Lecture 11: Mechanics of Materials I

- Announcements:
- Module 7 on "Mechanics of Materials" online
- HW#3 due Tuesday, 2/27, at 10 a.m.
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- Reading: Senturia Chpt. 3, Jaeger Chpt. 11, Handouts: "Bulk Micromachining of Silicon"
- Lecture Topics:
  - Bulk Micromachining
  - Anisotropic Etching of Silicon
  - Boron-Doped Etch Stop
  - Electrochemical Etch Stop
  - Isotropic Etching of Silicon
  - Deep Reactive Ion Etching (DRIE)
  - Wafer Bonding
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- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
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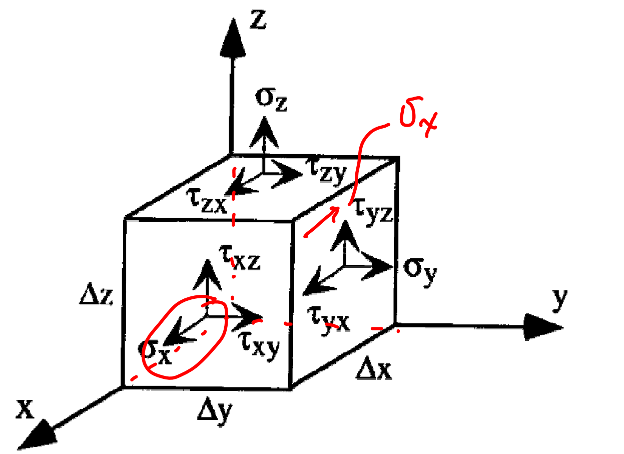
- Last Time: Going thru Module 6 … finish this
- Move on to Module 7

Example. Exercise the "terms"

⇒ Determine the volume change ΔV resulting from a uniaxial stress σ_x (along the x-direction)



Upon application of σ_x, what is ΔV?

Before Stress		After σ_x
Δx	$\longrightarrow$	$\Delta x(1+\epsilon_x)$
Δy	$\longrightarrow$	$\Delta y(1-\nu\epsilon_x)$
Δz	$\longrightarrow$	$\Delta z(1-\nu\epsilon_x)$

Assume isotropic material
↓
same ν along y & z

The resulting change in volume, ΔV:

$$\Delta V = \underbrace{\Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2}_{\text{volume after application of } \sigma_x} - \Delta x \Delta y \Delta z$$

$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$

[Assume small strains] → $(1+mx)^n \approx 1+nmx$

$\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

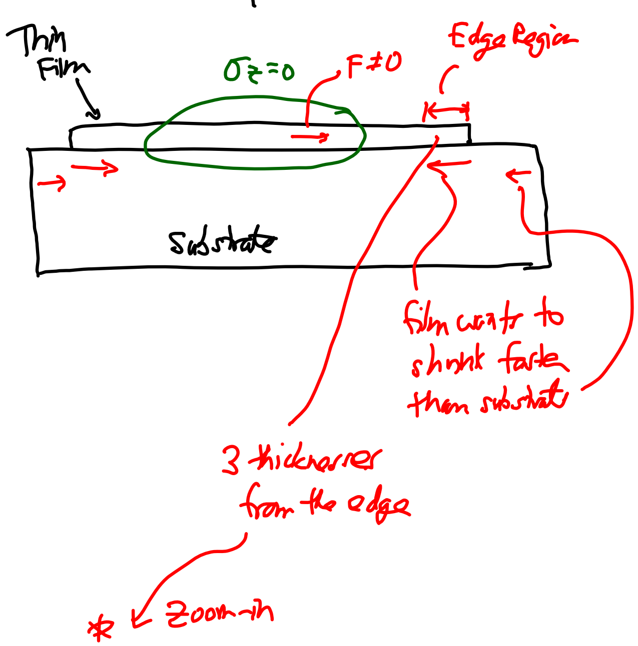
$$\boxed{\Delta V = \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

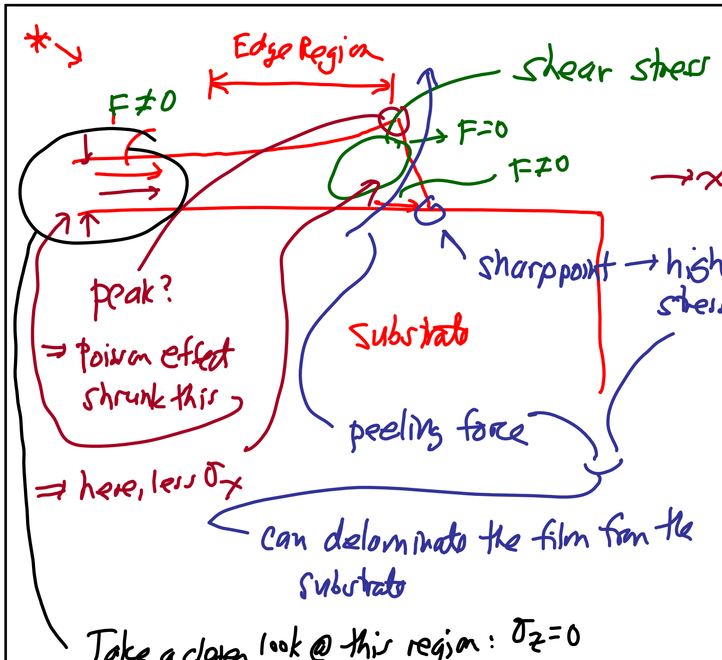
For $\nu = 0.5$ (rubber) → no ΔV!

$\nu < 0.5$ → finite ΔV

Important Case: Plane Stress

⇒ common case for a thin-film coating on a rigid substrate:





Take a closer look @ this region: $\sigma_z = 0$

Get two components of stress:

$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + 0)\right]$

$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + 0)\right]$

Assume: Plane Stress → isotropic → $\sigma_x = \sigma_y = \sigma$ → $\epsilon_x = \epsilon_y = \epsilon$

(symmetry in the xy-plane)

$\epsilon_x = \frac{1}{E}\left[\sigma - \nu\sigma\right]$

$= \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} \Rightarrow \epsilon_x = \frac{\sigma}{E'}$

where $\boxed{E' \triangleq \text{Biaxial Modulus} = \frac{E}{1-\nu}}$

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition. linear thermal expansion coefficient

$$\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Exp. Coefficient}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{Kelvin}^{-1}]$$

Remarks.

① α_T values tend to be in the 10^{-6} to 10^{-7} range

② $10^{-6}\ K^{-1}$ = 1 μstrain/K

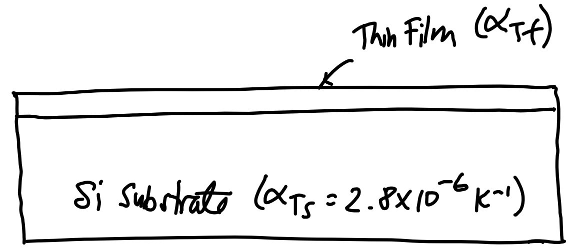
③ In 3D, get a volume thermal exp. coefficient

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

④ For moderate ΔT's → $\alpha_T \approx$ constant

for larger ΔT, then $\alpha_T = f(T)$

Ex. Thin-Film Thermal Stress



Assume.

① Substrate is much thicker than the film.

② Film is deposited stress-free @ T_d. ← deposition temperature

③ Then the whole thing is cooled to room temperature, T_r.

Thermal Strain of the Substrate: (in one plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate:

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But the film is attached to the substrate

⇒ thickness substrate >> thickness of the film

∴ substrate wins!

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thus, the actual strain experienced by the film is that of the substrate:

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} : \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

Note: This is biaxial strain (assuming the film is deposited isotropically onto the substrate)

$$\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{E'} \epsilon_{f,mismatch}$$