Lecture 27: Gyro Minimum Detectable Signal

- Announcements:
- Project Slide Set #3 due Friday, April 28
- Sign up sheet for Project Outbriefs on my door: please sign up
  - Wednesday, 5/3, 4-5:20 p.m.
  - Thursday, 5/11, 11-12, 2-2:40 p.m.
- Wrap up course
- Go through Final Exam info
- Pass out three sample final exams
- HKN will come at end for course evaluations
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- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
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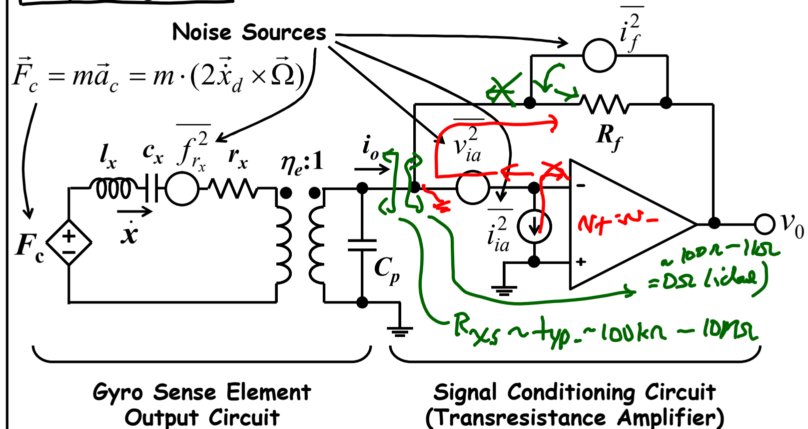
- Last Time:
- Started with gyro MDS analysis
- Continue with this …

Determine MDS



① Determine $\overline{v_o^2}$

⇒ use superposition one source at a time

$\overline{i_{ia}^2}$: $v_{o1} = i_{ia}R_f \longrightarrow \overline{v_{o1}^2} = \overline{i_{ia}^2}R_f^2$; $\frac{4kT\Delta f}{R_f}R_f^2 = 4kTR_f\Delta f$

$\overline{i_f^2}$: $v_{o2} = -i_f R_f \longrightarrow \overline{v_{o2}^2} = \overline{i_f^2}R_f^2$

$\overline{v_{ia}^2}$: unity gain buffer; $v_{o3} \rightarrow v_{o3} = v_{ia}$

$\left(\frac{\omega_s Q_s}{k_s} H_s(j\omega_d)\right)\overline{v_{o3}^2} = \overline{v_{ia}^2}\left|\frac{\omega_s Q_s}{k_s} H_s(j\omega_d)\right|^2$

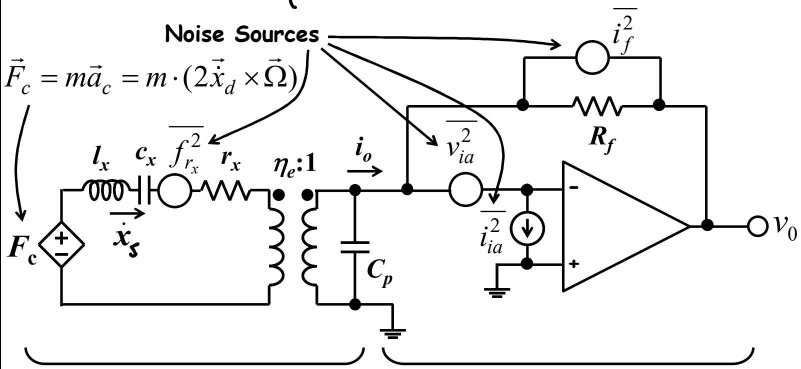
$\overline{f_{rx}^2}$: $v_{o4} = -\frac{f_x}{r_x}\eta_e R_f \rightarrow \overline{v_{o4}^2} = \frac{\overline{f_x^2}}{r_x^2}\eta_e^2 R_f^2$

$= \frac{4kTr_x\Delta f}{r_x^2}\eta_e^2 R_f^2 \quad \left|\frac{\omega_s Q_s}{k_s} H_s(j\omega_d)\right|^2$

$$\overline{v_o^2} = \overline{i_{ia}^2}R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1 + \eta_e^2\frac{R_f}{r_x}\right)\Delta f$$

Total output mean-square voltage noise

② Find v_o in terms of rotation rate Ω:



Gyro Sense Element Output Circuit

Signal Conditioning Circuit (Transresistance Amplifier)

⇒ Find the rotation-to-i_o transfer function

$$\dot{x}_s = F_c k_s^{-1}(j\omega) = F_c\left(\frac{\omega_s Q_s}{k_s}\Theta_s(j\omega_d)\right)$$

$$[F_c = m a_c = 2\omega_d x_d \Omega m]$$

$$= \frac{\omega_s Q_s}{k_s}\cdot 2\omega_d x_d \Omega m\,\Theta_s(j\omega_d)$$

$\frac{m}{k_s} = \frac{1}{\omega_s^2}$

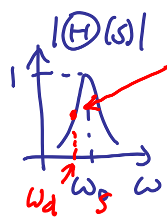
$$\dot{x}_s = 2\frac{\omega_d}{\omega_s}Q_s x_d\,\Theta_s(j\omega_d)\cdot\Omega$$

$$\Theta(s) = \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

$s=0$: $\Theta(0) = 0$

$s=j\omega_0$: $\Theta(j\omega_0) = 1$

$s=\infty$: $\Theta(\infty) = 0$



$$i_o = \eta_e \dot{x}_s = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e\,\Theta_s(j\omega_d)\cdot\Omega$$

∴ $$|v_o| = i_o R_f = \underbrace{2R_f\frac{\omega_d}{\omega_s}Q_s x_d \eta_e\,\Theta_s(j\omega_d)}_{A \triangleq \text{ scale factor}}\cdot\Omega$$

rotation rate

$$|v_o| = A\Omega$$

rms noise voltage

③ $\Omega = \Omega_{min}$ when $|v_o| = \sqrt{\overline{v_o^2}}$

minimum detectable rotation rate (MDS)

$$2R_f\frac{\omega_d}{\omega_s}Q_s x_d \eta_e\,\Theta_s(j\omega_d)\cdot\Omega_{min} = \sqrt{\left(\overline{i_{ia}^2}R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1+\eta_e^2\left|\frac{\omega_s Q_s}{k_s}\Theta_s(j\omega_d)\right|^2\frac{R_f}{r_x}\right)\right)\Delta f}$$

Solve for Ω_{min}:

$$\Omega_{min} = \frac{\sqrt{\left(\overline{i_{ia}^2}R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1+\eta_e^2\left|\frac{\omega_s Q_s}{k_s}\Theta_s(j\omega_d)\right|^2\frac{R_f}{r_x}\right)\right)\Delta f}}{2R_f\frac{\omega_d}{\omega_s}Q_s x_d \eta_e\,\Theta_s(j\omega_d)}$$

$$\times\left(\frac{3600\,s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) \rightarrow \left[(^\circ/hr)/\sqrt{Hz}\right]$$

$$\boxed{\text{Angle Random Walk} = ARW = \frac{1}{60}\Omega_{min}\ \left[^\circ/\sqrt{hr}\right]}$$

Easier to determine directional error as a function of elapsed time.

- Related courses at UC Berkeley:
  - EE 143: Microfabrication Technology
  - EE 147/247A: Introduction to MEMS
  - ME 119: Introduction to MEMS (mainly fabrication)
  - BioEng 121: Introduction to Micro and Nano Biotechnology and BioMEMS
  - ME C219 – EE C246: MEMS Design
  - EE 290M?