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Brute Force Methods for Resonance Frequency Determination

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Basic Concept: Scaling Guitar Strings

Guitar String

Vib. Amplitude vs. Freq. (110 Hz)

Low Q vs. High Q

Vibrating "A" String (110 Hz)

Stiffness

Freq. Equation:

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_r}{m_r}}$$

Freq. Mass

μMechanical Resonator

Metallized Electrode
Anchor
Polysilicon Clamped-Clamped Beam

[Bannon 1996]

Performance:
 $L_r = 40.8 \mu\text{m}$
 $m_r \sim 10^{-13} \text{ kg}$
 $W_r = 8 \mu\text{m}, h_r = 2 \mu\text{m}$
 $d = 1000 \text{ \AA}, V_p = 5 \text{ V}$
 $\text{Press.} = 70 \text{ mTorr}$

$f_o = 8.5 \text{ MHz}$
 $Q_{vac} = 8,000$
 $Q_{air} \sim 50$

Transmission [dB] vs. Frequency [MHz]

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Anchor Losses

Fixed-Fixed Beam Resonator

Elastic Wave Radiation

Anchor Electrode Gap Anchor

Q = 300 at 70MHz

Problem: direct anchoring to the substrate ⇒ anchor radiation into the substrate ⇒ lower Q

Solution: support at motionless nodal points ⇒ isolate resonator from anchors ⇒ less energy loss ⇒ higher Q

Free-Free Beam Resonator

Supporting Beams
Anchor Free-Free Beam

Q = 15,000 at 92MHz

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92 MHz Free-Free Beam μResonator

- Free-free beam μmechanical resonator with non-intrusive supports ⇒ reduce anchor dissipation ⇒ higher Q

Drive Electrode Support Beams Flexural-Mode Beam (13.1 μm) Anchor

Ground Plane and Sense Electrode (10.4 μm)

Design/Performance:
 $L_r = 13.1 \mu\text{m}, W_r = 6 \mu\text{m}$
 $h = 2 \mu\text{m}, d = 1000 \text{ \AA}$
 $V_p = 28-76 \text{ V}, W_e = 2.8 \mu\text{m}$
 $f_o \sim 92.25 \text{ MHz}$
 $Q \sim 7,450 @ 10 \text{ mTorr}$

[Wang, Yu, Nguyen 1998]

Transmission [dB] vs. Frequency [MHz]

92.25 MHz
 $Q = 7,450$

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Higher Order Modes for Higher Freq.

2nd Mode Free-Free Beam

3rd Mode Free Free Beam

Distinct Mode Shapes

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Flexural-Mode Beam Wave Equation

Derive the wave equation for transverse vibration:
Dynamic Equilibrium Condition for Forces in the y-direction:

$$F - (F + \frac{\partial F}{\partial x} dx) - \rho A dx \frac{\partial^2 u}{\partial t^2} = 0 \quad (1)$$

and the moment equilibrium condition: $-F dx + \frac{\partial M}{\partial x} dx \approx 0 \quad (2)$

Combining (1) & (2):

$$\frac{\partial^2 M}{\partial x^2} dx = -\rho A dx \frac{\partial^2 u}{\partial t^2} \rightarrow \frac{\partial^2}{\partial x^2} (-EI \frac{\partial^2 u}{\partial x^2}) = -\rho A \frac{\partial^2 u}{\partial t^2} \rightarrow \frac{\partial^4 u}{\partial x^4} = \left(\frac{EI}{\rho A} \right) \frac{\partial^4 u}{\partial t^4}$$

$I_y = \frac{Wh^3}{12}$

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Example: Free-Free Beam

- Determine the resonance frequency of the beam
- Specify the lumped parameter mechanical equivalent circuit
- Transform to a lumped parameter electrical equivalent circuit
- Start with the flexural-mode beam equation:

$$\frac{\partial^2 u}{\partial t^2} = \left(\frac{EI}{\rho A} \right) \frac{\partial^4 u}{\partial x^4}$$

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Free-Free Beam Frequency

- Substitute $u = u_1 e^{j\omega t}$ into the wave equation:

$$\frac{\partial^4 u}{\partial x^4} = \left(\omega^2 \frac{\rho A}{EI} \right) u \quad (1)$$

- This is a 4th order differential equation with solution:

$$u(x) = A \cosh kx + B \sinh kx + C \cos kx + D \sin kx \quad (2)$$

Give the mode shape during resonance vibration.

Boundary Conditions:

At $x = 0$	At $x = l$	
$\frac{\partial^2 u}{\partial x^2} = 0$	$\frac{\partial^2 u}{\partial x^2} = 0$	$M = 0$ (Bending moment)
$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial M}{\partial x} = 0$ (Shearing force)

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Free-Free Beam Frequency (cont)

- Applying B.C.'s, get $A=C$ and $B=D$, and

$$\begin{bmatrix} (\cosh kl - \cos kl) & (\sinh kl - \sin kl) \\ (\sinh kl + \sin kl) & (\cosh kl - \cos kl) \end{bmatrix} \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} = 0 \quad (3)$$
- Setting the determinant = 0 yields

$$\cos kl = \frac{1}{\cosh kl}$$
- Which has roots at

$$k_1 l = 4.730 \quad k_2 l = 7.853 \quad k_3 l = 10.996$$

These values of kl correspond to the different modes of vibration!
- Substituting (2) into (1) finally yields:

$$k^4 = \frac{\rho A}{EI} \omega^2 \rightarrow f_n = \frac{(k_n l)^2}{2\pi l^2} \sqrt{\frac{EI}{\rho A}} \quad \left[\text{Free-Free Beam Frequency Equation} \right]$$

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Higher Order Free-Free Beam Modes

Mode	n	Nodal Points	$k_n l$	f_n/f_1
Fundamental (f_1)	1	2	4.730	1.000
1st Harmonic	2	3	7.853	2.757
2nd Harmonic	3	4	10.996	5.404
3rd Harmonic	4	5	14.137	8.932
4th Harmonic	5	6	17.279	13.344

← More than 10x increase

X_1 Fundamental Mode ($n=1$)
(a)

X_2 1st Harmonic ($n=2$)
(b)

X_3 2nd Harmonic ($n=3$)

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Mode Shape Expression

- The mode shape expression can be obtained by using the fact that $A=C$ and $B=D$ into (2), yielding

$$u_x = \mathcal{A} \left[\left(\frac{\mathcal{A}}{\mathcal{B}} \right) (\cosh kx + \cos kx) + (\sinh kx + \sin kx) \right]$$
- Get the amplitude ratio by expanding (3) [the matrix] and solving, which yields

$$\frac{\mathcal{A}}{\mathcal{B}} = \frac{\sin kl - \sinh kl}{\cosh kl - \cos kl}$$
- Then just substitute the roots for each mode to get the expression for mode shape

X_1 Fundamental Mode ($n=1$)
[Substitute $k_1 l = 4.730$]

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