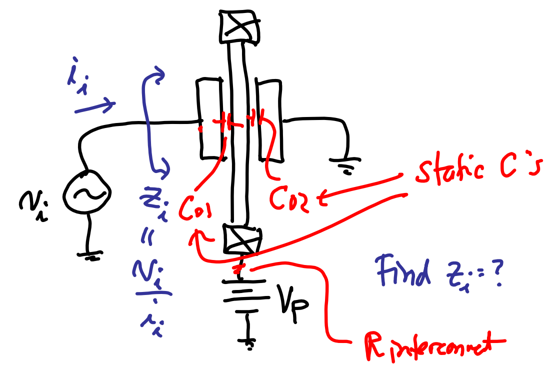
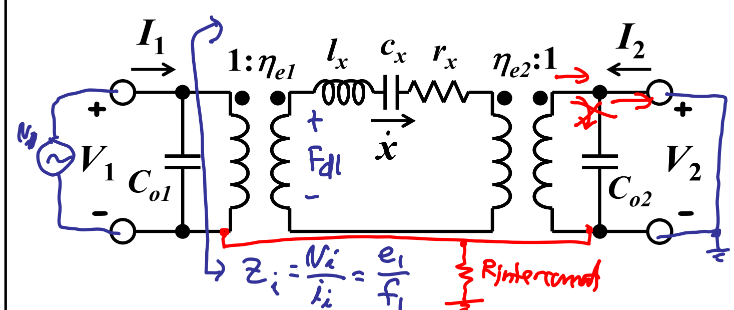
Lecture 23: Mechanical Circuit Analysis & Gyroscopes

- **Announcements**:
- Module 14 on Sensing Circuits online
- Module 15 on Gyros, Noise, & MDS online
- HW#6 online and due Thursday, April 27
- Project Slide Set #1 due Friday, April 14
- ------------------------------------
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - —Force-to-Velocity Equiv. Ckt.
    - —Input Equivalent Ckt.
  - Current Modeling
    - —Output Current Into Ground
    - —Input Current
    - —Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- ------------------------------------
- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - —Velocity Sensing
    - —Position Sensing
- ------------------------------------
- **Last Time**:
- Derived the complete equivalent circuit for a capacitively transduced mechanical structure

Input Impedance Into Port 1





For now, neglect C_{o1} & C_{o2}.
(Most interested in the mechanical input impedance.)

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{matrix} e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \\ f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2 \end{matrix}$$

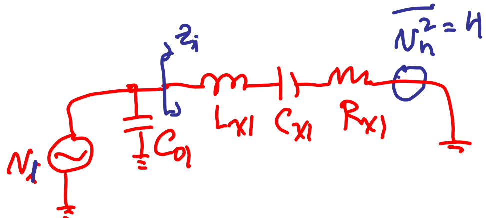
$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \rightarrow \frac{v_i}{i_i} = Z_i = -\frac{1}{\eta_{e1}^2}\frac{F_{d1}}{(-\dot{x})} = \frac{1}{\eta_e^2} Z_x$$

$$z_i = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

$$= \underbrace{j\omega\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \underbrace{\frac{1}{j\omega(\eta_{e1}^2 c_x)}}_{C_{x1}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$

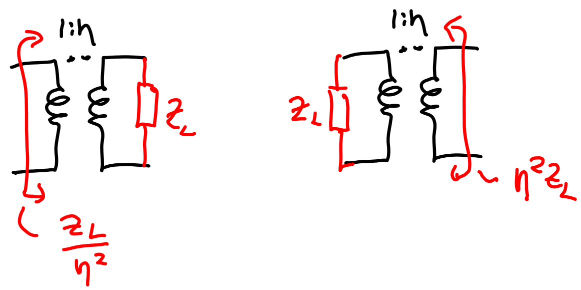
to model noise!

$$\overline{v_n^2} = 4kTR_x$$

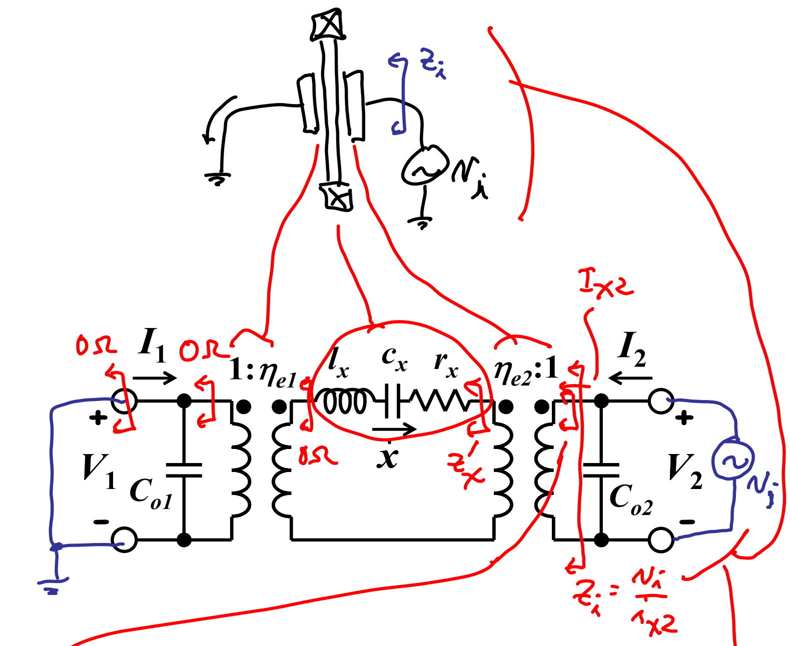


Purely Electrical Equiv. Ckt.

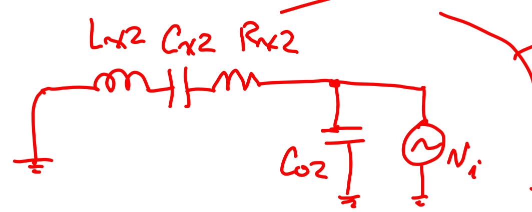
Xformer Inspection Analysis



Input Impedance Into Port 2

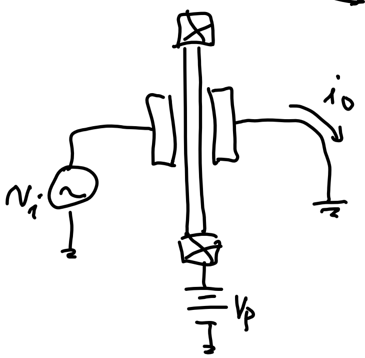


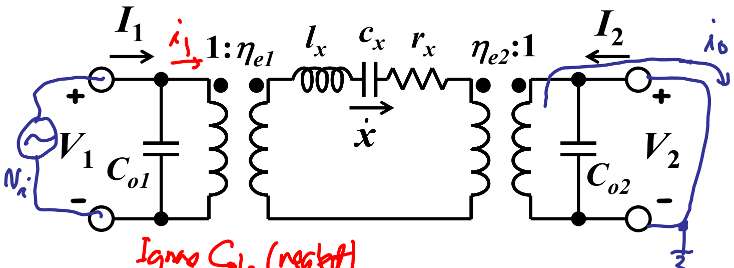
$$z_i = \frac{v_i}{i_{x2}} = \frac{z_x}{\eta_{e2}^2} = \underbrace{j\omega\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \underbrace{\frac{1}{j\omega(\eta_{e2}^2 c_x)}}_{C_{x2}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$



Port 1 to Port 2 Transconductance

$G = \frac{i_o}{v_i}$





Ignore C_{o1}. (neglect)

$\dot{x} = \frac{1}{\eta_{e1}} i_1$, $i_o = \eta_{e2}\dot{x}$ $\rightarrow$ $i_o = \frac{\eta_{e2}}{\eta_{e1}} i_1 = \frac{\eta_{e2}}{\eta_{e1}}\left(\frac{v_i}{Z_i}\right)$

$$= \frac{\eta_{e2}}{\eta_{e1}} v_i \left[\eta_{e1}^2 \frac{1}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}\right]$$

$$\therefore \frac{i_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}$$

$$\frac{i_o}{v_i}(j\omega) = \frac{1}{j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}}$$

$L_{x12} = \frac{l_x}{\eta_{e1}\eta_{e2}}$, $C_{x12} = \eta_{e1}\eta_{e2}c_x$, $R_{x12} = \frac{r_x}{\eta_{e1}\eta_{e2}}$

Separate freq. response & magnitude

$$\frac{i_o}{v_i}(s) = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{s\left(\frac{1}{L_x}\right)}{s^2 + \frac{1}{L_xC_x} + s\left(\frac{R_x}{L_x}\right)}$$

$\left[\frac{1}{L_xC_x} = \omega_o^2,\ Q = \frac{\omega_o L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o}{Q}\right]$

$$\frac{i_o}{v_i}(s) = \frac{1}{R_x}\,\frac{s\left(\frac{\omega_o}{Q}\right)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2} = \frac{1}{R_x}\,Ⓗ(s)$$

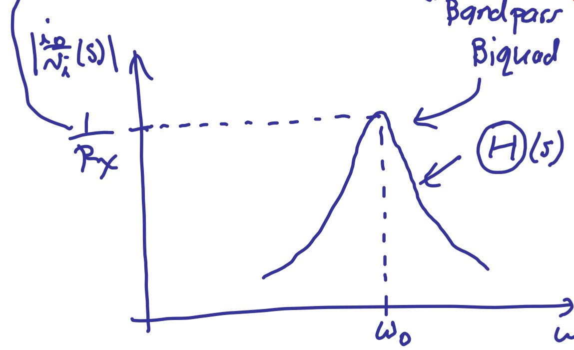
Gain Term — Freq. Shaping Term → Bandpass Biquad

Ⓗ(s) drops out @ resonance, i.e., =1

left w/ this @ resonance

↓

Just solve the ckt @ resonance, then multiply by modified Ⓗ(s).



- Now, go through slides 21-22 in Module 13
- Then, start gyroscopes by going through slides 1-16 in Module 15