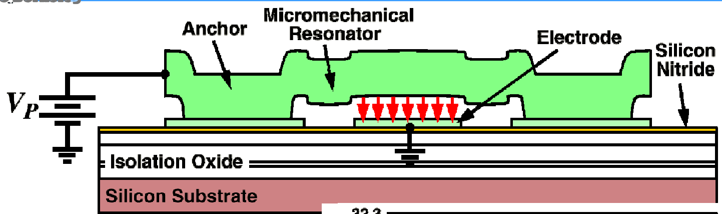
Voltage-Controllable Center Frequency



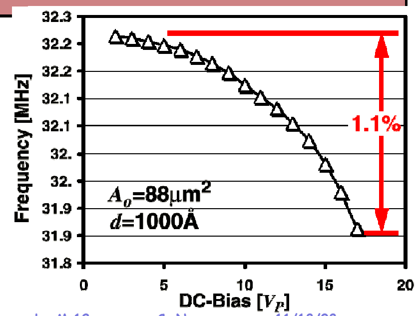
- Quadrature force ➠ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

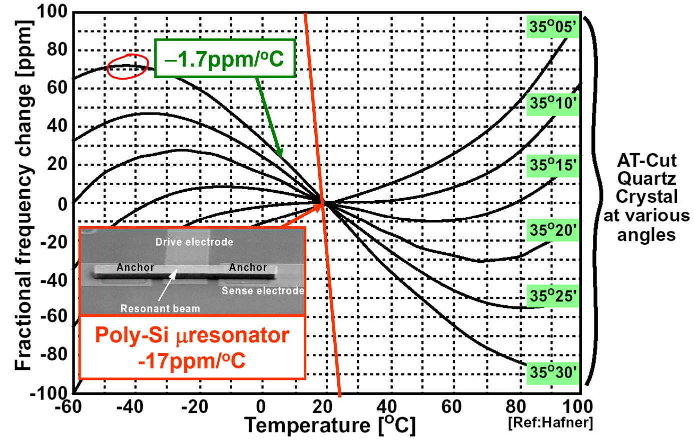
Electrode Overlap Area

Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



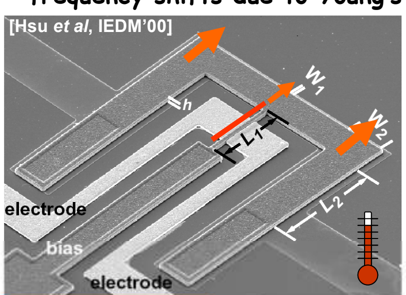
Microresonator Thermal Stability

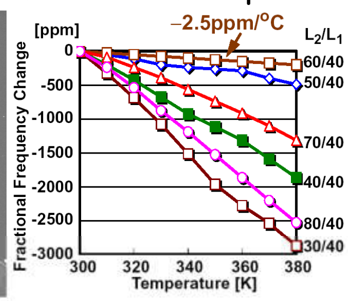


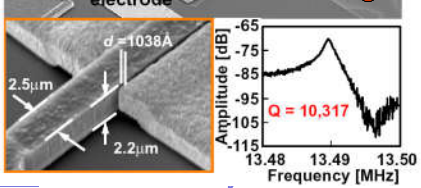
- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

Geometric-Stress Compensation

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.



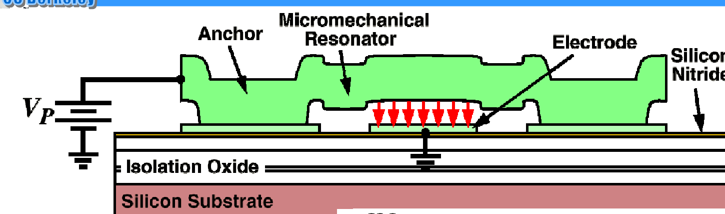




- *Problems*:
  - stress relaxation
  - compromised design flexibility

[Hsu et al IEDM 2000]

Voltage-Controllable Center Frequency



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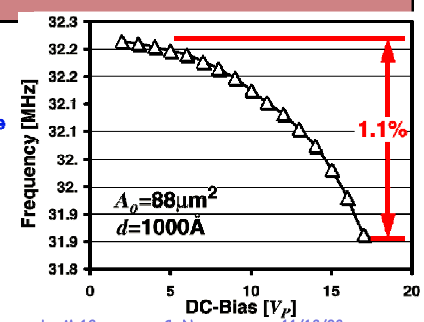
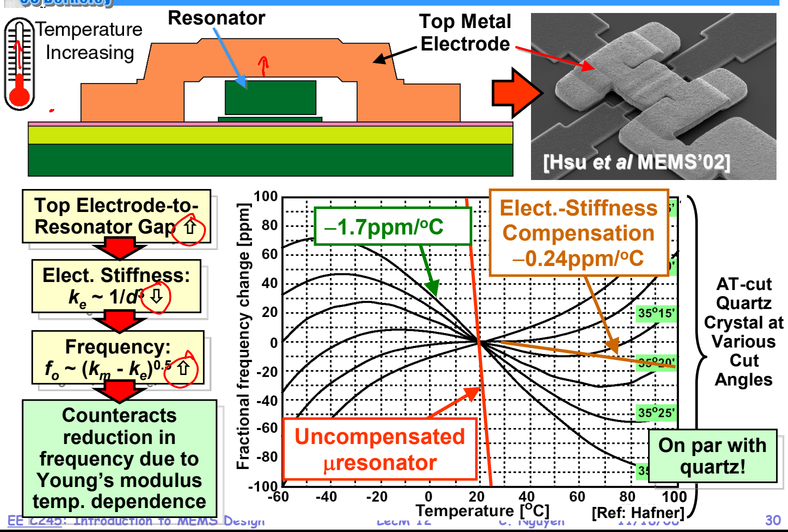
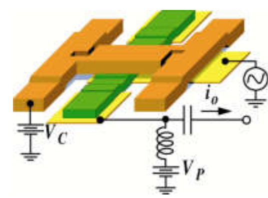


Excellent Temperature Stability

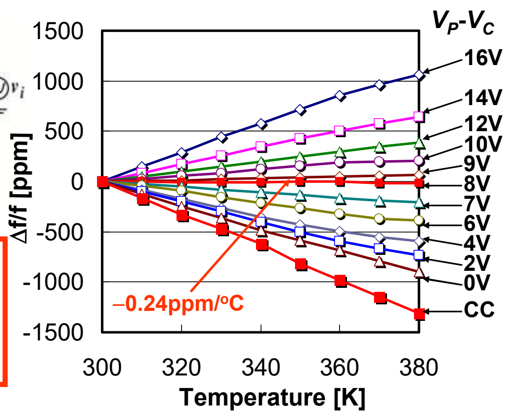


Measured Δf/f vs. T for k_e-Compensated μResonators



Design/Performance:
f_o =10MHz, Q=4,000
V_P=8V, h_e=4μm
d_o=1000Å, h=2μm
W_r=8μm, L_r=40μm

[Hsu et al MEMS'02]



- Slits help to release the stress generated by lateral thermal expansion → linear TC_f curves → -0.24ppm/°C!!!

Can One Cancel k_e w/ Two Electrodes?

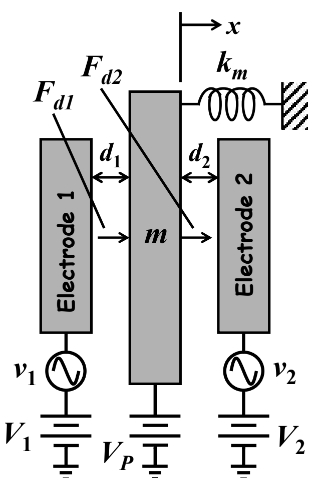
- What if we don't like the dependence of frequency on V_P?
- Can we cancel k_e via a differential input electrode configuration?
- If we do a similar analysis for F_{d2} at Electrode 2:

Subtracts from the F_{d1} term, as expected

$$F_{d2}\Big|_{\omega_o} = -V_{P2}\frac{C_{o2}}{d_2}|v_2|\cos\omega_o t$$

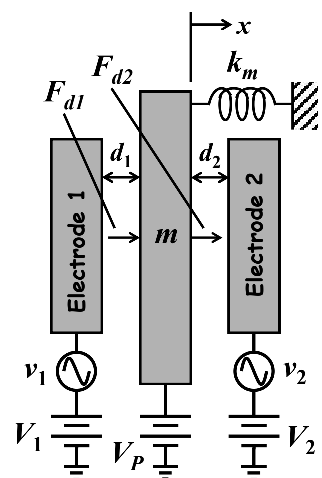
$$+V_{P2}^2\frac{C_{o2}}{d_2^2}|x|\sin\omega_o t$$

Adds to the quadrature term → k_e's add, no matter the electrode configuration!



Problems With Parallel-Plate C Drive

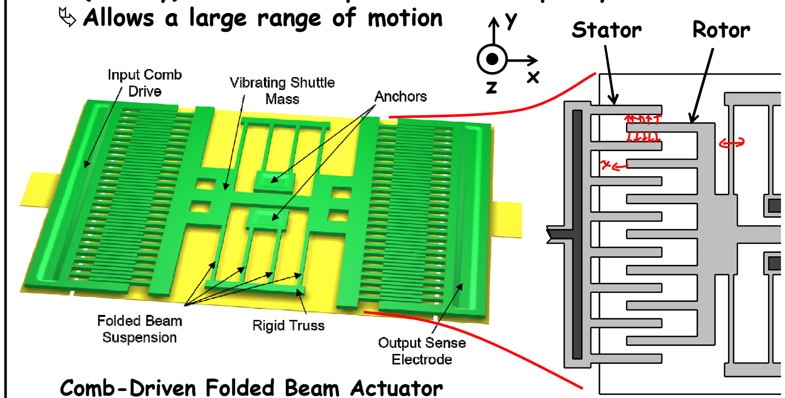
- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_o)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap … but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)



Electrostatic Comb Drive

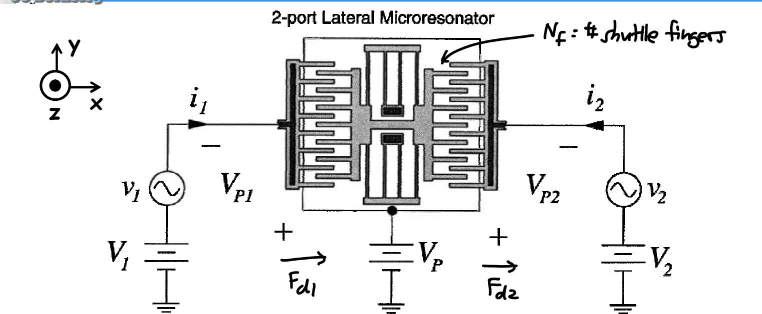
Electrostatic Comb Drive

- Use of comb-capacitive tranducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion



Comb-Driven Folded Beam Actuator

Typical Drive & Sense Configuration



Simple Analysis:

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(v_i - V_{P1})^2 = \frac{1}{2}\left(-\frac{\epsilon_o h}{d_1}\right)(v_i^2 - 2V_{P1}v_i + V_{P1}^2)(2N_f)$$

$$F_{d2} = \frac{1}{2}\frac{\partial C_2}{\partial x}(v_2 - V_{P2})^2 = \frac{1}{2}\left(\frac{\epsilon_o h}{d_2}\right)(v_2^2 - 2V_{P2}v_2 + V_{P2}^2)(2N_f)$$

$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2}\left(\frac{\epsilon_o h}{d}\right)\left(v_2^2 - v_i^2 - 2(V_{P2}v_2 - V_{P1}v_i) + V_{P2}^2 - V_{P1}^2\right)(2N_f)$$

For $V_1 = V_2$, $v_i = -v_2$

$$F_{net} = 2(2N_f)\left(\frac{\epsilon_o h}{d}\right)V_{P1}v_i$$

Comb-Drive Force Equation (2nd Pass)

- In our 1st pass, we accounted for
  - Parallel-plate capacitance between stator and rotor
- … but neglected:
  - Fringing fields
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!