Lecture 12: Mechanics of Materials II

- **Announcements:**
- HW#3 online and due Tuesday, 2/27, at 10 a.m.
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- Reading: Senturia, Chpt. 8
- Lecture Topics:
- Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
- ---------------------------------
- **Last Time:**
- Defined the linear thermal expansion coefficient
- Now, continue with this

over

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition: linear thermal expansion coefficient

$$\boxed{\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Exp. Coefficient}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{Kelvin}^{-1}]}$$

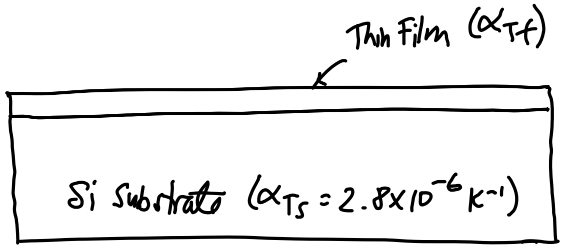
Remarks:

1. α_T values tend to be in the 10^{-6} to 10^{-7} range
2. $10^{-6}\ K^{-1} = 1\,\mu\text{strain}/K$
3. In 3D, get a volume thermal exp. coefficient

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

4. For moderate ΔT's → $\alpha_T \approx$ constant
   for large ΔT, then $\alpha_T = f(T)$

Ex. Thin-Film Thermal Stress



Assume.

1. Substrate is much thicker than the film.
2. Film is deposited stress-free @ T_d. ← deposition temperature
3. Then the whole thing is cooled to room temperature, T_r.

Thermal Strain of the Substrate: (in one plane dimension)

$\epsilon_s = -\alpha_{Ts}\Delta T$, where $\Delta T = T_d - T_r$

If the film were not attached to the substrate:

$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$

But the film is attached to the substrate

⇒ thickness substrate >> thickness of the film

∴ substrate wins!

*

thus, the actual strain experienced by the film is that of the substrate:

$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

Note: This is biaxial strain (assuming the film is deposited isotropically onto the substrate)

$$\boxed{\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{E'} \epsilon_{f,mismatch}}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\,K^{-1}$

$E' = 4\,GPa$

$\underbrace{\text{deposited @ } 250^\circ C\text{, then cooled to RT} = 25^\circ C}_{\Delta T = 225K}$

$\epsilon_{f,mismatch} = (70 - 2.8)\mu(225) = 1.5\times10^{-2}$

$[\mu = 10^{-6}, m = 10^{-3}, k = 10^{3}, G = 10^{9}]$

$\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = 60.5\,MPa$

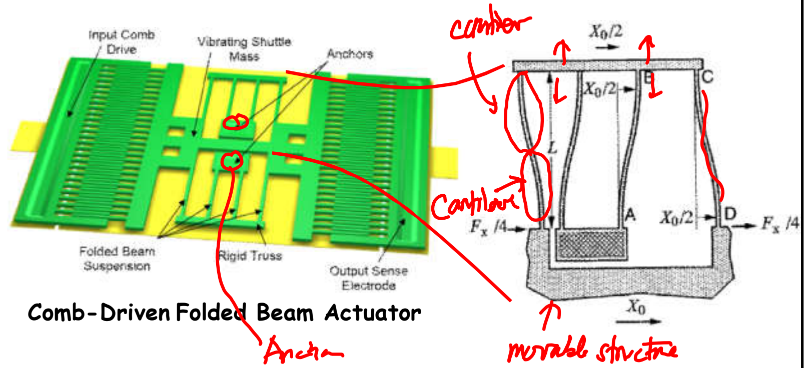
stress is (+) → tensile

[(−) would be compressive]

SiO_2

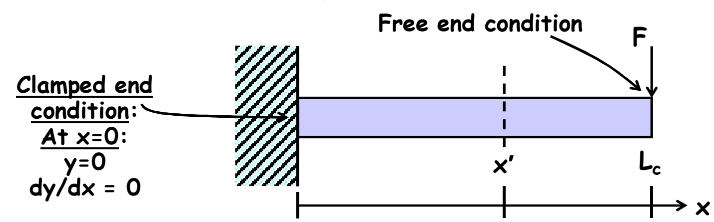
- Now go through Module 7 slides 21-55

- Springs and suspensions very common in MEMS
- Coils are popular in the macro-world; but not easy to make in the micro-world
- Beams: simpler to fabricate and analyze; become "stronger" on the micro-scale → use beams for MEMS



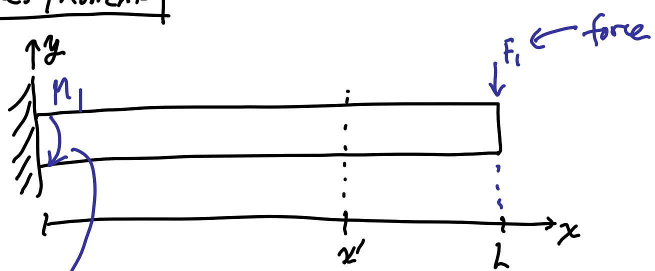
Comb-Driven Folded Beam Actuator

Problem: Bending a Cantilever Beam



- Objective: Find relation between tip deflection $y(x=L_c)$ and applied load F
- Assumptions:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
  3. Shear stresses are negligible

Forces & Moments



Moment due to $F_1 \rightarrow M_1 = F_1 L$