





Equivalent Dynamic Mass

• We know the mode shape, so we can write expressions for displacement and velocity at resonance

Displacement: $u(x) = B \left[S(\cosh kx + \cos kx) + (\sinh kx + \sin kx) \right]$, $S = \frac{A}{B}$

$[V(x) = \omega u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2} [V(x)]^2} = \frac{\frac{1}{2} \rho A \int_0^l \omega^2 [u(x')]^2 dx'}{\frac{1}{2} \omega^2 [u(x)]^2}$

$$M_{eq}(x) = \frac{\rho A \int_0^l B^2 [S(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')]^2 dx'}{B^2 [S(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}$$

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Equivalent Dynamic Stiffness & Damping

• Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_o = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_o^2 M_{eq}(x)$$

• And damping also follows readily

$$Q = \frac{\omega_o M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$

↑ damping

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Equivalent Lumped Mechanical Circuit

$K_{eq}(x) = \omega_o^2 M_{eq}(x)$

$M_{eq}(x) = \frac{\rho A \int_0^l [u(x')]^2 dx'}{[u(x)]^2}$

$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$

The diagram shows a mechanical circuit with a spring $K_{eq}(x)$, a damper $C_{eq}(x)$, and a mass $M_{eq}(x)$ connected in parallel to a fixed support.

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Equivalent Lumped Mechanical Circuit

Example: Polysilicon w/ $l=14.9\mu\text{m}$, $W=6\mu\text{m}$, $h=2\mu\text{m} \rightarrow 70\text{ MHz}$

$K_{eq}(0) = 19,927\text{ N/m}$
 $M_{eq}(0) = 1.03 \times 10^{-13}\text{ kg}$
 $C_{eq}(0) = 5.66 \times 10^{-9}\text{ kg/s}$

$K_{eq}(l/2) = 53,938\text{ N/m}$
 $M_{eq}(l/2) = 2.78 \times 10^{-13}\text{ kg}$
 $C_{eq}(l/2) = 1.53 \times 10^{-8}\text{ kg/s}$

$K_{eq}(\text{node}) = \infty$
 $M_{eq}(\text{node}) = \infty$
 $C_{eq}(\text{node}) = \infty$

The diagram shows two mechanical circuits. The first is at $x=0$ and the second is at $x=l/2$. Each circuit consists of a spring, a damper, and a mass connected in parallel to a fixed support.

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