Lecture 22: Equivalent Circuits II

- **Announcements**:
- Module 13 on Equivalent Circuits II online
- HW#5 online and due Thursday, April 13
- Project Slide Set #1 due Friday, April 14
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- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
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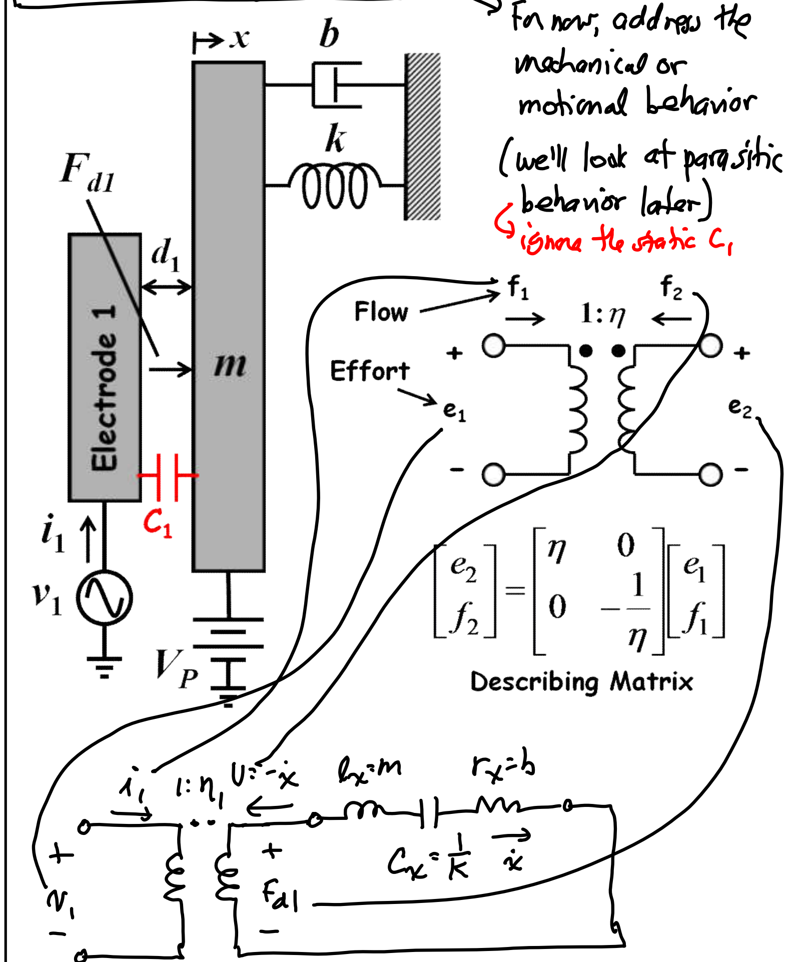
- **Last Time**:
- Finished Capacitive Transducers
- Now, model the transducer in an equivalent circuit

Input Electrical Equivalent Ckt.



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

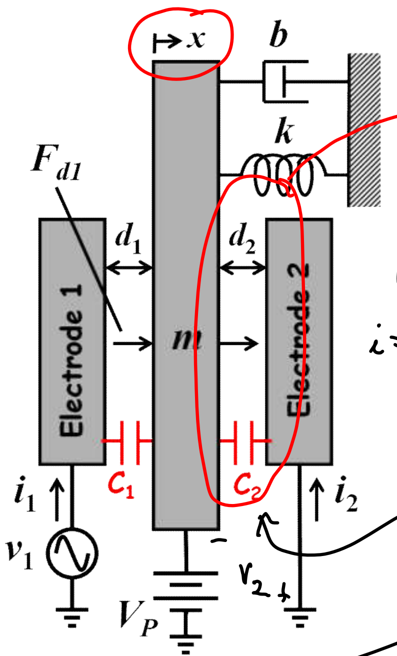
$$\underbrace{F_{d1}}_{e_2} = -\underbrace{V_P \frac{\partial C_1}{\partial x}}_{\eta_1} \underbrace{v_1}_{e_1}$$

$$\boxed{\eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|}$$

$\frac{\partial C_1}{\partial x} \sim \frac{\epsilon_0 A}{d_1^2}$ → gap↓ → η_1↑↑

V_P↑ → η_1↑

Output Current Into Ground



Want to model this!

$[q = CV]$

$$i = \frac{dq}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

time

$C_2 = f(t)$

$$i_2 = \cancel{C_2(x,t)\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$$

$$[V_2(t) = -V_P] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

In phasor form: $I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$

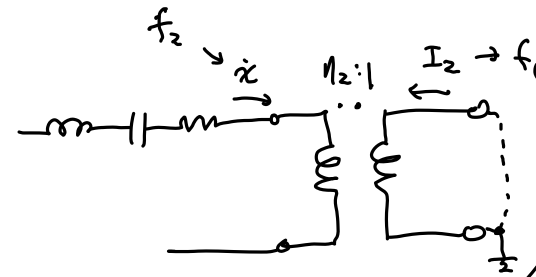
$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

motional current

$$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C_2}{\partial x}\dot{x}$$

90° phase lag (+) (+) velocity

$I_2 = (-)$ when $x = (+)$ ✓



$$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$$

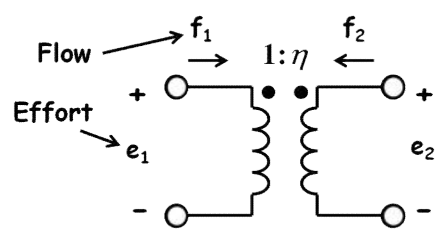
$$[f_1 = I_2,\ f_2 = \dot{x}] \Rightarrow I_2 = -\eta_2\dot{x}$$

$$\therefore\ \boxed{\eta_2 = \left|V_P\frac{\partial C_2}{\partial x}\right|}$$

$\frac{\varepsilon_o A}{d^2}$

DC-bias

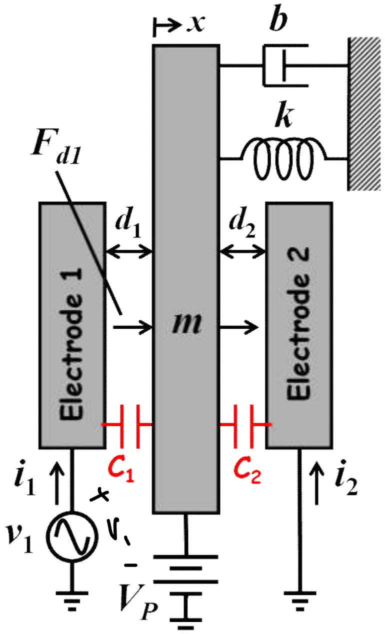
change in C per unit displacement



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

Input Current Expression



Get $I_1(j\omega)$:

$$i_1(t) = C_1(x,t)\frac{dV_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

$$\left(V_1(t) = v_1 - V_P\right) \Rightarrow i_1 = C_1\frac{dv_1}{dt} + (v_1 - V_P)\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

f(t)

$$\therefore I_1(j\omega) = \underbrace{j\omega C_1 v_1}_{\text{Feedthrough Current}} + \cancel{j\omega v_1\frac{\partial C_1}{\partial x}X} \underbrace{- j\omega V_P\frac{\partial C_1}{\partial x}X}_{\text{Motional Current}} \quad *$$

(due to motion of structure)

@ DC: $x = \frac{F_{d1}}{k} = -\frac{1}{k}V_P\frac{\partial C_1}{\partial x}v_1$

@ resonance: $x = \frac{QF_{d1}}{jk} = -\frac{Q}{jk}V_P\frac{\partial C_1}{\partial x}v_1 = X$

90° phase lag

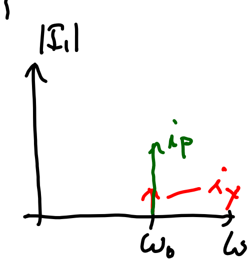
Thus: (@ resonance, ω_o)

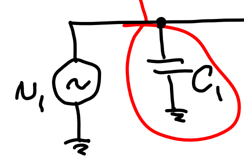
$$* \quad I_1(j\omega) = j\omega_o C_1 v_1 + j\omega_o\left(V_P\frac{\partial C_1}{\partial x}\right)^2\frac{Q}{jk}v_1$$

$$= \underbrace{j\omega_o C_1 v_1}_{i_P} + \underbrace{\omega_o\frac{Q}{k}\eta_{e1}^2 v_1}_{i_x}$$

i_P: 90° phase-shifted from v_1 → This is a capacitor in shunt w/ the input!

i_x: in phase w/ v_1 → This is an effective resistance seen @ resonance "looking into electrode 1!"





← resistance

"Motional Resistance"

Motional Resistance:

$$R_{x1} = \frac{V_1}{I_1} = \frac{k}{\omega_o Q \eta_{e1}^2} = \frac{m\omega_o}{Q\eta_{e1}^2} = \boxed{\frac{b}{\eta_{e1}^2} = R_{x1}}$$

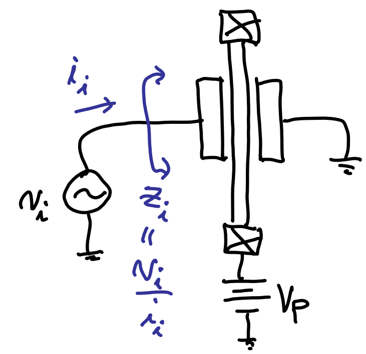
"motion"

$\omega_o^2 = \frac{k}{m}$

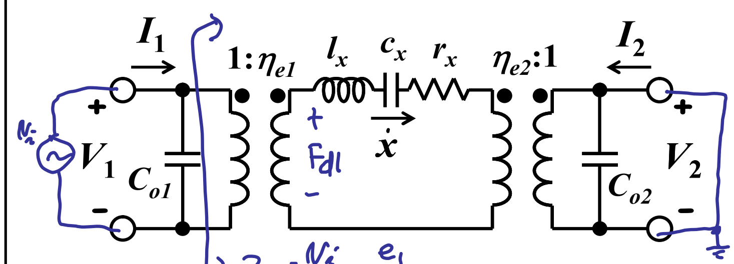
In design, this is a very important parameter!

- Look at slide 16 in Module 13

Input Impedance Into Port 1



Find $z_i = ?$



$z_i = \frac{v_i}{i_i} = \frac{e_1}{f_1}$

For now, neglect C_{o1} & C_{o2}.

(Most interested in the mechanical input impedance.)

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{array}{l} e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \\ f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2 \end{array}$$

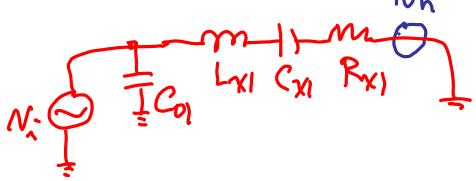
$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \rightarrow \frac{v_i}{i_i} = z_i = -\frac{1}{\eta_{e1}^2}\frac{F_{d1}}{(-\dot{x})} = \frac{1}{\eta_e^2} z_x$$

$$z_i = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

$$= \underbrace{j\omega\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \underbrace{\frac{1}{j\omega(\eta_{e1}^2 c_x)}}_{C_{x1}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$

to model noise!

$$\overline{v_n^2} = 4kTR_x$$



Purely Electrical Equiv. Ckt.