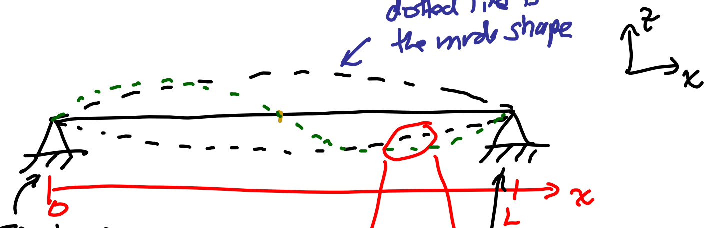
Lecture 2: Benefits of Scaling I

- Announcements:
- The notes from last time will soon be online, as well as the video – both in the Lecture link table
- Modules 1 & 2 will be online (also, in the Lecture link table)
- As announced last time, I will be traveling next week (at the IEEE MEMS Conference)
  - Next week's lectures will be by recorded video
  - The videos will be online in the Lecture link table in the far right column
  - Please watch the videos before the week after next to avoid falling behind
- Get your computer accounts by following the instructions at the end of the Course Info Sheet (the new one recently uploaded)
- Please sign up for the Piazza course page:
- piazza.com/berkeley/spring2017/eec247bmec218
- -------------------------------------
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
- -------------------------------------
- Start going through Module 2

Scaling of Guitar Strings

guitar string ≡ transversely vibrating stretched wire

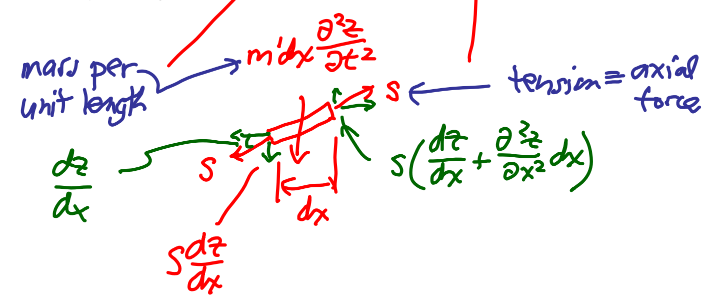


Simple Support $z(x=0)=0$ } Boundary Condition

Simple Support $z(x=L)=0$ } B.C.

⇒ want equation for resonance freq. f_0 (fundamental mode)

Free-Body Diagram



⇒ condition for dynamic equilibrium:

$$S\left(\frac{\partial z}{\partial x}+\frac{\partial^2 z}{\partial x^2}dx\right) - S\frac{\partial z}{\partial x} - m'dx\frac{\partial^2 z}{\partial t^2} = 0$$

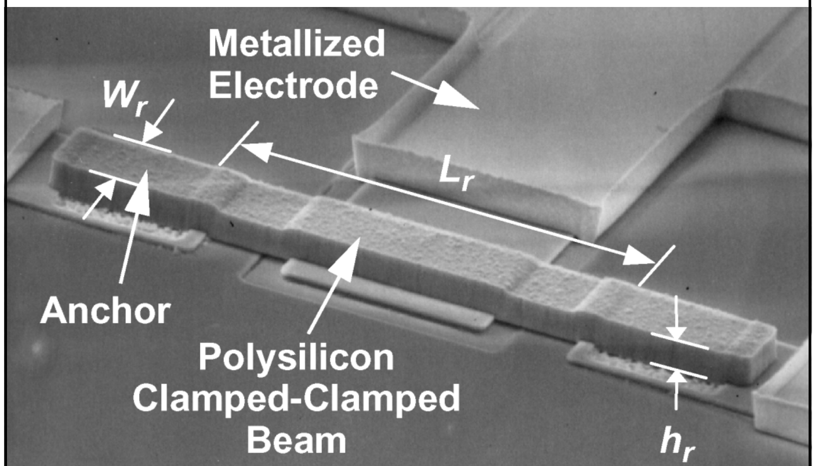
Solve

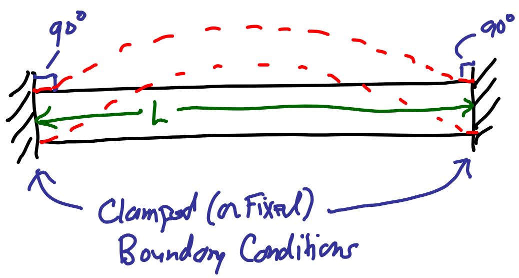
$$\boxed{f_i = \frac{i}{2L}\sqrt{\frac{S}{m'}}}$$ ← frequency

if $L\downarrow \rightarrow f_i\uparrow$

i = mode = 1, 2, 3, ...

Clamped-Clamped Beam





⇒ Eq. for resonance:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \qquad (1)$$

$L\downarrow \rightarrow f_o\uparrow\uparrow$

where $E \triangleq$ Young's modulus [GPa]
$\rho \triangleq$ density [kg/m³]
$h \triangleq$ thickness [m]
$L \triangleq$ length [m]

Example. $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150\,GPa$, $\rho = 2300\,kg/m^3$

$$\therefore f_o = (1.03)\underbrace{\sqrt{\frac{150G}{2300}}}\frac{2\mu}{(40\mu)^2} \Rightarrow \boxed{f_o = 10.4\,MHz}$$

$\sqrt{\frac{E}{\rho}}$ = acoustic velocity = 8076 m/s

Scaling:

① Scale all dimensions equally by a factor S (↑ 2X, ½X)

$f_o \sim \frac{S}{S^2} = \frac{1}{S}$

② If scale L only: $f_o = \frac{1}{S^2}$ → even faster rise in f_o
...but... problems! ...

Example.

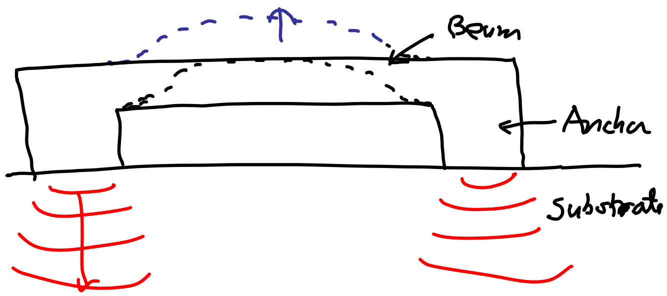
$L = 4\mu m \rightarrow f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} \rightarrow \boxed{f_o = 1.04\,GHz!}$

↑ ignore width effects (for now)
↳ questionable thing to do → $f_o \sim 800\,MHz$

Remarks.

① Eq. (1) not accurate when $L \approx h \approx W$.

② when $L \approx h$ → get anchor loss problems



Energy radiates into substrate! } anchor loss

$$Q = \frac{\text{Energy per cycle}}{\text{Energy lost per cycle}} \downarrow$$

sharpness of the peak

Problem!

③ Solution: nanodimensions ✓

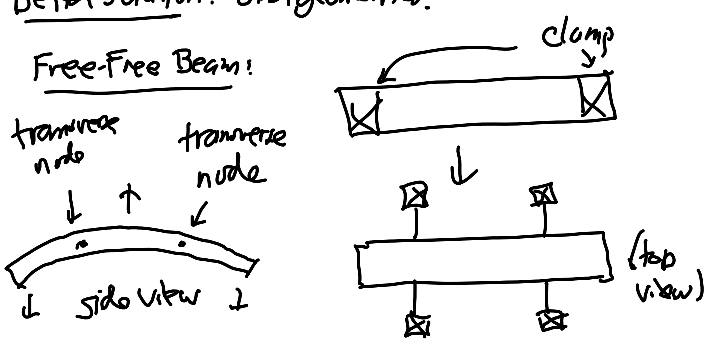
this means: h~300nm, L~1µm

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad k\downarrow,\ m\downarrow$$

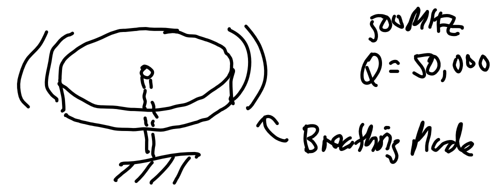
Problem: power handling ↓ when size ↓

④ Better Solution: other geometries! ✓✓

Free-Free Beam:

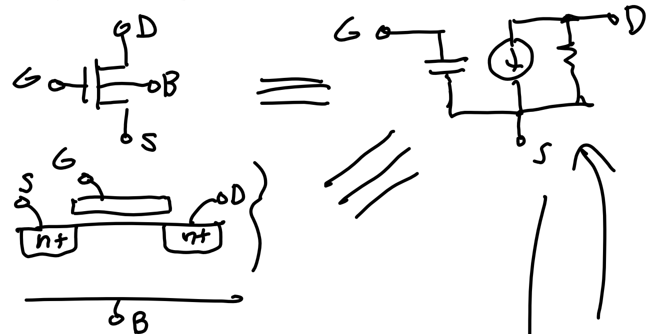


Radial Mode Disk:



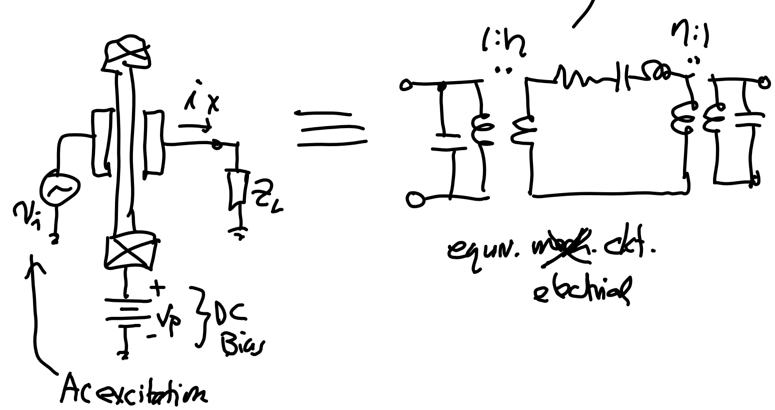
Circuit Design

Transistor Ckt.



Mechanical Ckt.

SPICE



equiv. ~~mech.~~ electrical ckt.

