Let’s do some analysis on the resonance behavior of this comb-drive structure.
**x-Direction Resonance Frequency**

First, let’s find $f_0$ when all ports are grounded

\[ \omega_0 = \sqrt{\frac{k_s}{m_s}} \]

\[
k_s = k_c = \frac{2Ew^2h}{L^3} = \frac{2(150G)(2\mu)(2\mu)}{50\mu} = 38.4 \text{ N/m} \]

\[
m_s = M_s + \frac{1}{4} M_t + \frac{12}{35} M_b = 20.35 \text{ ng} \]

\[
M_s = \rho h A_s = 18.4 \text{ ng} \]

\[
M_t = 2\rho h A_t = 2.76 \text{ ng} \]

\[
M_b = 8\rho h LW = 3.68 \text{ ng} \]

\[
\omega_0 = \sqrt{\frac{38.4}{20.3}} = 1.37 \text{ Mrad} \Rightarrow f_0 = 218.618 \text{ kHz} \]
Now, let’s find $f_0$ when ports 1, 2 & 4 are grounded, and all other ports are biased to 50 V

$$f_0' = f_0 \left(1 - \frac{k_{equ}}{k_t}\right)^{1/2}$$

Electrical stiffness from port 4

Stiffness @ truss

$$k_{equ} = V_p^2 C_0 \frac{d_0^2}{d_0^2} = V_p^2 \varepsilon_0 A_{04} = V_p^2 \varepsilon_0 \left(2N_{41} h L_0\right)$$

$$= \left(50\right)^2 \left(8.85 \times 10^{-12}\right) \left(2\right) \left(8\right) \left(2\pi\right) \left(10\mu\right) \left(1\mu\right)$$

$$= 7.08 \text{ N/m}$$

$$f_0' = (218k) \left(1 - \frac{7.08}{153.6}\right)^{1/2}$$

$$f_0 = 0.9767 f_0'$$

$$f_0 = 213.517 \text{ kHz}$$
Now draw the transformer-based equivalent circuit (between ports 1 & 2) when ports 1, 2 & 4 are grounded and all other ports are biased to 50 V.

\[ L_x = m_s = 2.035 \times 10^{-11} \text{ kg (or} \text{ t)} \]

\[ C_x = \frac{1}{k_3} = 0.024 \frac{m}{N} \text{ (or} \text{ F)} \]

\[ r_x = b_x = \frac{\sqrt{k_3 m_s}}{Q} = 0.2795 \frac{\text{kg}}{\text{m}^2} \text{ (or} \text{ Nm)} \]

\[ \eta_{e1} = \eta_{e2} = \frac{V_p}{2\epsilon} = 2\epsilon \frac{2N_a \epsilon_0 (2\mu)}{1\mu} \]

\[ C_0 = C_{02} = \frac{\epsilon_0 h L_0 (2N_4)}{d_0} \]

\[ C_0 = 2.48 \text{ ff} = C_{02} \]
Next, draw the transformer-based equivalent circuit between ports 1 & 3 with all other ports are biased to 50 V.

\[ i_o = V_P \left( \frac{\partial C_1}{\partial x} + \frac{\partial C_2}{\partial x} \right) \frac{dx}{dt} = 0 \]

\[ F_d = V_P \left( \frac{\partial C_1}{\partial x} + \frac{\partial C_2}{\partial x} \right) \sigma_i = 0 \]

\[ \eta_{e3} = V_P \left( \frac{2C_1}{2x} + \frac{2C_2}{2x} \right) = 0 \]

\[ C_{03} = \frac{2N_3 \varepsilon_0 h L_0}{d_0} \]
Sensing

\[
\frac{|u_o(s)|}{|u_i(s)|} = \frac{v_o(s)}{v_i(s)} = \frac{C_x/C_D}{1 - C_x/C_D} \frac{(\omega_0')^2}{s^2 + \left(\frac{\omega_0'}{Q'}\right)s + (\omega_0')^2} \]

DC gain

low-pass

biqquad

\[
\omega_0' = \omega_0 \sqrt{1 + \frac{C_x}{C_D}}
\]

\[
Q' = Q \sqrt{1 + \frac{C_x}{C_D}}
\]

\[
\omega_0' = 227.2 \text{ kHz}
\]

\[
Q' = 104,100
\]

\[
C_x = \eta \varepsilon_0 C_x = 4 \mu \text{F}
\]

\[
\frac{C_x/C_D}{1 - C_x/C_D} = 0.074
\]

\[
|\frac{v_o(s)}{v_i(s)}| \times Q'
\]

\[
|f_o| f_o'
\]

\[
C_D = 0.05 \mu \text{F}
\]
Sensing

\[ C_{\text{eff}} = C_p + (1 + A_o)C_f \]

\[ f'_o = f_o \left( 1 + \frac{C_X}{C_{\text{eff}}} \right) \Rightarrow \Delta f_o \downarrow \]

\[ \left| \frac{V_o(s)}{V_i(s)} \right| = \frac{C_X/C_{\text{eff}}}{1 + C_X/C_{\text{eff}}} \frac{(\omega'_o)^2}{s^2 + \left( \frac{\omega'_o}{Q'_o} \right)s + (\omega'_o)^2} \]