Lecture 26: Equivalent Input Noise

- Announcements:
- HW#6 due Thursday, April 27
- Project Slide Set #3 due Friday, April 28
- Module 17 on Noise & MDS has been online
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- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
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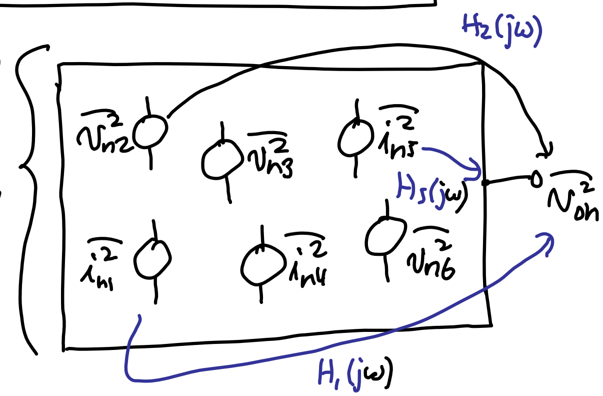
- Last Time:
- Circuit noise calculations
- Now, move to noise sources and examples …

Systematic Noise Calculation Procedure

General Ckt. w/ Several Noise Sources



Assume noise sources are uncorrelated.

① For $\overline{i_{n1}^2}$, replace w/ a source i_{n1}.

② Calculate $v_{on1}(\omega) = i_{n1}(\omega) H_1(j\omega)$
(treating it like a deterministic signal)

③ Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H_1(j\omega)|^2$

④ Repeat for each noise source:
$\overline{v_{on2}^2} = f(\overline{v_{n2}^2})$, $\overline{v_{o3}^2} = f(\overline{v_{n3}^2})$, …

⑤ Add noise power (mean-square values)

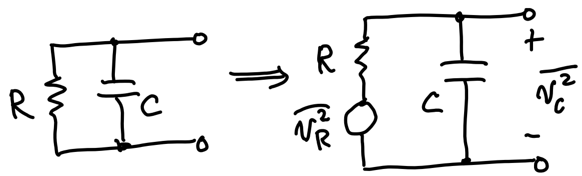
$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \ldots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \ldots}$$

↑ total rms value

Why $\frac{\overline{v_{nR}^2}}{\Delta f} = 4kTR$? (a heuristic argument)

Consider an RC ckt:



$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_c^2} \rightarrow \boxed{\overline{v_c^2} = \frac{kT}{C}}$ ← integrated noise over all freqs.

(total mean-square voltage integrated over all freqs.)

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ gives us this (assume white noise)

$$\overline{v_c^2} = \int_0^\infty \left|\frac{1}{1+j\omega RC}\right|^2 \frac{\overline{v_R^2}}{\Delta f}\, df = \frac{kT}{C}$$

[noise is white] [$\omega_b = \frac{1}{RC}$]

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega^2}\, d\omega$$

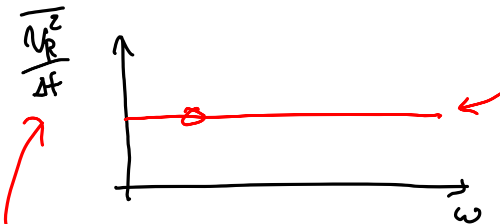
$$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\frac{\omega_b^2}{\omega_b}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Big|_0^\infty$$

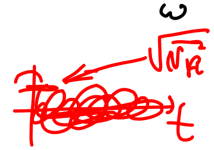
$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\left(\frac{\pi}{2}\omega_b - 0\right) = \frac{1}{4}\omega_b\frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C} \rightarrow *$$

$* \rightarrow \frac{\overline{v_R^2}}{\Delta f} = 4kT\left(\frac{\omega_b}{C}\right) \Rightarrow \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR}$ ✓

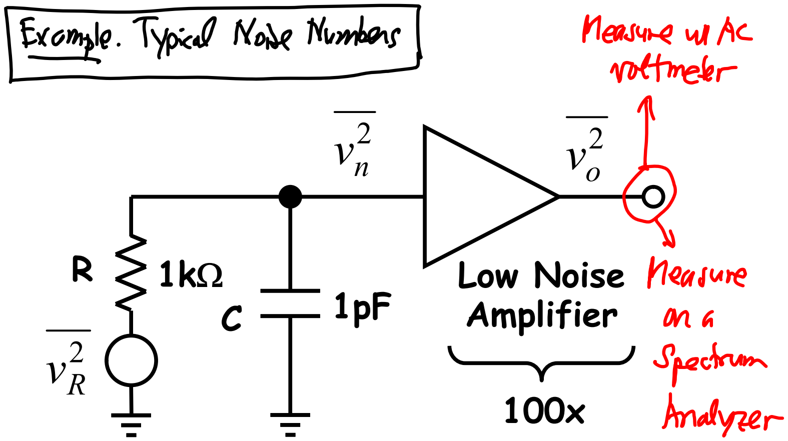
(↑ R)

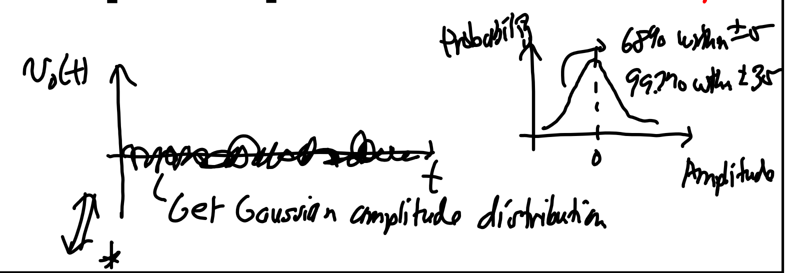


$\overline{v_R^2} = 4kT\Delta f$ (↑ bandwidth) →



Example. Typical Noise Numbers





$R = 1k\Omega \rightarrow \sqrt{(1.66\times10^{-20})(1k)}$

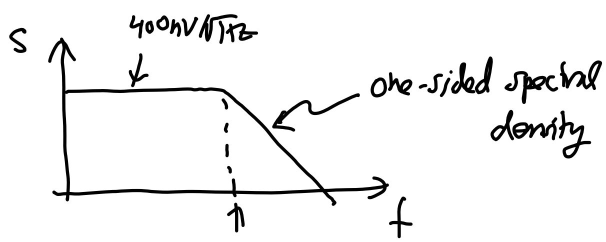
$1k\Omega$: $4nV/\sqrt{Hz}$ (for every 1k of R)

$1pF$: $\sqrt{\frac{kT}{C}} = 64\mu V\,rms$

Case: AC Voltmeter

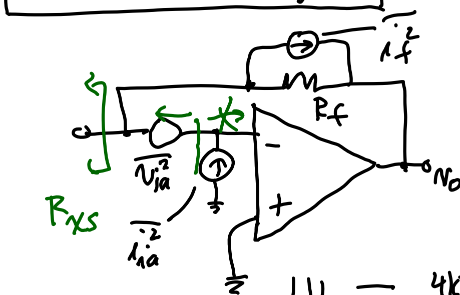
$\sqrt{\overline{v_o^2}} = (100)(64\mu V rms) = 6.4\,mV\,rms$

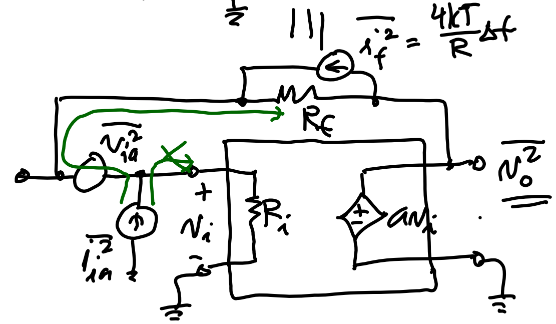
Case: Spectrum Analyzer



$\frac{1}{2\pi(1k)(1p)} = 60nHz$

Noise in a TransR Amplifier





Get $\overline{v_o^2}$: use superposition → handle one noise source at a time

$\overline{i_{ia}^2}$: $v_{o1} = i_{ia} R_f \rightarrow \overline{v_{o1}^2} = \overline{i_{ia}^2} R_f^2$