Lecture 21: Electrical Stiffness & Comb Drive

- Announcements:
- Module 12 on Capacitive Transducers online
- HW#6 online and due Tuesday, April 17
- Project Slide Set #1 due Friday, April 13
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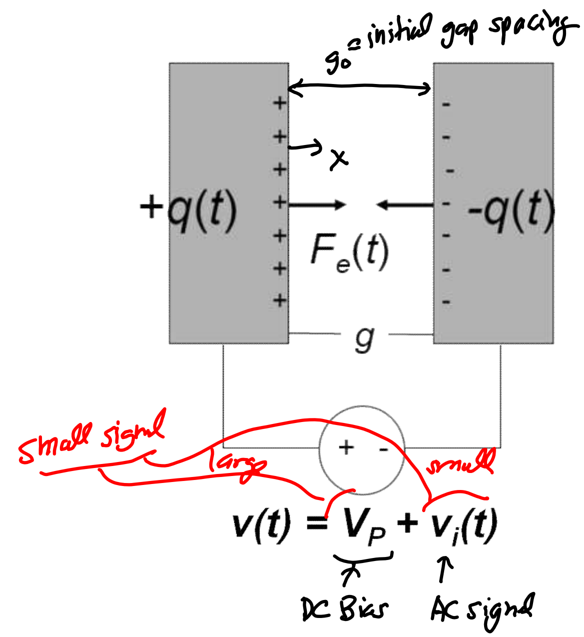
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- -----------------------------------
- Last Time:
- Linearizing capacitive transducers
- Continue with this

over

Linearizing the Voltage-to-Force Transfer Function



$$F_e(t) = \frac{\partial W'}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{2}C\,[v(t)]^2\right]$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}[v(t)]^2 = \frac{1}{2}\frac{\partial C}{\partial x}\left[V_P + v_i(t)\right]^2$$

$$= \frac{1}{2}\left[V_P^2 + 2V_P v_i(t) + \cancel{[v_i(t)]^2}\right]\frac{\partial C}{\partial x}$$

$$[V_P >> v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2}V_P^2\frac{\partial C}{\partial x}}_{\text{DC offset}} + \underbrace{V_P\frac{\partial C}{\partial x}v_i(t)}_{\text{AC Drive Signal}} \longrightarrow *$$

$C_0 = \frac{\epsilon A}{g_0}$ ← plate overlap area

static

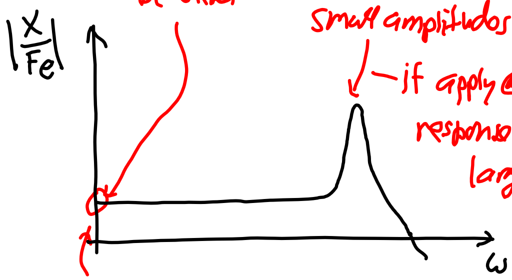
$C(x) = \frac{\epsilon A}{g_0 - x} = C_0\left(1 - \frac{x}{g_0}\right)^{-1}$

$(x \ll g_0) \Rightarrow \approx C_0\left(1 + \frac{x}{g_0}\right)$

$\therefore \frac{\partial C}{\partial x} = \frac{C_0}{g_0} = \frac{\epsilon A}{g_0^2}$ ← gain is a strong fcn of g_0!

* $F_e(t) = \underbrace{\frac{1}{2}\frac{C_0}{g_0}V_P^2}_{\text{DC offset}} + \underbrace{V_P\frac{C_0}{g_0}}_{\sim\text{constant for small amplitudes}} v_i(t)$

} $v_i \to F_e$ gain term



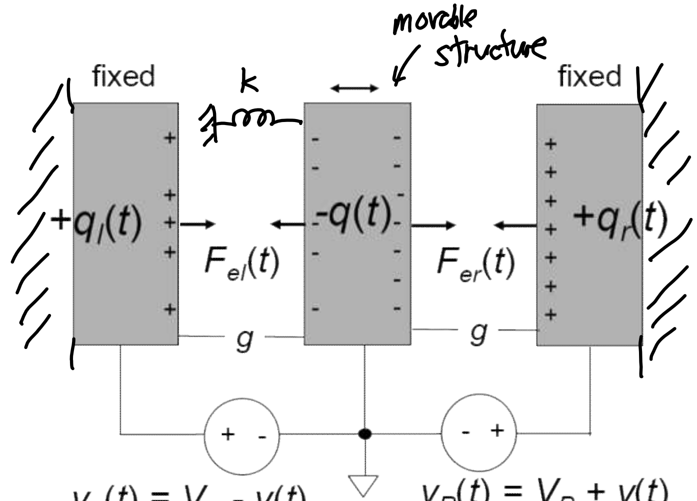
– if apply @ resonance, response can still be large!

very small response

↳ But must still worry about pullin, V_{PI}

as long as $v_i(t)$ is small, the response is fairly linear!

Cancel the DC Offset Via Differential Symmetry



$v_L(t) = V_P - v(t)$ $\quad$ $v_R(t) = V_P + v(t)$

signal voltages are opposite polarity → differential

$F_{net}(t) = F_{er}(t) - F_{el}(t)$

$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{[v_R(t)]^2 - [v_L(t)]^2\right\}$

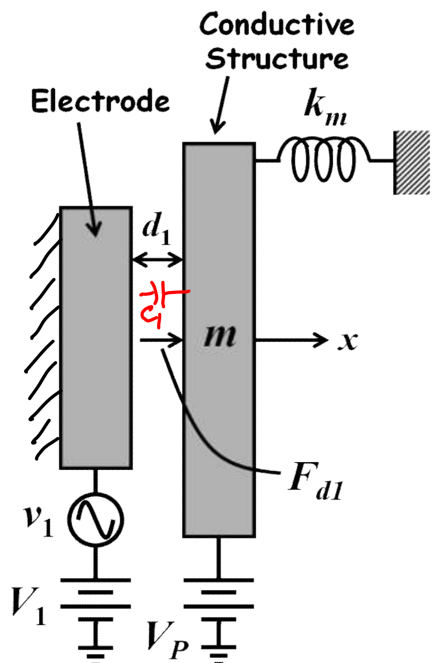
$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{\cancel{V_P^2} + 2V_P v(t) + \cancel{[v(t)]^2} - \left(\cancel{V_P^2} - 2V_P v(t) + \cancel{[v(t)]^2}\right)\right\}$

$\therefore \boxed{F_{net}(t) = 2V_P\frac{\partial C}{\partial x}v(t) = 2V_P\frac{C_0}{g_0}v(t)}$

no dc term ∴ less concern for pull-in → but, still concerning, since can't be perfectly balanced ← in the real world

Linear w/ $v(t)$!

Non-Linearity Still Effects Us → transducer nonlinearity



More Complete Force Expression

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_{01}\left(1+\frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1+\frac{x}{d_1}\right)^{-2}$$

[Expand into Taylor series]

$$\frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots\right)$$

where $A_1 = -\frac{2}{d_1}$, $A_2 = \frac{3}{d_1^2}$, $A_3 = -\frac{4}{d_1^3}$, ...

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(V_P - V_1 - v_1)^2 = \frac{1}{2}\frac{\partial C}{\partial x}(V_{P1} - v_1)^2$$

[small displacement: $x \ll d_1$] $V_{P1} = V_P - V_1$

$$F_{d1} = \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)(1 + A_1 x)\left(V_{P1}^2 - 2V_{P1}v_1 + v_1^2\right)$$

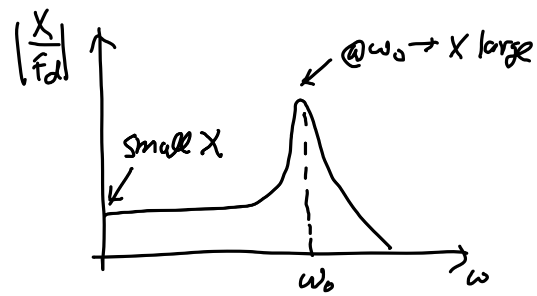
$$= \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)\left\{V_{P1}^2 - 2V_{P1}v_1 + v_1^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x v_1 + A_1 x v_1^2\right\}$$

$\frac{1}{2}(1 + \cos 2\omega_o t)$ not @ resonance

$\cos^2\omega_o t$

ignore this one since it's small!

Resonance:



@ resonance:

$$x = \frac{Q F_{d1}}{jk} = \frac{Q}{jk}\frac{\partial C}{\partial x}V_{P1}v_1$$

90° phase shift

$$v_1 = |v_1|\cos\omega_o t \longrightarrow x = |x|\sin\omega_o t$$

90° phase shift

Focus on force terms @ ω_o

$k_e \rightarrow$ electrical stiffness

$$F_{d1}\Big|_{\omega_o} = V_{P1}\frac{C_{o1}}{d_1}|v_i|\cos\omega_o t + V_{P1}^2\frac{C_{o1}}{d_1^2}|x|\sin\omega_o t$$

drive force term

proportional to x

90° phase-shifted fl.

in phase w/ displacement!

∴ it's a stiffness!

Electrical Stiffness:

① A negative spring constant!

② Derives from V_P:

$$\boxed{k_e = V_{P1}^2\frac{C_{o1}}{d_1^2} = V_{P1}^2\frac{\epsilon A}{d_1^3}}$$

overlap area of C

DC-Bias

3rd power dependence on gap!

$k_e \rightarrow$ can effect resonance freq., f_o!

$\omega_o \triangleq$ radian resonance freq. w/ no V_p applied (i.e., $V_p = 0V$)

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$

mechanical stiffness

electrical stiffness

$$\omega_o = \sqrt{\frac{k_m}{m}}$$ *

*

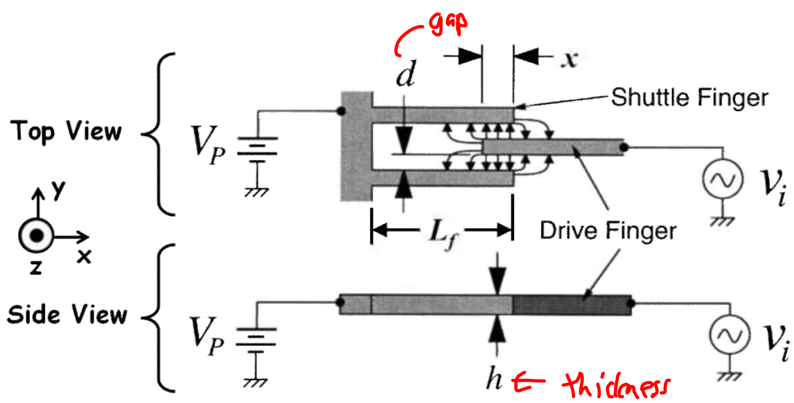
$$\omega_o' = \sqrt{\frac{k_m}{m}}\left(1 - \frac{k_e}{k_m}\right)^{1/2}$$

$$\boxed{\omega_o' = \omega_o\left[1 - \frac{V_{P1}^2}{k_m}\frac{\epsilon A}{d_1^3}\right]^{1/2}}$$

now a fcn of DC Bias!

(voltage-controllable!)

Electrostatic Comb-Drive



$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2$$

$$\left[C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d} \right]$$ ← not a fcn of x! (ideally)

$$F_d = \cancel{\frac{1}{2}}\,\frac{\cancel{2}\epsilon_0 h}{d}\left(\cancel{V_P^2} - 2V_P v_i + \cancel{v_i^2}\right)$$

can balance out via symmetrically placed electrodes

$v_i \ll V_P$

$$\boxed{F_d = -2V_P\frac{\epsilon_0 h}{d}v_i}$$

linear

∴ no electrical stiffness (no k_e!)