Lecture 22: Equivalent Circuits II

- **Announcements:**
- Module 13 on Equivalent Circuits II online
- HW#6 online and due Tuesday, April 17
- Project Slide Set #1 due Friday, April 13
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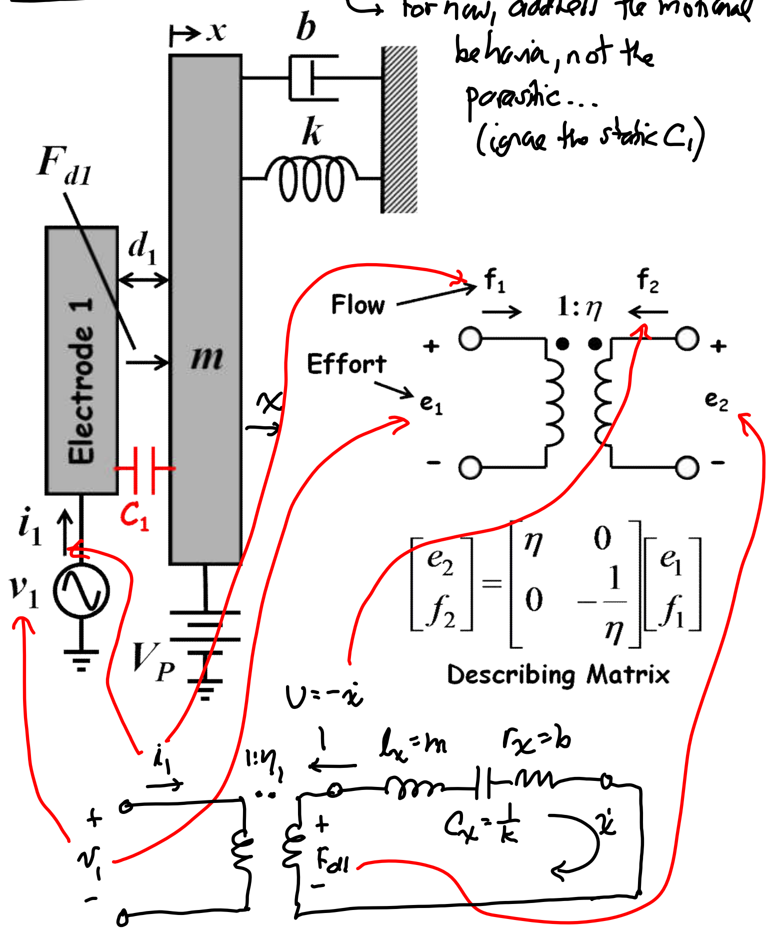
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- ------------------------------------
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- ------------------------------------
- Last Time: Finishing up Module 12 … continue …

Input Electrical Equiv. Ckt.

For now, address the motional behavior, not the parasitic... (ignore the static C_1)



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

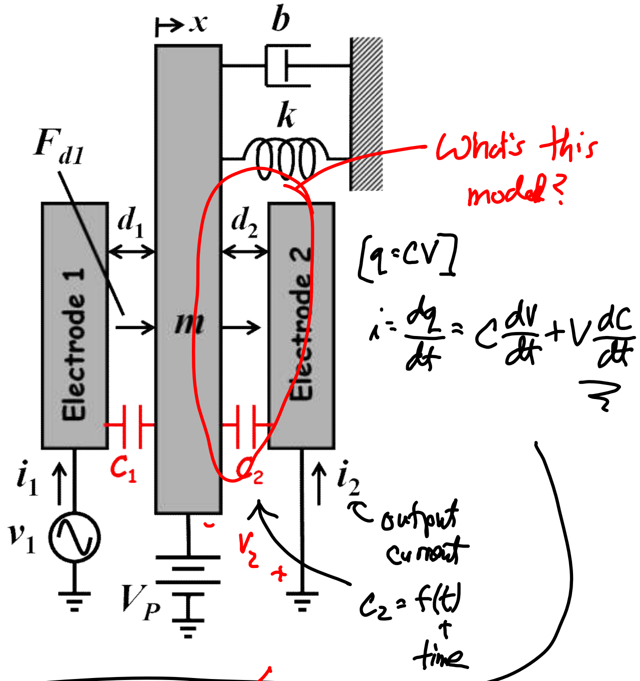
$$\underbrace{F_{d1}}_{e_2} = -\underbrace{V_P \frac{\partial C_1}{\partial x}}_{\eta_1} \underbrace{v_1}_{e_1}$$

$$\boxed{\eta_1 = \left|V_P \frac{\partial C_1}{\partial x}\right|}$$

$\frac{\epsilon_0 A}{d_0^2}$ → gap↓ → η_1↑↑

V_P↑ → η_1↑

Output Current Into Ground



$[q = CV]$

$$i = \frac{dq}{dt} = C\frac{dv}{dt} + V\frac{dC}{dt}$$

$$i_2 = C_2(x,t)\cancel{\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$$

$$\left[V_2(t) = -V_p \neq f(t)\right] \Rightarrow i_2 = -V_p\frac{dC_2}{dt} = -V_p\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

In phasor form: $I_2(j\omega) = -V_p\frac{\partial C_2}{\partial x}(j\omega X)$

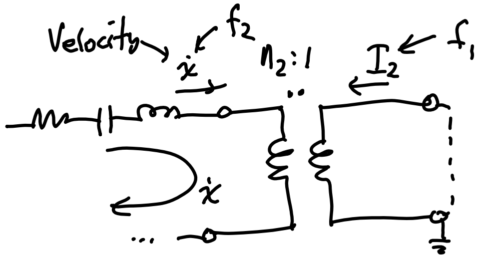
$$\boxed{I_2(j\omega) = -j\omega V_p\frac{\partial C_2}{\partial x}X}$$

output motional current

$$I_2(j\omega) = \underbrace{-j}_{\text{90° phase lag}}\omega V_p\underbrace{\frac{\partial C_2}{\partial x}}_{(+)}\underbrace{X}_{(+)} = -V_p\frac{\partial C}{\partial x}\dot{x}$$

velocity

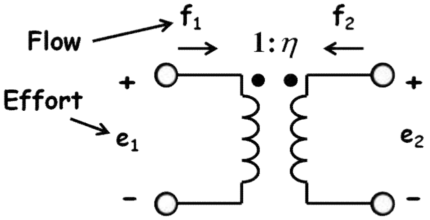
} $I_2 = (-)$ when $X = (+)$



$$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$$

$$[f_1 = I_2, f_2 = \dot{x}] \Rightarrow I_2 = -\eta_2\dot{x}$$

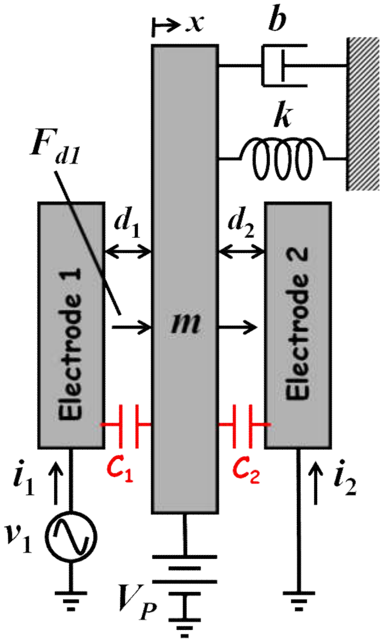
$$\therefore \boxed{\eta_2 = \left|V_p\frac{\partial C_2}{\partial x}\right|}$$



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

Input Current Expression ⇒ then yields complete equivalent input ckt



Get $I_1(j\omega)$!

$$i_1(t) = C_1(x,t)\frac{dv_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

$$\left[V_1(t) = v_1 - V_P\right] \Rightarrow i_1 = C_1\frac{dv_1}{dt} + (v_1 - V_P)\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

↑ f(t)

magnitude of $v_1 = |v_1|\cos\omega t$

$$\therefore\ I_1(j\omega) = \underbrace{j\omega C_1|v_1|}_{\text{Feedthrough Current}} + \cancel{j\omega|v_1|\frac{\partial C_1}{\partial x}X} - \underbrace{j\omega V_P\frac{\partial C_1}{\partial x}X}_{\text{Motional Current due to mass motion}} \longrightarrow \ast$$

@DC: $x = \frac{F_{d1}}{k} = -\frac{1}{k}V_P\frac{\partial C_1}{\partial x}v_1$

@ resonance: $x = \frac{QF_{d1}}{k} = -\frac{Q}{jk}V_P\frac{\partial C_1}{\partial x}v_1 = X$

↑ 90° phase lag

plug in to ★

Thus: (@ resonance) → ω_0

$$I_1(j\omega) = j\omega_0 C_1|v_1| + j\omega_0\Big(\overbrace{V_P\frac{\partial C_1}{\partial x}}^{\eta_{e1}}\Big)^2\frac{Q}{jk}|v_1|$$

$$= \underbrace{j\omega_0 C_1|v_1|}_{\text{90° phase-shifted from } v_1} + \underbrace{\omega_0\frac{Q}{k}\eta_{e1}^2|v_1|}_{\text{In phase w/ } v_1\text{!}}$$

↓ This is a capacitor in shunt w/ input!

↑ This is an effective resistance @ ω_0 seen looking into the electrode!