x(t) = X\cos\omega t$

Equation of Motion:

$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$

⇒ using phasor concepts:

$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$

⇒ by analogy:

$\boxed{F \rightarrow v \quad m_{eq} \rightarrow l_x \quad c_{eq} \rightarrow r_x \quad \dot{x} \rightarrow i \quad k_{eq} \rightarrow \frac{1}{c_x}}$

[Parameter Relationships in the Current Analogy]

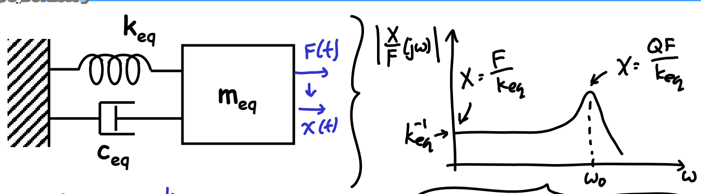
$v(t) = V\cos\omega t \rightarrow i(t) = I\cos\omega t$

Impedance looking in:

$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$

$v = j\omega l_x i + \frac{(1/c_x)}{j\omega} i + r_x i$

Bandpass Biquad Transfer Function



$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$

⇒ Converting to full phasor form:

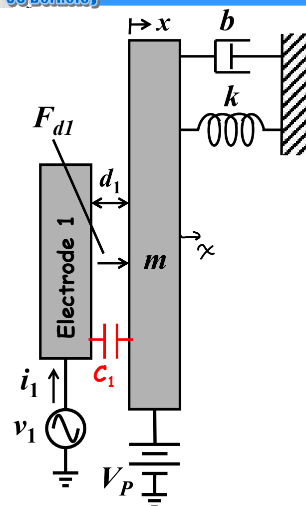
$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega}X) + c_{eq}(j\omega X)$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1-\left(\frac{\omega}{\omega_o}\right)^2 + j\frac{\omega}{Q\omega_o}}}$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}\omega}{k_{eq}}\right]^{-1} = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_o}\right)^2 + 1 + j\frac{\omega}{Q\omega_o}\right]^{-1}$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_o^2,\; Q = \frac{m_{eq}\omega_o}{c_{eq}} = \frac{k_{eq}}{\omega_o c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_o\right]$$

Force-to-Velocity Relationship



- The relationship between input voltage v_1 and force F_{d1}:

$$F_{d1} \approx -V_P \frac{\partial C_1}{\partial x} v_1$$

- When displacement x is the mechanical output variable:

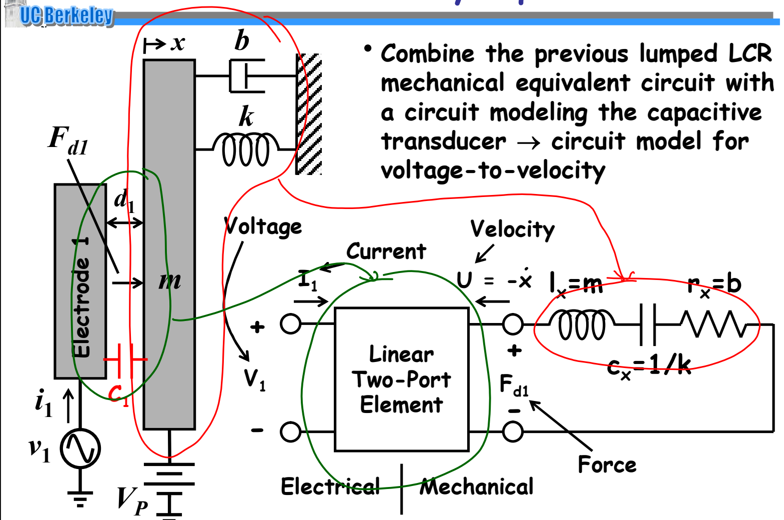
$$\frac{X(s)}{F_{d1}(s)} = \frac{1}{k}\frac{\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

$s = j\omega$

- When velocity υ is the mechanical output variable:

$$\frac{\upsilon(s)}{F_{d1}(s)} = \frac{sX(s)}{F_{d1}(s)} = \frac{1}{k}\frac{\omega_o^2 s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

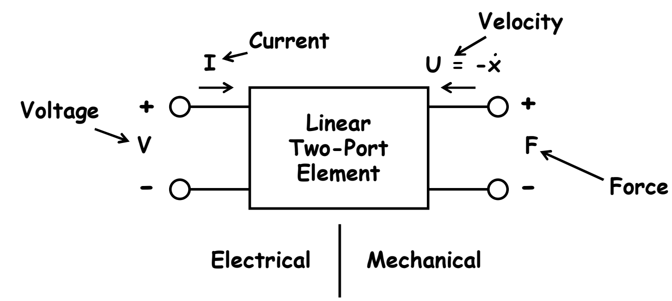
Force-to-Velocity Equiv. Ckt.



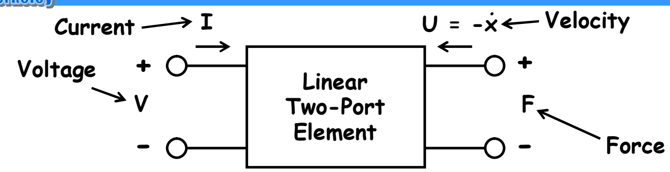
- Combine the previous lumped LCR mechanical equivalent circuit with a circuit modeling the capacitive transducer → circuit model for voltage-to-velocity

Equiv. Circuit for a Linear Transducer

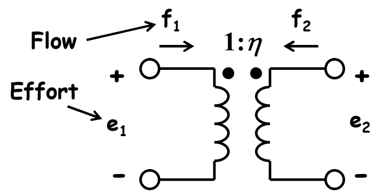
- A transducer …
  - converts energy from one domain (e.g., electrical) to another (e.g., mechanical)
  - has at least two ports
  - is not generally linear, but is virtually linear when operated with small signals (i.e., small displacements)



Equiv. Circuit for a Linear Transducer



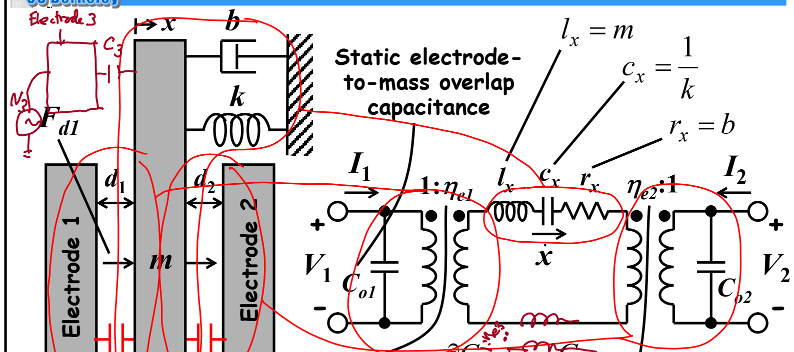
- For physical consistency, use a transformer equivalent circuit to model the energy conversion from the electrical domain to mechanical domain



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

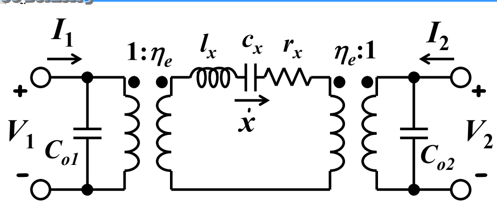
Complete Electrical-Port Equiv. Circuit



$$\eta_{e1} = V_P \frac{\partial C_1}{\partial x} = V_P \frac{C_{o1}}{d_1}$$

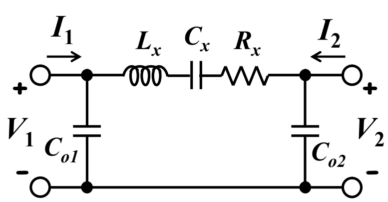
$$\eta_{e2} = V_P \frac{\partial C_2}{\partial x} = V_P \frac{C_{o2}}{d_2}$$

Condensed Equiv. Circuit (Symmetrical)



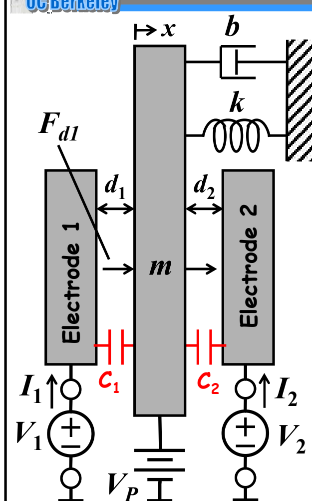
Holds for the symmetrical case, where port 1 and port 2 are identical

If $\eta_{e1} = \eta_{e2}$, then ...



where
$$L_x = \frac{m}{\eta_e^2}, \quad C_x = \frac{\eta_e^2}{k}, \quad R_x = \frac{b}{\eta_e^2}$$

Phasings of Signals



- **Below**: plots of resonance electrical and mechanical signals vs. time, showing the phasings between them