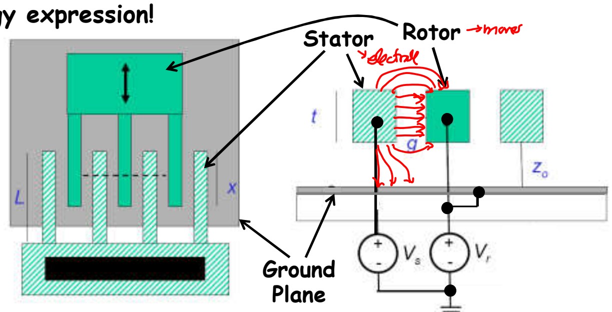
Comb-Drive Force Equation (2nd Pass)

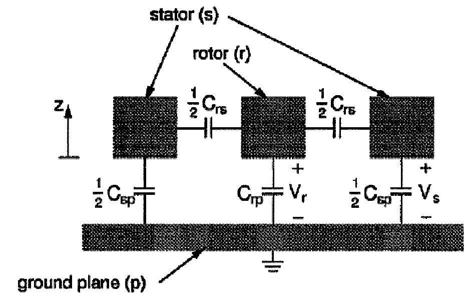
- In our 1st pass, we accounted for
  - Parallel-plate capacitance between stator and rotor
- … but neglected:
  - Fringing fields
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!



Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

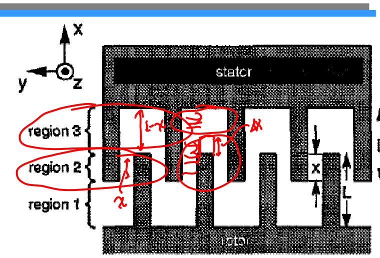
$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$



[Gary Fedder, Ph.D., UC Berkeley, 1994]

Capacitance Expressions

- Case: $V_r = V_p = 0V$
- C_{sp} depends on whether or not fingers are engaged

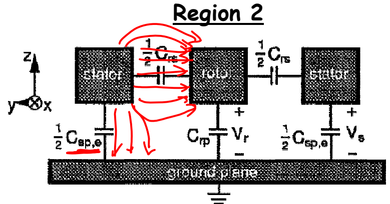


$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

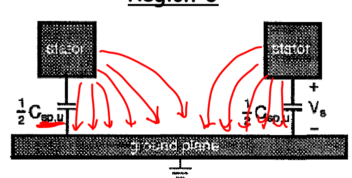
$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length

Region 2



Region 3



[Gary Fedder, Ph.D., UC Berkeley, 1994]

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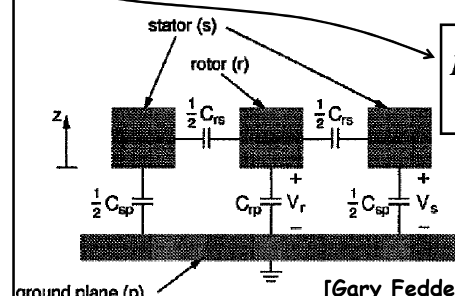
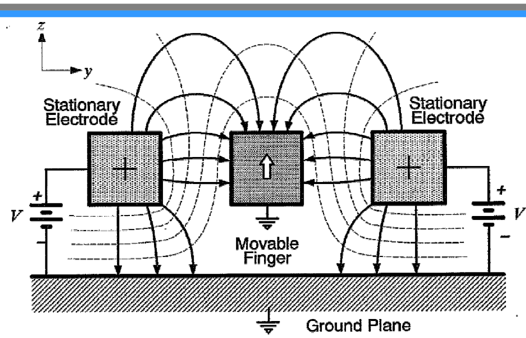


$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$

(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]

Vertical Force (Levitation)

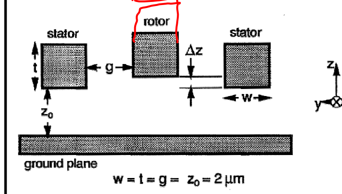


$$F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2}\frac{dC_{sp}}{dz}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dz}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dz}(V_s - V_r)^2$$

- For V_r = 0V (as shown): $F_{e,z} = \frac{1}{2}Nx\left[\frac{d(C'_{sp,e} + C'_{rs})}{dz}\right]V_s^2$

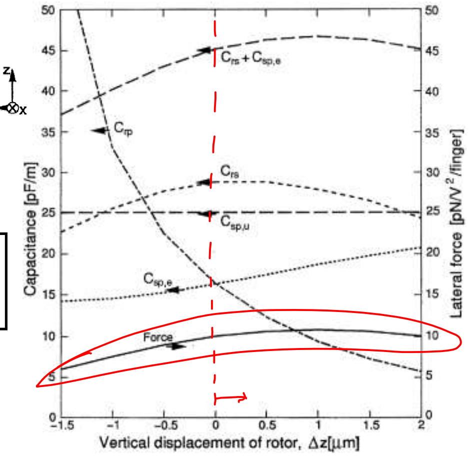
Simulate to Get Capacitors → Force

- Below: 2D finite element simulation



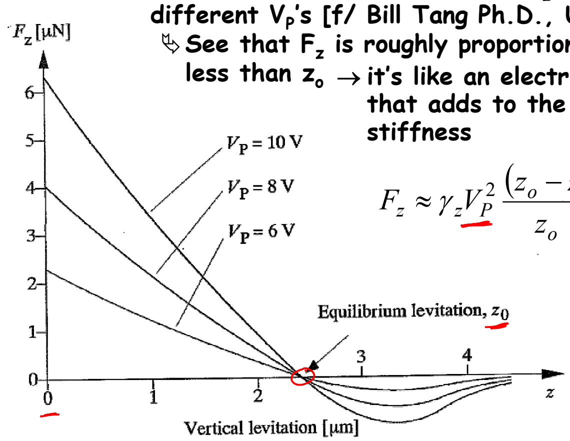
$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$

20-40% reduction of $F_{e,x}$



Simulated Levitation Force

- Below: simulated vertical force F_z vs. z at different V_P's [f/ Bill Tang Ph.D., UCB, 1990]
  - See that F_z is roughly proportional to –z for z less than z_o → it's like an electrical stiffness that adds to the mechanical stiffness



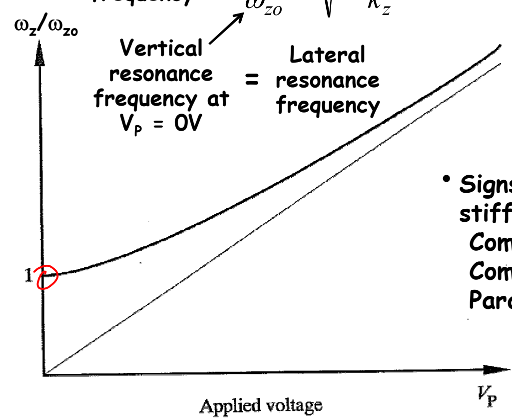
$$F_z \approx \gamma_z V_P^2 \frac{(z_o - z)}{z_o} = k_e(z_o - z)$$

Electrical Stiffness

Vertical Resonance Frequency

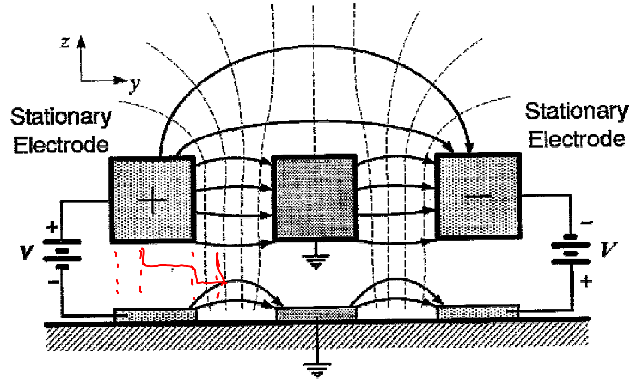
Vertical resonance frequency → $\frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}}$ where $k_e = \left(\frac{\gamma_z}{z_o}\right)V^2$

Vertical resonance frequency at V_P = 0V = Lateral resonance frequency



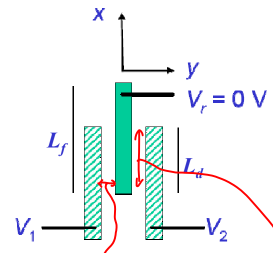
- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → k_e = 0
  - Comb (z-axis) → k_e > 0
  - Parallel Plate → k_e < 0

Suppressing Levitation



- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

Force of Comb-Drive vs. Parallel-Plate



Gap = d_o = 1 μm
Thickness = h = 2 μm
Finger Length = L_f = 100 μm
Finger Overlap = L_d = 75 μm

- Comb drive (x-direction)
  - $V_1 = V_2 = V_s = 1V$

$$F_{e,x} = \frac{1}{2}\frac{\varepsilon_o h}{d_o}V_s^2$$

- Differential Parallel-Plate (y-direction)
  - $V_1 = 0V,\ V_2 = 1V$

$$F_{e,y} = \frac{1}{2}\frac{\varepsilon_o h L_d}{d_o^2}V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{1}{2}\frac{\varepsilon_o h L_d}{d_o^2}V_2^2}{\frac{1}{2}\frac{\varepsilon_o h}{d_o}V_s^2} = \frac{L_d}{d_o}$$

Parallel-plate generates a much larger force; but at the cost of linearity