Lecture 14: Beam Combos I

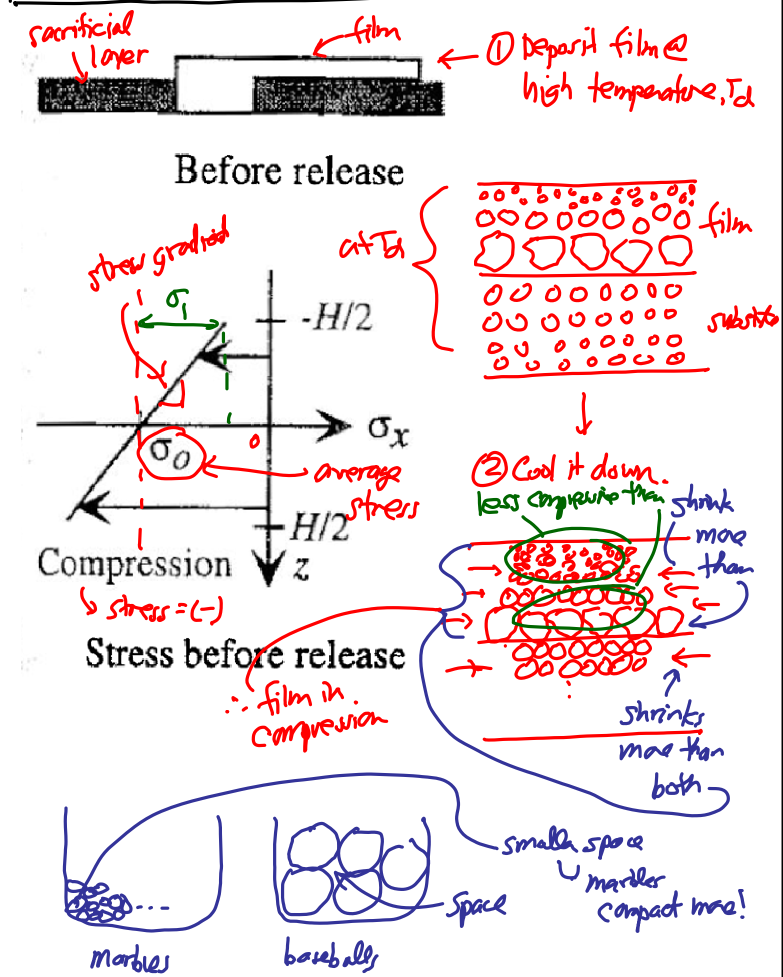
- **Announcements**:
- HW#3 online, due Thursday, next week, 10 a.m.
  - shorter time span than before
- Midterm Exam 3 weeks away, Thursday, March 22, 11-12:30 p.m., 3109 Etcheverry (right here)
- ---------------------------------------
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- ---------------------------------------
- **Last Time**:
- Working through stress gradients
- Continue with this

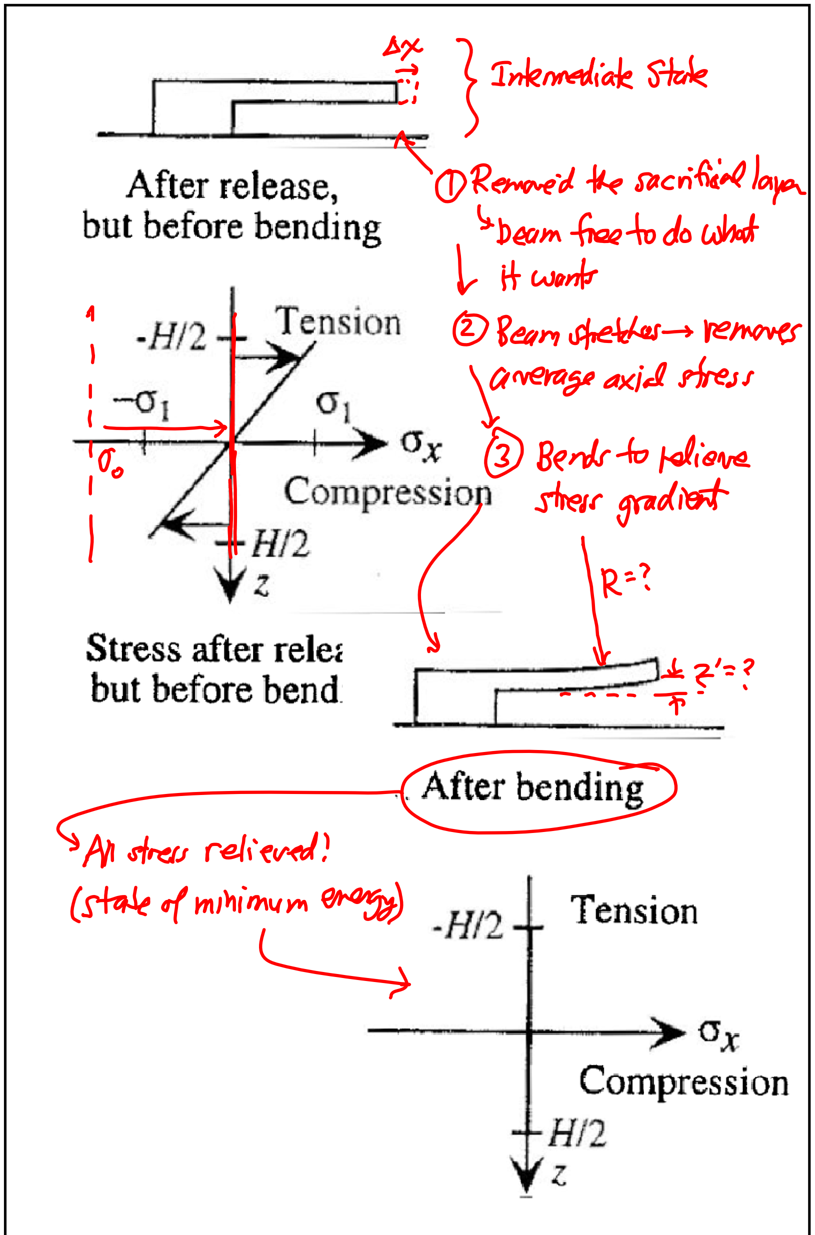
over

$$\boxed{\sigma_{max} = \varepsilon_{max} E = \frac{6L}{Wh^2} F}$$

(maximum stress in a bent cantilever subjected to a force F at its tip)

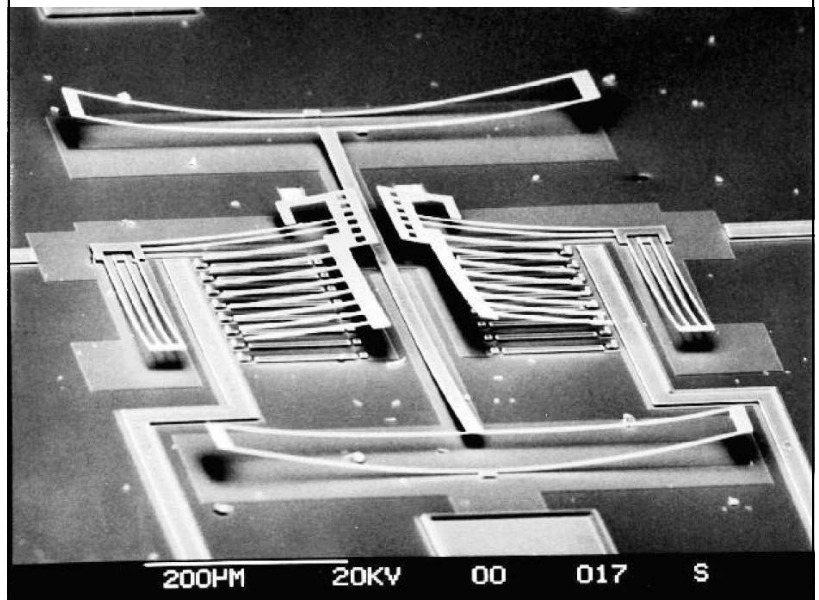
Stress Gradient in Cantilevers





After bending

- … and the result:



- Need to quantify this

Bending Due to Stress Gradients

Find the radius of curvature.

Prior to release, axial stress: $\sigma = \sigma_0 - \frac{\sigma_1}{(H/2)} z$

The internal moment:

$$M_x = \int_{-\frac{H}{2}}^{\frac{H}{2}} [(W \cdot dz)\cdot \sigma]\cdot z = \int_{-\frac{H}{2}}^{\frac{H}{2}} W\left(z\sigma_0 - \frac{\sigma_1 z^2}{(H/2)}\right) dz$$

$$= W\left(\frac{1}{2}\sigma_0 z^2 - \frac{2\sigma_1 z^3}{3H}\right)\Bigg|_{-\frac{H}{2}}^{\frac{H}{2}}$$

$$= W\left(\cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\sigma_1\frac{H^2}{8} - \cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\frac{\sigma_1 H^2}{8}\right)$$

average stress cancels out

$$\boxed{M_x = -\frac{1}{6}\sigma_1 W H^2}$$

Thus, the radius of curvature:

$$\frac{1}{R} = -\frac{M_x}{E'I} \rightarrow R = \frac{E'I}{M_x} = \frac{1}{2}\frac{E'H}{\sigma_1}$$

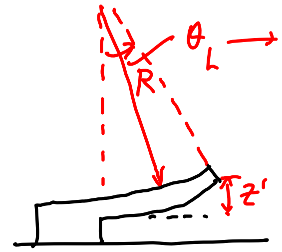
E': Biaxial Modulus

$$\left[I = \frac{1}{12}Wh^3\right]$$

$$\boxed{R = \frac{1}{2}\frac{E}{(1-\nu)}\frac{H}{\sigma_1}}$$

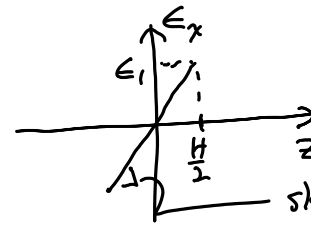
[Radius of Curvature for a Cantilever w/ a Stress Gradient]

Radius of Curvature → z'



θ_L → integrate over θ_L to get z'
→ Do this in homework...

Definition. Strain Gradient

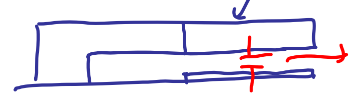


slope ≙ Strain Gradient = Γ

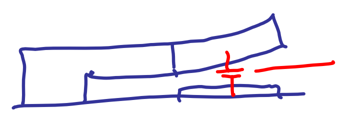
$$\Gamma = \frac{\epsilon_1}{(H/2)}$$

$$R = \frac{1}{2}\frac{E}{(1-\nu)}\frac{H}{\sigma_1} = \frac{H}{2}\frac{E'}{\sigma_1} = \frac{(H/2)}{\epsilon_1} = \frac{1}{\Gamma} \rightarrow \boxed{\Gamma = \frac{1}{R}}$$

proof mass

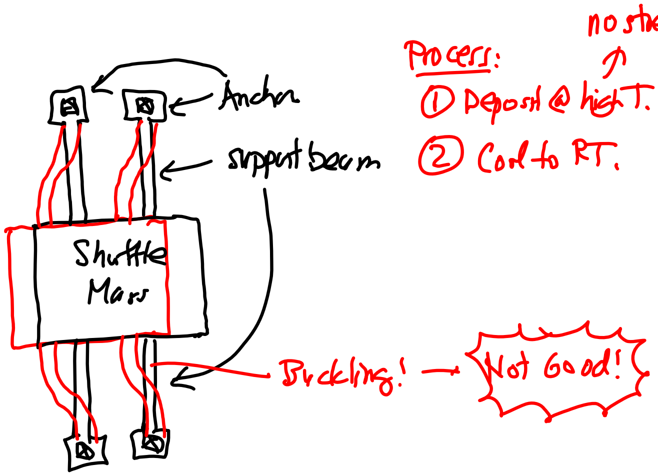


ΔC → acceleration



zero acceleration offset

Simple Method to Support a Structure

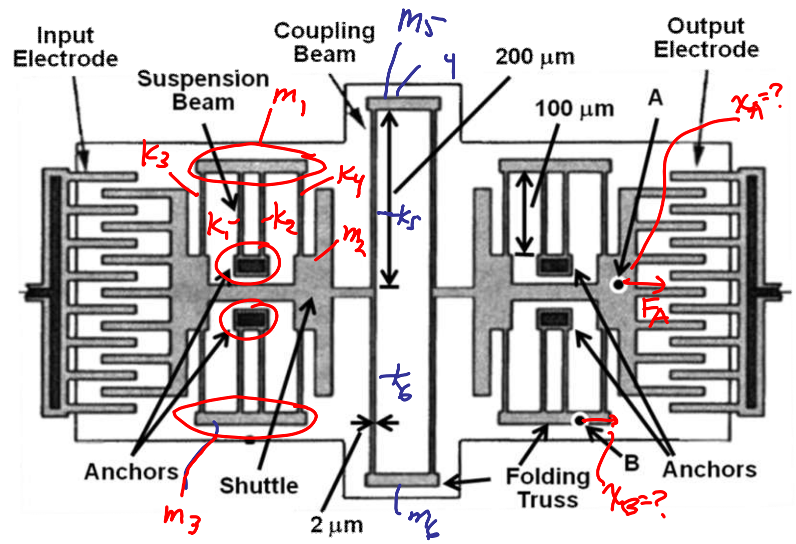


How to Defend Against this:

① Change process parameters to reduce stress (e.g., change deposition recipe)
↳ can't always do this

② Design → folded beam!

Analyzing an Interconnected Ensemble of Beams (Springs) & Masses



Typical Questions: → all demand that we know $x = f(F)$

① How does the structure move in response to a force at a specific location. ↳ = that we know

② What is the frequency response to an AC force applied at a specific location. ↳ the stiffness?

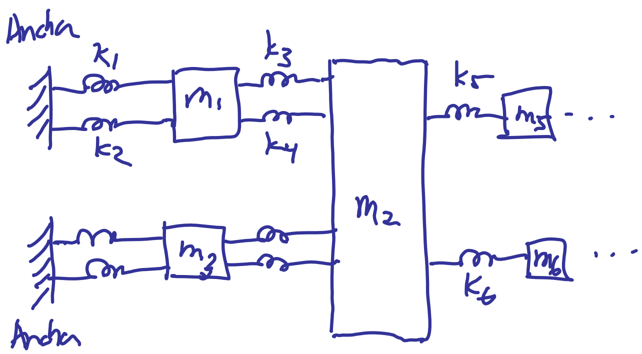
③ Noise?

④ Response to environmental stimuli? (e.g., rotation)

⑤ How does stress affect the behavior of the structure?

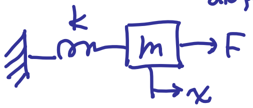
Procedure:

① Build the ckt. (Extract the ckt.) → in the x-direction (for this example)



② Analyze to get $x = f(F)$ ← force

displacement

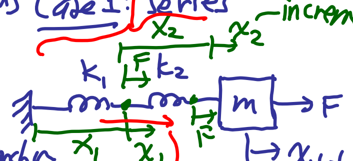


$\Rightarrow F = kx \rightarrow x = \frac{F}{k} = \left(\frac{1}{k}\right)F$

compliance

(a) Case 1: Series

series because one must go through both k_1 & k_2 to get from the anchor to the forcing pt.

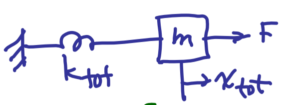


x_{tot} ← want this ∴ need k_{tot}

F → thru variable

across variable

$x_{tot} = \left(\frac{1}{k_{tot}}\right)F$



$x_1 = \frac{F}{k_1}$ $\quad x_2 = \frac{F}{k_2}$

$$\underbrace{x_{tot} = x_1 + x_2}_{\text{identifies series.}} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{F}{(k_1 \| k_2)} = \frac{F}{k_{tot}}$$

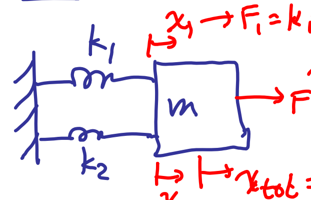
*

* $\left[\text{"}\|\text{" operator} \triangleq A\|B = \frac{1}{\frac{1}{A} + \frac{1}{B}} = \frac{AB}{A+B}\right]$

$\boxed{k_{tot} = k_1 \| k_2}$ (for k_1 & k_2 in series)

(b) Case 2: Parallel Springs

$x_1 \rightarrow F_1 = k_1 x_{tot}$



$F = k_{tot} x_{tot}$

want k_{tot}

$\underbrace{x_{tot} = x_1 = x_2}$

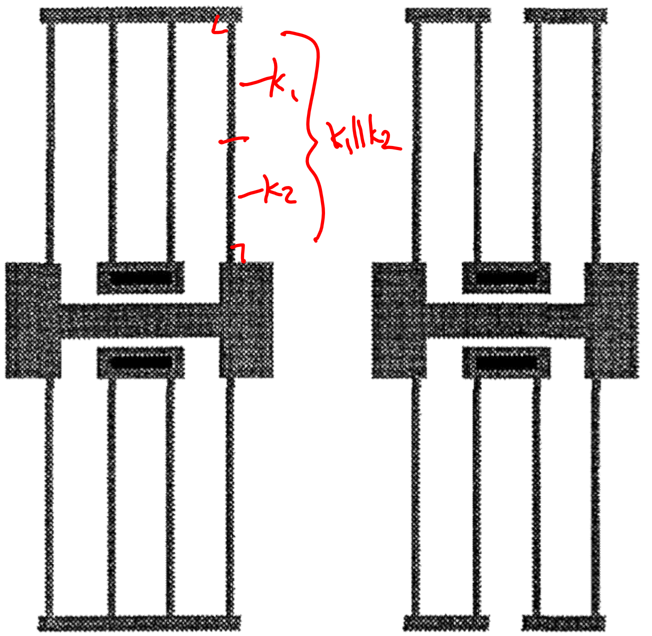
indicates parallel

or → only need to go through one of the springs to get from the anchor to the forcing pt.

$F_2 = k_2 x_{tot}$

$F = F_1 + F_2 = \underbrace{(k_1 + k_2)}_{k_{tot}} x_{tot}$

$\boxed{k_{tot} = k_1 + k_2}$ (for k_1 & k_2 in parallel)



(a) Inner fold, continuous truss

(b) Inner fold, discontinuous truss