**Vertical Stress Gradients**

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction

**Measurement of Stress Gradient**

- Use cantilever beams
  - Strain gradient ($\Gamma = \text{slope of strain-thickness curve}$)
  - Assumptions: linear strain gradient $\Gamma$, $z = rL^2/2$

**Folded-Beam Suspension**

- Use of folded-beam suspension brings many benefits
  - Stress relief: folding truss is free to move in y-direction, so beams can expand and contract more readily to relieve stress
  - High y-axis to x-axis stiffness ratio

[Diagram of folded-beam suspension]
### Beam End Conditions

**From Reddy, Finite Element Method**

<table>
<thead>
<tr>
<th>Type of support</th>
<th>Displacement boundary conditions</th>
<th>Force boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREE</td>
<td>$u = 0$</td>
<td>$w = 0$, as specified</td>
</tr>
<tr>
<td>PINNED</td>
<td>$u = 0$</td>
<td>Moment is specified</td>
</tr>
<tr>
<td>ROLLER (vertical)</td>
<td>$u = 0$</td>
<td>Transverse force and moment are specified</td>
</tr>
<tr>
<td>ROLLER (horizontal)</td>
<td>$w = 0$</td>
<td>Horizontal force and bending moment are specified</td>
</tr>
<tr>
<td>FIXED or CLAMPED</td>
<td>$w = 0$</td>
<td>$\partial w/\partial x = 0$</td>
</tr>
</tbody>
</table>

### Common Loading & Boundary Conditions

- Displacement equations derived for various beams with concentrated load $F$ or distributed load $f$


- **Concentrated load**
  - cantilever: $x = \frac{F}{EI}$, $y = \frac{Fw}{EI}$, $z = \frac{Fw^2}{2EI}$
  - guided-end: $x = \frac{F}{EI}$, $y = \frac{Fw}{EI}$, $z = \frac{Fw^2}{2EI}$
  - fixed-fixed: $x = \frac{F}{EI}$, $y = \frac{Fw}{EI}$, $z = \frac{Fw^2}{2EI}$

- **Distributed load**
  - cantilever: $x = \frac{5F}{6EI}$, $y = \frac{5Fw}{6EI}$, $z = \frac{5Fw^2}{24EI}$
  - guided-end: $x = \frac{5F}{6EI}$, $y = \frac{5Fw}{6EI}$, $z = \frac{5Fw^2}{24EI}$
  - fixed-fixed: $x = \frac{5F}{6EI}$, $y = \frac{5Fw}{6EI}$, $z = \frac{5Fw^2}{24EI}$