

**Lecture 25: Sensing Circuits II and Noise**

- Announcements:
- HW#6 online and due Thursday, April 27
- Project Slide Set #2 due Friday, April 21
- Module 17 on Noise & MDS online
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- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - ↳ Detection Circuits
    - Velocity Sensing
    - Position Sensing
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- Reading: Senturia Chpt. 16
- Lecture Topics:
  - ↳ Minimum Detectable Signal
  - ↳ Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - ↳ Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
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- Last Time:
- Single-ended position detection using Module 14
- Now, move to differential sensing ...

**Differential Position Sensing** (of an accelerometer)

**Proof Mass**

**Sense Finger**

$F_{\text{inertial}} = ma \rightarrow x$

$x$

$a \downarrow$

**Suspension Beam in Tension**

Tethers with fixed ends

$C_1$   $C_2$

Applied Acceleration

$F = ma$

$x$

Smaller bigger!

Fixed Capacitor Plates

$V_p$   $-V_p$   $V_o$

$$V_o = -V_p + (2V_p) \frac{C_1}{C_1 + C_2}$$

$$= \frac{-V_p C_1 - V_p C_2 + 2V_p C_1}{C_1 + C_2}$$

$$\Rightarrow V_o = V_p \left( \frac{C_1 - C_2}{C_1 + C_2} \right) \quad (\text{ideal})$$

Issue: parasitic  $C_p$ !

$$V_o = \left( \frac{C_1 C_2}{C_1 + C_2 + C_p} \right) V_p$$

↳ if  $C_p \gg C_1$  or  $C_2 \rightarrow$  degrade sensitivity!

**Problem!**

Solution: use an op amp!

bootstrapped out

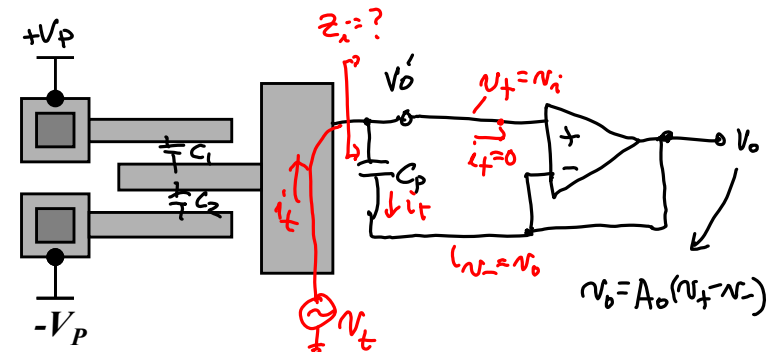
interconnect (electrode & access nodes output)

**Remarks:**

- Works best if op amp is ideal and we access to all C terminals.
- Any shunt C to ground avoids cancellation

③ Finite op amp gain reduces the  $C_p$  cancellation.

Case: Finite Op Amp Gain (an op amp non-ideality)



Get  $z_i = \frac{N_t}{i_t}$ :

$$i_t = (V_i - V_o) s C_p = V_i \left( 1 - \frac{A_o}{1 + A_o} \right) s C_p$$

$$= V_i \frac{1}{1 + A_o} s C_p$$

$$\therefore z_i = \frac{N_t}{i_t} = \frac{V_i}{i_t} = \frac{1}{s \left[ \frac{C_p}{1 + A_o} \right]} \rightarrow C_{\text{eff}} = \frac{C_p}{1 + A_o}$$

Ex.  $A_o = 100, C_p = 2 \text{ pF}$

$$\Rightarrow C_{\text{eff}} = \frac{2 \text{ pF}}{101} = 20 \text{ fF}$$

ADXL-50  $C_i \approx 100 \text{ fF}$  not negligible!  
↳ would like high  $A_o$ !

### Circuit Noise Calculations

Inputs:  $N_i(j\omega)$  (Deterministic),  $S_i(\omega)$  (Random)

Block:  $H(j\omega)$

Outputs:  $N_o(j\omega)$  (Deterministic),  $S_o(\omega)$  (Random)

**Deterministic Signals:**

$N_o(j\omega) = H(j\omega)N_i(j\omega)$

**Random Signal:**

Mean-Square Spectral Density

$S_o(\omega) = [H(j\omega)H^*(j\omega)]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$

$\sqrt{S_o(\omega_0)} = |H(j\omega)| \sqrt{S_i(\omega)}$  (root mean-square amplitude)

### Noise Source Correlation

**Case ①: Single Noise Source**

This is correlated w/ this, since it derives from it!

Thus, can write:

$N_{on1} = H_1(j\omega)N_{n1}$  (can work w/ the actual signal as usual)

**Case ②: Multiple Noise Sources**

⇒ in general, the noise sources are not correlated

Can write:  $N_{on1} = H_1(j\omega)N_{n1}$  ← these are not correlated ∴ cannot write

$N_{on2} = H_2(j\omega)N_{n2}$

$N_{OT} \neq N_{on1} + N_{on2}$

Rather:  $\overline{N_{OT}^2} = \overline{N_{on1}^2} + \overline{N_{on2}^2}$  (must add powers!)

Systematic Noise Calculation Procedure

General Ckt. w/ Several Noise Sources

Assume noise sources are correlated.

- ① For  $\overline{i_{n1}^2}$ , replace w/ a source  $i_{n1}$ .
- ② Calculate  $v_{on1}(\omega) = i_{n1}(\omega)H_1(j\omega)$   
 (treating it like a deterministic signal)
- ③ Determine  $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H_1(j\omega)|^2$
- ④ Repeat for each noise source:  
 $\overline{v_{on2}^2} = f(\overline{v_{n2}^2})$ ,  $\overline{v_{on3}^2} = f(\overline{v_{n3}^2})$ , ...
- ⑤ Add noise power (mean-square values)  
 $\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots$   
 $\overline{v_{onTOT}} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots}$   
 ↑  
 total rms value